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Some Types of Soft Ordered Maps via Soft Pre open Sets

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Abstract: In this study, we define the concepts of soft x-pre continuous, soft x-pre open, soft x-pre closed and soft x-pre homeomorphism maps, where $x \in \{I, D, B\}$. These concepts are formulated depending on the increasing, decreasing and balancing soft pre open sets. We characterize these concepts and illustrate the relationships among them with the help of examples. Among the interesting obtained results are those associate concepts with their counterparts on topological ordered spaces.

Keywords: soft $I(D,B)$-pre continuous map, soft $I(D,B)$-pre open map, soft $I(D,B)$-pre homeomorphism map

1 Introduction

In 1965, Nachbin [1] presented the concept of topological ordered space which consists of a topological space (X, τ) equipped with a partial order relation $\preceq$. McCartan [2] in 1968, studied separation axioms via topological ordered spaces. Kumar [3] defined the concepts of continuous and homeomorphism maps via topological ordered spaces.

In 1999, Molotdsov [4] introduced the concept of soft sets for dealing with uncertainties and vagueness. Then Maji et al. [5] established the basis of the soft set theory by defining some operators like soft subset and equality relations and soft union and intersection of two soft sets. Shabir and Naz [6] initiated the idea of soft topological spaces and studied soft separation axioms. Later on, many researchers carried out various studies to present and discuss topological notions on soft topologies (see, for example [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], and [20]). Arockiarani and Lancy [21] presented a concept of soft pre-open sets and studied its main properties; and Akdag and Ozkan [22] carried out a detailed study on soft pre-separation axioms. [23] studied soft pre connected spaces and [24] investigated the properties of soft pre continuity.

In this work, we aim to give other applications of soft pre open sets by defining some soft ordered maps, namely soft x-pre continuous, soft x-pre open, soft x-pre closed and soft x-pre homeomorphism maps for $x \in \{I, D, B\}$. We describe each one of these maps and construct some examples to show the relationships among them. Also, we demonstrate a role of extended soft topologies on studying the interrelations between these soft maps and their counterparts of maps in topological ordered spaces.

2 Preliminaries

2.1 Soft sets

Let X, Y be the universe sets and A, C, E, F be the sets of parameters.

Definition 2.1.[4] A notation G_E is said to be a soft set over X if G is a map of a set of parameters E into 2^X and it is written as a set of ordered pairs $G_E = \{(e, G(e)) : e \in E \text{ and } G(e) \in 2^X\}$. The set of all soft sets over X with a set of parameters E is denoted by $S(X_E)$.

Definition 2.3.[5] A soft set G_E over X is called a null soft set, denoted by $\widetilde{\emptyset}$, if $G(e) = \emptyset$ for each $e \in E$; and it is called an absolute soft set, denoted by $\widetilde{X}$, if $G(e) = X$ for each $e \in E$.

Definition 2.2.[6] For $x \in X$ and a soft set G_E over X, we say that $x \in G_E$ if $x \in G(e)$ for each $e \in E$ and $x \notin G_E$ if $x \notin G(e)$ for some $e \in E$.

Definition 2.4.[25] The relative complement of a soft set G_E is denoted by G^c_E, where $G^c : E \to 2^X$ is a mapping

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defined by $G^c(e) = X \setminus G(e)$ for each $e \in E$.

Definition 2.5.[26] A soft set G_A is a soft subset of a soft set F_C, denoted by $G_A \widetilde{\subseteq} F_C$, if $A \subseteq C$ and for all $a \in A$, we have $G(a) \subseteq F(a)$.
The soft sets G_A and G_C are soft equal if each one of them is a soft subset of the other.

Definition 2.6.[5] The union of two soft sets G_A and F_C over X, denoted by $G_A \widetilde{\bigcup} F_C$, is a soft set H_D, where $D = A \bigcup C$ and a mapping $H : D \to 2^X$ is given as follows:

$$H(d) = \begin{cases} G(d) & : \quad d \in A - C \\ F(d) & : \quad d \in C - A \\ G(d) \bigcup F(d) & : \quad d \in A \bigcap C \end{cases}$$

Definition 2.7.[25] The intersection of two soft sets G_A and F_C over X, denoted by $G_A \widetilde{\bigcap} F_C$, is a soft set H_D, where $D = A \bigcap C \neq \emptyset$, and a mapping $H : D \to 2^X$ is given by $H(d) = G(d) \bigcap F(d)$.

Definition 2.8.[27] Consider $f : X \to Y$ and $\phi : A \to C$ are two maps and let $f_\phi : S(X_A) \to S(Y_C)$ be a soft map. Let G_K and H_L be soft subsets of $S(X_A)$ and $S(Y_C)$, respectively. Then

(i)$f_\phi(G_K) = (f_\phi(G))_C$ is a soft subset of $S(Y_C)$ such that

$$f_\phi(G)(c) = \begin{cases} \bigcup\limits_{a \in \phi^{-1}(c) \bigcap K} f(G(a)) & : \quad \phi^{-1}(c) \bigcap K \neq \emptyset \\ \emptyset & : \quad \phi^{-1}(c) \bigcap K = \emptyset \end{cases}$$

for each $c \in C$.

(ii)$f_\phi^{-1}(H_L) = (f_\phi^{-1}(H))_A$ is a soft subset of $S(X_A)$ such that

$$f_\phi^{-1}(H)(a) = \begin{cases} f^{-1}(H(\phi(a))) & : \quad \phi(a) \in L \\ \emptyset & : \quad \phi(a) \notin L \end{cases}$$

for each $a \in A$.

Remark 2.9. Henceforth, a soft map $f_\phi : S(X_A) \to S(Y_C)$ implies that a map f of the universe set X into the universe set Y and a map ϕ of the set of parameters A into the set of parameters C.

Definition 2.10.[27] A soft map $f_\phi : S(X_A) \to S(Y_C)$ is said to be injective (resp. surjective, bijective) if f and ϕ are injective (resp. surjective, bijective).

Definition 2.11. [28] Consider a soft map $f_\phi : S(X_A) \to S(Y_C)$ and let G_A and H_C be two soft subsets of $S(X_A)$ and $S(Y_C)$, respectively. Then we have the following results:

(i)$G_A \widetilde{\subseteq} f_\phi^{-1} f_\phi(G_A)$ and the equality relation holds if f_ϕ is injective.

(ii)$f_\phi f_\phi^{-1}(H_C) \widetilde{\subseteq} H_C$ and the equality relation holds if f_ϕ is surjective.

Definition 2.12.([28], [29]) A soft set P_E over X is called soft point if there exists $e \in E$ and there exists $x \in X$ such that $P(e) = \{x\}$ and $P(a) = \emptyset$ for each $a \in E \setminus \{e\}$. A soft point will be shortly denoted by P_e^x and we say that $P_e^x \in G_E$ if $x \in G(e)$.

Definition 2.13.[30] Let $\preceq$ be a partial order relation on a non-empty set X and let E be a set of parameters. A triple $(X, E, \preceq)$ is said to be a partially ordered soft set.

Definition 2.14.[30] We define an increasing soft operator $i : (S(X_E), \preceq) \to (S(X_E), \preceq)$ and a decreasing soft operator $d : (S(X_E), \preceq) \to (S(X_E), \preceq)$ as follows: For each soft subset G_E of $S(X_E)$

(i)$i(G_E) = (iG)_E$, where iG is a mapping of E into X given by $iG(e) = i(G(e)) = \{x \in X : y \preceq x$ for some $y \in G(e)\}$.

(ii)$d(G_E) = (dG)_E$, where dG is a mapping of E into X given by $dG(e) = d(G(e)) = \{x \in X : x \preceq y$ for some $y \in G(e)\}$.

Definition 2.15.[30] A soft subset G_E of a partially ordered soft set $(X, E, \preceq)$ is said to be increasing (resp. decreasing) if $G_E = i(G_E)$(resp. $G_E = d(G_E)$).

Theorem 2.16.[30] If a soft map $f_\phi : (S(X_A), \preceq_1) \to (S(Y_C), \preceq_2)$ is increasing, then the inverse image of each increasing (resp. decreasing) soft subset of $\widetilde{Y}$ is an increasing (resp. a decreasing) soft subset of $\widetilde{X}$.

2.2 Soft topologies

Definition 2.17.[6] A soft topology on a non-empty set X is a collection τ of soft sets over X under a fixed parameters set E such that τ contains absolute and null soft sets and it is closed under finite soft intersection and arbitrary soft union.
A triple (X, τ, E) is called a soft topological space. Every member of τ is called soft open and its relative complement is called soft closed.

Proposition 2.18.[6] Let (X, τ, E) be a soft topological space. Then $\tau_e = \{G(e) : G_E \in \tau\}$ defines a topology on X for each $e \in E$. Henceforth, τ_e is called a parametric topology.

Definition 2.19.[28] Consider a soft topological space (X, τ, E) and let τ_e be a parametric topology on X. Then $\tau^\star = \{G_E : G(e) \in \tau_e$ for each $e \in E\}$ is a soft topology on X finer than τ.
In this work, we term $\tau^\star$ an extended soft topology.

Definition 2.20.[21] A soft subset H_E of (X, τ, E) is said to be soft pre open if $H_E \widetilde{\subseteq} int(cl(H_E))$. Its relative complement is said to be soft pre closed.

Definition 2.21.([6] [21]) For a soft subset H_E of (X,τ,E), we define the following four operators:

(i)$int(H_E)$ (resp. $int_p(H_E)$) is the largest soft open (resp. soft pre open) set contained in H_E.
(ii)$cl(H_E)$ (resp. $cl_p(H_E)$) is the smallest soft closed (resp. soft pre closed) set containing H_E.

Definition 2.22.[24] A soft map $f_\phi : (X,\tau,A) \to (Y,\theta,C)$ is said to be:

(i)Soft pre continuous if the inverse image of each soft open subset of (Y,θ,C) is a soft pre open subset of (X,τ,A).
(ii)Soft pre open (resp. soft pre closed) if the image of each soft open (resp. soft closed) subset of (X,τ,A) is a soft pre open (resp. soft pre closed) subset of (Y,θ,C).
(iii)Soft pre homeomorphism if it is bijective, soft pre continuous and soft pre open.

Definition 2.23.[30] A quadrable system $(X,\tau,E,\preceq)$ is said to be a soft topological ordered space, where (X,τ,E) is a soft topological space and $(X,E,\preceq)$ is a partially ordered soft set. Henceforth, the two notations $(X,\tau,E,\preceq_1)$ and $(Y,\theta,F,\preceq_2)$ stand for soft topological ordered spaces.

Definition 2.24.[31] The composition of two soft maps $f_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ and $g_\lambda : (Y,\theta,F,\preceq_2) \to (Z,\upsilon,K,\preceq_3)$ is a soft map $f_\phi \circ g_\lambda : (X,\tau,E,\preceq_1) \to (Z,\upsilon,K,\preceq_3)$ and is given by $(f_\phi \circ g_\lambda)(P_e^x) = f_\phi(g_\lambda(P_e^x))$.

Definition 2.25.[32] A map $(X,\tau,\preceq_1) \to (Y,\theta,\preceq_2)$ is said to be:

(i)I (resp. D, B) -pre continuous if the inverse image of each open set is I (resp. D, B) -pre open.
(ii)I (resp. D, B) -pre open if the image of each open set is I (resp. D, B) -pre open.
(iii)I (resp. D, B) -pre closed if the image of each open set is I (resp. D, B) -pre closed.
(iv)I (resp. D, B) -pre homeomorphism if it is bijective, I (resp. D, B) -pre continuous and I (resp. D, B) -pre open.

3 New types of soft pre ordered maps

3.1 Soft $I(D,B)$-pre continuity

This section introduces the concepts of $I(D,B)$-pre continuity with respect to soft point, ordinary point and the universe set. We give the equivalent terms for each one of these concepts at the ordinary points and provide some illustrative examples.

Definition 3.1. A soft subset H_E of $(X,\tau,E,\preceq_1)$ is said to be:

(i)Soft I (resp. Soft D, Soft B) -pre open if it is soft pre open and increasing (resp. decreasing, balancing).
(ii)Soft I (resp. Soft D, Soft B) -pre closed if it is soft pre closed and increasing (resp. decreasing, balancing).

Definition 3.2. A soft map $f_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ is called:

(i)Soft I (resp. Soft D, Soft B) -pre continuous at $P_e^x \in \widetilde{X}$ if for each soft open set H_F containing $f_\phi(P_e^x)$, there exists a soft I (resp. soft D, soft B) -pre open set G_E containing P_e^x such that $f_\phi(G_E)\widetilde{\subseteq}H_F$.
(ii)Soft I (resp. Soft D, Soft B) -pre continuous at $x \in X$ if it is soft I (resp. soft D, soft B) -pre continuous at each P_e^x.
(iii)Soft I (resp. Soft D, Soft B) -pre continuous if it is soft I (resp. soft D, soft B) -pre continuous at each $x \in X$.

Theorem 3.3. A soft map $f_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ is soft I (resp. soft D, soft B) -pre continuous if and only if the inverse image of each soft open subset of $\widetilde{Y}$ is a soft I (resp. soft D, soft B) -pre open subset of $\widetilde{X}$.

Proof. We prove the theorem in the case of f_ϕ is soft D-pre continuous and the other cases can be achieved similarly.
Necessity: Let G_F be a soft open subset of $\widetilde{Y}$, Then we have the following two cases:

(i)Either $f_\phi^{-1}(G_F) = \widetilde{\emptyset}$.
(ii)Or $f^{-1}(G_F) \neq \widetilde{\emptyset}$. By choosing $P_e^x \in X$ such that $P_e^x \in f_\phi^{-1}(G_F)$, we obtain that $f_\phi(P_e^x) \in G_F$. So there exists a soft D-pre open set H_E containing P_e^x such that $f_\phi(H_E)\widetilde{\subseteq}G_F$. Since P_e^x is chosen arbitrary, then $f_\phi^{-1}(G_F) = \widetilde{\bigcup}_{P_e^x \in f_\phi^{-1}(G_F)} H_E$.

From the two cases above, we conclude that $f_\phi^{-1}(G_F)$ is a soft D-pre open subset of $\widetilde{X}$.
Sufficiency: Let G_F be a soft open subset of $\widetilde{Y}$ containing $f_\phi(P_e^x)$. Then $P_e^x \in f_\phi^{-1}(G_F)$. By hypothesis, $f_\phi^{-1}(G_F)$ is a soft D-pre open set. Since $f_\phi(f_\phi^{-1}(G_F))\widetilde{\subseteq}G_F$, then f_ϕ is a soft D-pre continuous map at $P_e^x \in X$ and since P_e^x is chosen arbitrary, then f_ϕ is a soft D-pre continuous map.□

Remark 3.4 From Definition 3.2, we can note the following:

(i)Every soft I (soft D, soft B) -pre continuous map is always soft pre continuous.
(ii)Every soft B-pre continuous map is soft I-pre continuous or soft D-pre continuous.

The two examples below elucidate that the converse of the two results of the above remark does not need to be true in general.

Example 3.5. Let $E=\{e_1,e_2\}$ be a parameters set and $X=\{1,2,3,4\}$ be the universe set. Consider $\phi: E\to E$ and $f: X\to X$ are two identity maps. Let $\preceq=\triangle\bigcup\{(1,3)\}$ be a partial order relation on X and consider that $\tau=\{\widetilde{\emptyset},\widetilde{X},F_E,G_E\}$ and $\theta=\{\widetilde{\emptyset},\widetilde{Y},H_E,L_E\}$ are two soft topologies on X, where $F_E=\{(e_1,\{1\}),(e_2,\{3,4\})\}$, $G_E=\{(e_1,\emptyset),(e_2,\{3\})\}$, $H_E=\{(e_1,\{1\}),(e_2,\{2,3\})\}$ and $L_E=\{(e_1,\{1\}),(e_2,\{3\})\}$. For a soft map $f_\phi:(X,\tau,E,\preceq)\to(X,\theta,E,\preceq)$, we note that $f_\phi^{-1}(H_E)=H_E$ and $f_\phi^{-1}(L_E)=L_E$ are soft pre open sets. So f_ϕ is a soft pre continuous map. On the other hand, $f_\phi^{-1}(H_E)$ is neither a soft D-pre open set nor a soft I-pre open set. Hence f_ϕ is not soft I (soft D, soft B)-pre continuous.

Example 3.6. In the example above, if we replace only the partial order relation by $\preceq=\triangle\bigcup\{(2,4)\}$(resp. $\preceq=\triangle\bigcup\{(4,1)\}$), then the soft map f_ϕ is soft D-pre continuous (resp. soft I-continuous), but is not soft B-pre continuous.

Definition 3.7. For a soft subset H_E of $(X,\tau,E,\preceq)$, we define the following six operators:

(i)H_E^{ipo}(resp. H_E^{dpo},H_E^{bpo}) is the largest soft I (resp. soft D, soft B) -pre open set that is contained in H_E.
(ii)H_E^{ipcl}(resp. H_E^{dpcl},H_E^{bpcl}) is the smallest soft I (resp. soft D, soft B) -pre closed set that contains H_E.

Lemma 3.8. For any soft subset H_E of $(X,\tau,E,\preceq)$, the following statements hold:

(i)$(H_E^{dpcl})^c=(H_E^c)^{ipo}$.
(ii)$(H_E^{ipcl})^c=(H_E^c)^{dpo}$.
(iii)$(H_E^{bpcl})^c=(H_E^c)^{bpo}$.

Proof.

(i)$(H_E^{dpcl})^c=\{\widetilde{\bigcup}F_E: F_E$ is a soft D-pre closed set containing $H_E\}^c$
$=\widetilde{\bigcap}\{F_E^c: F_E^c$ is a soft I-pre open set contained in $H_E^c\}$
$=(H_E^c)^{ipo}$.

By analogy with **(i)**, one can prove **(ii)** and **(iii)**.□

Theorem 3.9. The following five properties of a soft map $f_\phi:(X,\tau,E\preceq_1)\to(Y,\theta,F,\preceq_2)$ are equivalent:

(i)f_ϕ is soft I-pre continuous;
(ii)$f_\phi^{-1}(L_F)$ is a soft D-pre closed subset of $\widetilde{X}$ for each soft closed subset L_F of $\widetilde{Y}$;
(iii)$(f_\phi^{-1}(M_F))^{dpcl}\widetilde{\subseteq}f_\phi^{-1}(cl(M_F))$ for every $M_F\widetilde{\subseteq}\widetilde{Y}$;
(iv)$f_\phi(N_E^{dpcl})\widetilde{\subseteq}cl(f_\phi(N_E))$ for every $N_E\widetilde{\subseteq}\widetilde{X}$;
(v)$f_\phi^{-1}(int(M_F))\widetilde{\subseteq}(f_\phi^{-1}(M_F))^{ipo}$ for every $M_F\widetilde{\subseteq}\widetilde{Y}$.

Proof. $(i)\Rightarrow(ii)$: Consider L_F as a soft closed subset of $\widetilde{Y}$. By hypothesis, $f_\phi^{-1}(L_F^c)$ is a soft I-pre open subset of $\widetilde{X}$ and by the fact that $f_\phi^{-1}(L_F^c)=(f_\phi^{-1}(L_F))^c$, we obtain that $f_\phi^{-1}(L_F)$ is soft D-pre closed as required.
$(ii)\Rightarrow(iii)$: It follows from statement **(ii)** that $f_\phi^{-1}(cl(M_E))$ is a soft D-pre closed subset of $\widetilde{X}$. So $(f_\phi^{-1}(M_F))^{dpcl}\widetilde{\subseteq}(f_\phi^{-1}(cl(M_F))^{dpcl}=f_\phi^{-1}(cl(M_F))$.
$(iii)\Rightarrow(iv)$: From (iii), we have $(f_\phi^{-1}(f_\phi(N_E))^{dpcl}$ $\widetilde{\subseteq}f_\phi^{-1}(cl(f_\phi(N_E))$. By using the fact that $N_E^{dpcl}\widetilde{\subseteq}(f_\phi^{-1}(f_\phi(N_E))^{dpcl}$, we have $f_\phi(N_E^{dpcl})\widetilde{\subseteq}cl(f_\phi(N_E))$.
$(iv)\Rightarrow(v)$: For any soft subset M_F of $\widetilde{Y}$, we obtain from Lemma 3.8 that $f_\phi(\widetilde{X}-(f_\phi^{-1}(N_E))^{ipo})=f_\phi(((f_\phi^{-1}(N_E))^c)^{dscl})$. It follows from statement (iv), that $f_\phi(((f_\phi^{-1}(N_E))^c)^{dpcl})$ $\widetilde{\subseteq}cl(f_\phi(f_\phi^{-1}(N_E))^c)=cl(f_\phi(f_\phi^{-1}(N_E^c)))\widetilde{\subseteq}cl(\widetilde{Y}-N_E)=\widetilde{Y}-int(N_E)$. Therefore $(\widetilde{X}-(f_\phi^{-1}(N_E))^{ipo})\widetilde{\subseteq}f_\phi^{-1}(\widetilde{Y}-int(N_E))=\widetilde{X}-f_\phi^{-1}(int(N_E))$. Thus $f_\phi^{-1}(int(N_E))\widetilde{\subseteq}(f_\phi^{-1}(N_E))^{ipo}$.
$(v)\Rightarrow(i)$: Consider M_F as a soft open subset of $\widetilde{Y}$. Then $f_\phi^{-1}(M_F)=f_\phi^{-1}(int(M_F))\widetilde{\subseteq}(f_\phi^{-1}(M_F))^{ipo}$. So $(f_\phi^{-1}(M_F))^{ipo}=f_\phi^{-1}(M_F)$ and this means that $f_\phi^{-1}(M_F)$ is a soft I-pre open subset of $\widetilde{X}$. Hence the desired result is proved.□

Theorem 3.10. The following five properties of a soft map $f_\phi:(X,\tau,E\preceq_1)\to(Y,\theta,F,\preceq_2)$ are equivalent:

(i)f_ϕ is soft D-pre continuous (resp. soft B-pre continuous);
(ii)$f_\phi^{-1}(L_F)$ is a soft I-pre closed (resp. soft B-pre closed) subset of $\widetilde{X}$ for each soft closed subset L_F of $\widetilde{Y}$;
(iii)$(f_\phi^{-1}(M_F))^{ipcl}\widetilde{\subseteq}f_\phi^{-1}(cl(M_F))$(resp. $(f_\phi^{-1}(M_F))^{bpcl}\widetilde{\subseteq}f_\phi^{-1}(cl(M_F))$ for every $M_F\widetilde{\subseteq}\widetilde{Y}$;
(iv)$f_\phi(N_E^{ipcl})\widetilde{\subseteq}cl(f_\phi(N_E))$(resp. $f_\phi(N_E^{bpcl})\widetilde{\subseteq}cl(f_\phi(N_E))$ for every $N_E\widetilde{\subseteq}\widetilde{X}$;
(v)$f_\phi^{-1}(int(M_F))\widetilde{\subseteq}(f_\phi^{-1}(M_F))^{dpo}$(resp. $f_\phi^{-1}(int(M_F))\widetilde{\subseteq}(f_\phi^{-1}(M_F))^{bpo}$ for every $M_F\widetilde{\subseteq}\widetilde{Y}$.

Proof. The proof is similar to that of Theorem 3.9.□

Definition 3.11. A map $(X,\tau,\preceq_1)\to(Y,\theta,\preceq_2)$ is said to be I (resp. D, B) -pre continuous if the inverse image of each open set is I (resp. D, B) -pre open.

Theorem 3.12. Let $\tau^{\star}$ be an extended soft topology on X. Then a soft map $g_{\phi} : (X, \tau^{\star}, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft I (resp. soft D, soft B) -pre continuous If and only if a map $g : (X, \tau^{\star}_e, \preceq_1) \to (Y, \theta_{\phi(e)}, \preceq_2)$ is I (resp. D, B) -pre continuous.

Proof. *Necessity*: Let U be an open subset of $(Y, \theta_{\phi(e)}, \preceq_2)$. Then there exists a soft open subset G_F of $(Y, \theta, F, \preceq_2)$ such that $G(\phi(e)) = U$. Since g_{ϕ} is a soft I (resp. soft D, soft B) -pre continuous map, then $g_{\phi}^{-1}(G_F)$ is a soft I (resp. soft D, soft B) -pre open set. From Definition 2.8, it follows that a soft subset $g_{\phi}^{-1}(G_F) = (g_{\phi}^{-1}(G))_E$ of $(X, \tau, E, \preceq_1)$ is given by $g_{\phi}^{-1}(G)(e) = g^{-1}(G(\phi(e)))$ for each $e \in E$. By hypothesis, $\tau^{\star}$ is an extended soft topology on X, we obtain that a subset $g^{-1}(G(\phi(e))) = g^{-1}(U)$ of $(X, \tau_e, \preceq_1)$ is I (resp. D, B) -pre open. Hence a map g is I (resp. D, B) -pre continuous.
Sufficiency: Let G_F be a soft open subset of $(Y, \theta, F, \preceq_2)$. Then from Definition 2.8, it follows that a soft subset $g_{\phi}^{-1}(G_F) = (g_{\phi}^{-1}(G))_E$ of $(X, \tau^{\star}, E, \preceq_1)$ is given by $g_{\phi}^{-1}(G)(e) = g^{-1}(G(\phi(e)))$ for each $e \in E$. Since a map g is I (resp. D, B) -pre continuous, then a subset $g^{-1}(G(\phi(e)))$ of $(X, \tau^{\star}_e, \preceq_1)$ is I (resp. D, B) -pre open. By hypothesis, $\tau^{\star}$ is an extended soft topology on X, we obtain that $g_{\phi}^{-1}(G_F)$ is a soft I (resp. soft D, soft B) -pre open subset of $(X, \tau^{\star}, E, \preceq_1)$. Hence a soft map g_{ϕ} is soft I (resp. soft D, soft B) -pre continuous.□

Proposition 3.13. If a bijective soft map $f_{\phi} : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft B-pre continuous such that θ is not the indiscrete soft topology, then $\preceq_1$ is not linearly ordered.

3.2 Soft $I(D,B)$-pre open and soft $I(D,B)$-pre closed maps

This part presents the notions of soft $I(D,B)$-pre open and soft $I(D,B)$-pre closed maps; and elucidates the relationships among them with the help of examples. Some properties and characterizations of each one of these notions are studied.

Definition 3.14. A soft map $f_{\phi} : (X, \tau, E, \preceq_1) \to (Y, \tau, F, \preceq_2)$ is called:

(i)Soft I (resp. Soft D, Soft B) -pre open if the image of every soft open subset of $\widetilde{X}$ is a soft I (resp. soft D, soft B) -pre open subset of $\widetilde{Y}$.
(ii)Soft I (resp. Soft D, Soft B) -pre closed if the image of every soft closed subset of $\widetilde{X}$ is a soft I (resp. soft D, soft B) -pre closed subset of $\widetilde{Y}$.

Remark 3.15. From Definition 3.14, we can note the following:

(i)Every soft I (D, B) -pre open map is soft pre open.
(ii)Every soft I (D, B) -pre closed map is soft pre closed.
(iii)Every soft B-pre open (resp. soft B-pre closed) map is soft I-pre open or soft D-pre open (resp. soft I-pre closed or soft D-pre closed).

We construct the following two examples to show that the converse of the three statements of the above remark fails.

Example 3.16. Let E, X, $\phi : E \to E$, $f : X \to X$ and $\preceq$ be the same as in Example 3.5. Consider that $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_E\}$ and $\theta = \{\widetilde{\emptyset}, \widetilde{Y}, L_E\}$ are two soft topologies on X, where $F_E = \{(e_1, \{1\}), (e_2, \{3,4\})\}$ and $L_E = \{(e_1, \{1\}), (e_2, \{3\})\}$. For a soft map $f_{\phi} : (X, \tau, E, \preceq) \to (X, \theta, E, \preceq)$, we note that $f_{\phi}(F_E) = F_E$ is a soft pre open set. So f_{ϕ} is a soft pre open map. On the other hand, $f_{\phi}(F_E)$ is neither a soft D-pre open set nor a soft I-pre open set. Hence f_{ϕ} is not soft I (soft D, soft B)-pre open. Also, f_{ϕ} is a soft pre closed map, but it is not soft I (soft D, soft B)-pre closed.

Example 3.17. In the example above, if we replace only the partial order relation by $\preceq = \triangle \bigcup \{(2,4)\}$, then the soft map f_{ϕ} is soft I-pre open and soft D-pre closed, but it is neither a soft B-pre open map nor a soft B-pre closed map. Also, if we replace only the partial order relation by $\preceq = \triangle \bigcup \{(1,2)\}$), then the soft map f_{ϕ} is soft D-open and soft I-pre closed, but it is neither a soft B-pre open map nor a soft B-pre closed map.

Theorem 3.18. The following three properties of a soft map $f_{\phi} : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ are equivalent:

(i)f_{ϕ} is soft I-pre open;
(ii)$int(f_{\phi}^{-1}(M_F)) \widetilde{\subseteq} f_{\phi}^{-1}(M_F^{ipo})$ for every $M_F \widetilde{\subseteq} \widetilde{Y}$;
(iii)$f_{\phi}(int(N_E)) \widetilde{\subseteq} (f_{\phi}(N_E))^{ipo}$ for every $N_E \widetilde{\subseteq} \widetilde{X}$.

Proof. $(i) \Rightarrow (ii)$: Given a soft subset M_F of $\widetilde{Y}$, it is obvious that $int(f_{\phi}^{-1}(M_F))$ is a soft open subset of $\widetilde{X}$. Then, by hypothesis, it follows that $f_{\phi}(int(f_{\phi}^{-1}(M_F)))$ is a soft I-pre open subset of $\widetilde{Y}$. Since $f_{\phi}(int(f_{\phi}^{-1}(M_F))) \widetilde{\subseteq} f_{\phi}(f_{\phi}^{-1}(M_F)) \widetilde{\subseteq} M_F$, then $int(f_{\phi}^{-1}(M_F)) \widetilde{\subseteq} f_{\phi}^{-1}(M_F^{ipo})$.
$(ii) \Rightarrow (iii)$: Given a soft subset N_E of $\widetilde{X}$, from (ii), we obtain that $int(f_{\phi}^{-1}(f_{\phi}(N_E))) \widetilde{\subseteq} f_{\phi}^{-1}((f_{\phi}(N_E))^{ipo})$. Since $int(N_E) \widetilde{\subseteq} f_{\phi}^{-1}(f_{\phi}(int(f_{\phi}^{-1}(f_{\phi}(N_E))))) \widetilde{\subseteq} f_{\phi}^{-1}((f_{\phi}(N_E))^{ipo})$, then $f_{\phi}(int(N_E)) \widetilde{\subseteq} (f_{\phi}(N_E))^{ipo}$ as required.
$(iii) \Rightarrow (i)$: Let G_E be a soft open subset of $\widetilde{X}$. Then $f_{\phi}(int(G_E)) = f_{\phi}(G_E) \widetilde{\subseteq} (f_{\phi}(G_E))^{ipo}$. Hence f_{ϕ} is a soft I-pre open map.□

In a similar manner, one can prove the following theorem.

Theorem 3.19. The following three properties of a soft map $f_\phi : (X,\tau,E \preceq_1) \to (Y,\theta,F,\preceq_2)$ are equivalent:

(i)f_ϕ is soft D-pre open (resp. soft B-pre open);
(ii)$int(f_\phi^{-1}(M_F))\widetilde{\subseteq}f_\phi^{-1}(M_F^{dpo})$(resp. $int(f_\phi^{-1}(M_F))\widetilde{\subseteq}f_\phi^{-1}(M_F^{bpo})$) for every $M_F\widetilde{\subseteq}\widetilde{Y}$;
(iii)$f_\phi(int(N_E))\widetilde{\subseteq}(f_\phi(N_E))^{dpo}$(resp. $f_\phi(int(N_E))\widetilde{\subseteq}(f_\phi(N_E))^{bpo}$) for every $N_E\widetilde{\subseteq}\widetilde{X}$.

Theorem 3.20. The following three statements hold for a soft map $f_\phi : (X,\tau,E \preceq_1) \to (Y,\theta,F,\preceq_2)$:

(i)f_ϕ is soft I-pre closed if and only if $(f_\phi(G_E))^{ipcl}\widetilde{\subseteq}f_\phi(cl(G_E))$ for every $G_E\widetilde{\subseteq}\widetilde{X}$.
(ii)f_ϕ is soft D-pre closed if and only if $(f_\phi(G_E))^{dpcl}\widetilde{\subseteq}f_\phi(cl(G_E))$ for every $G_E\widetilde{\subseteq}\widetilde{X}$.
(iii)f_ϕ is soft B-pre closed if and only if $(f_\phi(G_E))^{bpcl}\widetilde{\subseteq}f_\phi(cl(G_E))$ for every $G_E\widetilde{\subseteq}\widetilde{X}$.

Proof. We only prove the first statement and the others follow similar lines.
Necessity: Since f_ϕ is soft I-pre closed, then $f_\phi(cl(G_E))$ is a soft I-pre closed subset of $\widetilde{Y}$ and since $f_\phi(G_E)\widetilde{\subseteq}f_\phi(cl(G_E))$, then $(f_\phi(G_E))^{ipcl}\widetilde{\subseteq}f_\phi(cl(G_E))$.
Sufficiency: Consider H_E as a soft closed subset of $\widetilde{X}$. Then $f_\phi(H_E)\widetilde{\subseteq}(f_\phi(H_E))^{ipcl}\widetilde{\subseteq}f_\phi(cl(H_E)) = f_\phi(H_E)$. Therefore $f_\phi(H_E) = (f_\phi(H_E))^{ipcl}$. This means that $f_\phi(H_E)$ is a soft I-pre closed set. Hence the proof is complete. □

Theorem 3.21. The following three statements hold for a bijective soft map $f_\phi : (X,\tau,E \preceq_1) \to (Y,\theta,F,\preceq_2)$:

(i)f_ϕ is soft I (resp. soft D, soft B) -pre open if and only if f_ϕ is soft D (resp. soft I, soft B) -pre closed.
(ii)f_ϕ is soft I (resp. soft D, soft B) -pre open if and only if f_ϕ^{-1} is soft I (resp. soft D, soft B) -pre continuous.
(iii)f_ϕ is soft D (resp. soft I, soft B) -pre closed if and only if f_ϕ^{-1} is soft I (resp. soft D, soft B) -pre continuous.

Proof. For the sake of brevity, we only give proofs for the cases outside the parenthesis and the cases between parenthesis can be made similarly.

(i)To prove the necessary condition, let H_E be a soft closed subset of $\widetilde{X}$ and consider f_ϕ is a soft I-pre open map. Then H_E^c is soft open and $f_\phi(H_E^c)$ is soft I-pre open. It follows from the bijectiveness of f_ϕ, that $f_\phi(H_E^c) = [f_\phi(H_E)]^c$. This automatically implies that $f_\phi(H_E)$ is soft D-pre closed. Thus f_ϕ is a soft D-pre closed map. In a similar manner, we can prove the sufficient condition.
(ii)*Necessity*: Let G_E be a soft open subset of $\widetilde{X}$ and consider f_ϕ is a soft I-pre open map. Then $f_\phi(G_E)$ is soft I-pre open. It follows from the bijectiveness of f_ϕ, that $f_\phi(G_E) = (f_\phi^{-1})^{-1}(G_E)$. This automatically implies that $(f_\phi^{-1})^{-1}(G_E)$ is soft I-pre open. Thus f_ϕ^{-1} is a soft I-pre continuous map. In a similar manner, we can prove the sufficient condition.
(iii)The proof of this statement comes immediately from (i) and (ii) above.

□

Theorem 3.22. Let $\theta^\star$ be an extended soft topology on Y and ϕ is an injective map. Then a soft map $g_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta^\star,F,\preceq_2)$ is soft I (resp. soft D, soft B) -pre open If and only if a map $g : (X,\tau_e,\preceq_1) \to (Y,\theta^\star_{\phi(e)},\preceq_2)$ is I (resp. D, B) -pre open.

Proof. To prove the *necessary* part, let U be an open subset of $(X,\tau_e,\preceq_1)$ and $\phi(e) = f$. Then there exists a soft open subset G_E of $(X,\tau,E,\preceq_1)$ such that $G(e) = U$. Since g_ϕ is a soft I (resp. soft D, soft B) -pre open map, then $g_\phi(G_E)$ is a soft I (resp. soft D, soft B) -pre open set. From Definition 2.8, it follows that a soft subset $g_\phi(G_E) = (g_\phi(G))_F$ of $(Y,\theta,F,\preceq_2)$ is given by $g_\phi(G)(f) = \bigcup_{e\in\phi^{-1}(f)} g(G(e))$ for each $f \in F$. By hypothesis, $\theta^\star$ is an extended soft topology on Y, a subset $\bigcup_{e\in\phi^{-1}(f)} g(G(e)) = g(U)$ of $(Y,\theta_{\phi(e)},\preceq_2)$ is I (resp. D, B) -pre open. Hence a map g is I (resp. D, B) -pre open.
To prove the *sufficient* part, let G_E be a soft open subset of $(X,\tau,E,\preceq_1)$. Then from Definition 2.8, it follows that a soft subset $g_\phi(G_E) = (g_\phi(G))_F$ of $(Y,\theta^\star,F,\preceq_2)$ is given by $g_\phi(G)(f) = \bigcup_{e\in\phi^{-1}(f)} g(G(e))$ for each $f \in F$. Since a map g is I (resp. D, B) -pre open, then a subset $\bigcup_{e\in\phi^{-1}(f)} g(G(e))$ of $(Y,\theta^\star_{\phi(e)},\preceq_2)$ is I (resp. D, B) -pre open. By hypothesis, $\theta^\star$ is an extended soft topology on Y, $g_\phi(G_E)$ is a soft I (resp. soft D, soft B) -pre open subset of $(Y,\theta^\star,F,\preceq_2)$. Hence a soft map g_ϕ is soft I (resp. soft D, soft B) -pre open.□

The result above is restated in the case of soft I (resp. soft D, soft B) -pre closed maps and one can prove them similarly. So the proof will be omitted.

Theorem 3.23. Let $\theta^\star$ be an extended soft topology on Y and ϕ is an injective map. Then a soft map $g_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta^\star,F,\preceq_2)$ is soft I (resp. soft D, soft B) -pre closed If and only if a map $g : (X,\tau_e,\preceq_1) \to (Y,\theta^\star_{\phi(e)},\preceq_2)$ is I (resp. D, B) -pre closed.

Proposition 3.24. Consider that τ is not the indiscrete soft topology on X. If an injective soft map $f_\phi : (X,\tau,E \preceq_1) \to (Y,\theta,F,\preceq_2)$ is soft B-pre open or soft B-pre closed, then $\preceq_2$ is not linearly ordered.

Proposition 3.25. Let $f_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ and $g_\lambda : (Y,\theta,F,\preceq_2) \to (Z,\upsilon,K,\preceq_3)$ be two soft maps. Then for $x \in \{I,D,B\}$, the following properties hold.

(i)If f_ϕ is a soft x-pre continuous map and g_λ is a soft continuous map, then $g_\lambda \circ f_\phi$ is a soft x-pre continuous map.
(ii)If f_ϕ is a soft open (resp. soft closed) map and g_λ is a soft x-pre open (resp. x-pre closed) map, then $g_\lambda \circ f_\phi$ is a soft x-pre open (resp. x-pre closed) map.
(iii)If $g_\lambda \circ f_\phi$ is a soft x-pre open map and f_ϕ is surjective soft continuous, then g_λ is a soft x-pre open map.
(iv)If $g_\lambda \circ f_\phi$ is a soft closed map and g_λ is an injective soft x-pre continuous map, then f_ϕ is a soft x-pre closed map.

3.3 Soft $I(D,B)$-pre homeomorphism

We define and investigate in this section, the concepts of soft $I(D,B)$-pre homeomorphism maps. We discuss their main features and verify some findings related to them.

Definition 3.26. A bijective soft map $g_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ is called soft I (resp. soft D, soft B) -pre homeomorphism if it is soft I-pre continuous and soft I-pre open (resp. soft D-pre continuous and soft D-pre open, soft B-pre continuous and soft B-pre open).

Remark 3.27. From Definition 3.26, we can note the following:

(i)Every soft I (soft D, soft B) -pre homeomorphism map is soft pre homeomorphism.
(ii)Every soft B-pre homeomorphism map is soft I-pre homeomorphism or soft D-pre homeomorphism.

The two items of the above remark are not conversely as the following examples show.

Example 3.28. Let E, X, $\phi : E \to E$, $f : X \to X$ and $\preceq$ be the same as in Example 3.5. Consider that $\tau = \{\widetilde{\emptyset},\widetilde{X},F_E,L_E\}$ and $\theta = \{\widetilde{\emptyset},\widetilde{Y},L_E\}$ are two soft topologies on X, where $F_E = \{(e_1,\{1\}),(e_2,\{3,4\})\}$ and $L_E = \{(e_1,\{1\}),(e_2,\{3\})\}$. Then we find that $f_\phi : (X,\tau,E,\preceq) \to (X,\theta,E,\preceq)$ is a soft pre homeomorphism map, but it is neither a soft D-pre homeomorphism map nor a soft I-pre homeomorphism map. Hence f_ϕ is not soft I (soft D, soft B)-pre homeomorphism.

Example 3.29. In the example above, if we replace only the partial order relation by $\preceq = \triangle \bigcup \{(2,4)\}$, then the soft map f_ϕ is soft I-pre homeomorphism, but it is not a soft B-pre homeomorphism map. Also, if we replace only the partial order relation by $\preceq = \triangle \bigcup \{(1,2)\}$), then the soft map f_ϕ is soft D-homeomorphism, but it is not a soft B-pre homeomorphism map.

Theorem 3.30. Consider a bijective soft map $f_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ and let $(\gamma,\lambda) \in \{(I-pre,dpcl),(D-pre,ipcl),(B-pre,bpcl)\}$. If $(f_\phi(G_E))^\lambda = f_\phi(cl_p(G_E)) = cl_p(f_\phi(G_E)) = f_\phi(G_E^\lambda)$ for every $G_E \widetilde{\subseteq} \widetilde{X}$, then f_ϕ is soft γ-homeomorphism.

Proof. We make a proof for the theorem in the case of $(\gamma,\lambda) = (I-pre,dpcl)$ and the other follows similar line. The equality relation $(f_\phi(G_E))^{dpcl} = f_\phi(cl_p(G_E)) = cl_p(f_\phi(G_E)) = f_\phi(G_E^{dpcl})$ implies that $f_\phi(G_E^{dpcl}) \widetilde{\subseteq} cl_p(f_\phi(G_E)) \widetilde{\subseteq} cl(f_\phi(G_E))$ and $(f_\phi(G_E))^{dpcl} \widetilde{\subseteq} f_\phi(cl_p(G_E)) \widetilde{\subseteq} f_\phi(cl(G_E))$. So f_ϕ is soft I-pre continuous and soft D-pre closed map. Hence the desired result is proved.□

Theorem 3.31. If a bijective soft map $f_\phi : (X,\tau,E,\preceq_1) \to (Y,\theta,F,\preceq_2)$ is soft I-pre continuous (resp. soft D-pre continuous, soft B-pre continuous), then the following three statements are equivalent:

(i)f_ϕ is soft I-pre homeomorphism (resp. soft D-pre homeomorphism, soft B-pre homeomorphism).
(ii)f_ϕ^{-1} is soft I-pre continuous (resp. soft D-pre continuous, soft B-pre continuous).
(iii)f_ϕ is soft D-pre closed (resp. soft I-pre closed, soft B-pre closed).

Proof. $(i) \Rightarrow (ii)$ Since f_ϕ is a soft I-pre homeomorphism (resp. soft D-pre homeomorphism, soft B-pre homeomorphism) map, then f_ϕ is soft I-pre open (resp. soft D-pre open, soft B-pre open). It follows from item (ii) of Theorem 3.21, that f_ϕ^{-1} is soft I-pre continuous (resp. soft D-pre continuous, soft B-pre continuous).
$(ii) \Rightarrow (iii)$ The proof follows from item (iii) of Theorem 3.21.
$(iii) \Rightarrow (i)$ It is sufficient to prove that f_ϕ is a soft I-pre open (resp. soft D-pre open, soft B-pre open) map. This follows from item (i) of Theorem 3.21.□

Theorem 3.32. Let $\tau^\star$ and $\theta^\star$ be extended soft topologies on X and Y, respectively. Then a soft map $g_\phi : (X,\tau^\star,E,\preceq_1) \to (Y,\theta^\star,F,\preceq_2)$ is soft I (resp. soft D, soft B) -pre homeomorphism If and only if a map $g : (X,\tau_e^\star,\preceq_1) \to (Y,\theta_{\phi(e)}^\star,\preceq_2)$ is I (resp. D, B) -pre homeomorphism.

Proof. The proof is obtained immediately from Theorem 3.12 and Theorem 3.22. □

Proposition 3.33. Let the two soft topologies τ and θ on X and Y, respectively, such that τ and θ are not the indiscrete soft topology. If a soft map $f_\phi : (X,\tau,E \ \preceq_1) \to (Y,\theta,F,\preceq_2)$ is soft B-pre homeomorphism, then $\preceq_1$ and $\preceq_2$ are not linearly ordered.

4 Conclusion

In 2018, we have formulated the concept of soft topological ordered spaces as an extension of the concept of soft topological spaces [30]. Then we have utilized monotone soft sets to define some soft ordered maps and have investigated their main properties [31]. In this work, we have used soft pre open sets to give the concepts of soft x-pre continuous, soft x-pre open, soft x-pre closed and soft x-pre homeomorphism maps, where $x \in \{I, D, B\}$. We have given various characterizations for these concepts and have showed the relationships among them with the help of examples.

Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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