

A Study of Intuitionistic Fuzzy Associative Filter in Lattice Implication Algebra

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Abstract: In discrete mathematics, automated reasoning is quite useful for real-time applications involving autonomous vehicles and robots. A Lattice Implication Algebra (LIA) assumes implication values in a general complete mathematical lattice toward enhancing the representation of ambiguity in reasoning. Since real numbers stem from real-world measurements, the present study sets a ground for real-world applications of LIA. The aforementioned lattice of intervals follows a capacity to optimize LIA reasoning. In this paper, the notion of intuitionistic fuzzy associative filter in lattice implication algebra is introduced. The equivalent conditions for an intuitionistic fuzzy filter to become an intuitionistic fuzzy associative filter are explained. In addition, the relations among intuitionistic fuzzy associative filter, intuitionistic fuzzy fantastic filter and intuitionistic fuzzy positive implicative filter are elaborated..

Keywords: Lattice implication algebra, associative filter, fuzzy associative filter, intuitionistic fuzzy associative filter.

1 Introduction

Fuzzy sets were introduced by Zadeh [8] in 1965 to represent data and information possessing nonstatistical uncertainties. In 1986, on the basis of fuzzy set theory, Atanassov [1] proposed the intuitionistic fuzzy set characterized by a membership function and nonmembership function. Research behind the logical system depends mainly on lattice implication algebra proposed by Xu [6] in 1993. Following this Xu [6] have introduced and investigated the properties of filters in lattice implication algebra. In [3], Jun has introduced the notion of a fantastic filter in a lattice implication algebra and has given some results. Additionally, Zhan Jianming [7] has fuzzified the concept of fantastic filters of lattice implication algebras and has given the relations among fuzzy filter, fuzzy positive implicative filter and fuzzy fantastic filter. The present study mainly focus on the properties of intuitionistic fuzzy associative filters in lattice implication algebra and equivalent conditions for a intuitionistic fuzzy filter to be an intuitionistic fuzzy associative filter. This paper is structured as follows. Section 2 reproduces some basic definitions and results in lattice implication algebra. In section 3, the intuitionistic fuzzy associative filter is introduced, some of its

properties and comparisons with existing filters are given. The relation with some of the filters is shown in figure. The last section throws light of conclusion and future work that can extended from this present work.

2 Preliminary

Definition 2.1.[6] Let $(L, \vee, \wedge, \rightarrow, 0, 1)$ be a bounded lattice with order-reversing involution " \neg ", 1 and 0 are the greatest and least element of L respectively, $\rightarrow: L \times L \rightarrow L$ be a mapping. $(L, \vee, \wedge, \neg, \rightarrow, 0, 1)$ is called a lattice implication algebra if the following conditions hold for any $x, y, z \in L$.

$$(I1) \ x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$$

$$(I2) \ x \rightarrow x = 1$$

$$(I3) \ x \rightarrow y = y \rightarrow \neg x$$

$$(I4) \ \text{If } x \rightarrow y = y \rightarrow x = 1 \text{ then } x = y$$

$$(I5) \ (x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$$

$$(L1) \ (x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$$

$$(L2) \ (x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$$

A partial ordering " \leq " can be defined on a lattice implication algebra L by $x \leq y$ if and only if $x \rightarrow y = 1$.

Theorem 2.2.[6] In a lattice implication algebra L , for the accompanying hold:

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- (1) $0 \rightarrow x = 1, 1 \rightarrow x = x$ and $x \rightarrow 1 = 1$
- (2) $x' = x \rightarrow 0$
- (3) $x \rightarrow y \leq (y \rightarrow z) \rightarrow (x \rightarrow z)$
- (4) $x \vee y = (x \rightarrow y) \rightarrow y$
- (5) $x \leq y$ implies $y \rightarrow z \leq x \rightarrow z$ and $z \rightarrow x \leq z \rightarrow y$
- (6) $x \leq (x \rightarrow y) \rightarrow y$.

Definition 2.3.[1] An Intuitionistic Fuzzy Set (IFS) A of a non-empty set X is an object having the form $A = \{x, \mu_A(x), \nu_A(x) / x \in X\}$ where the function $\mu_A(x) : X \rightarrow [0, 1]$ denotes the degree of membership and $\nu_A(x) : X \rightarrow [0, 1]$ denotes the degree of non-membership respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for any $x \in X$.

Definition 2.4.[5] A subset F of a lattice implication algebra L is called a filter of L if it satisfies for all $x, y \in L$

- (i) $1 \in F$
- (ii) $x \in F$ and $x \rightarrow y \in F$ imply $y \in F$.

Definition 2.5.[5] A fuzzy set μ in a lattice implication algebra L is called a fuzzy filter of L if it satisfies

- (i) $\mu(1) \geq \mu(x)$ for all $x \in L$,
- (ii) $\mu(y) \geq \min\{\mu(x), \mu(x \rightarrow y)\}$ for all $x, y \in L$.

Definition 2.6.[3] A non empty subset F of a lattice implication algebra L is called a fantastic filter of L if it satisfies

- (i) $1 \in F$,
- (ii) $z \rightarrow (y \rightarrow x)$ and $z \in F$ imply $((x \rightarrow y) \rightarrow y) \rightarrow x \in F$, for all $x, y \in L$.

Definition 2.7.[8] A fuzzy set μ in L is called a fuzzy fantastic filter of L if it satisfies

- (i) $\mu(1) \geq \mu(x)$ for all $x \in L$,
- (ii) $\mu(((x \rightarrow y) \rightarrow y) \rightarrow x) \geq \min\{\mu(z \rightarrow (y \rightarrow x)), \mu(z)\}$ for all $x, y, z \in L$.

Definition 2.8.[2] Let A be an IFS of L and L be a lattice implication algebra. If A satisfies the following

- (i) For any $x \in L, \mu_A(1) \geq \mu_A(x)$ and $\nu_A(1) \leq \nu_A(x)$
 - (ii) For any $x, y \in L, \mu_A(y) \geq \min\{\mu_A(x \rightarrow y), \mu_A(x)\}$
 - (iii) For any $x, y \in L, \nu_A(y) \leq \max\{\nu_A(x \rightarrow y), \nu_A(x)\}$
- then A is called intuitionistic fuzzy filter of L .

Definition 2.9.[4] An IFS A of L is called an intuitionistic fuzzy fantastic filter of L if it satisfies the following conditions:

- (i) For any $x \in L, \mu_A(1) \geq \mu_A(x)$ and $\nu_A(1) \leq \nu_A(x)$
- (ii) For any $x, y, z \in L, \mu_A(((x \rightarrow y) \rightarrow y) \rightarrow x) \geq \min\{\mu_A(z \rightarrow (y \rightarrow x)), \mu_A(z)\}$
- (iii) For any $x, y, z \in L, \nu_A(((x \rightarrow y) \rightarrow y) \rightarrow x) \leq \max\{\nu_A(z \rightarrow (y \rightarrow x)), \nu_A(z)\}$.

Definition 2.10.[4] An IFS A of L is called an intuitionistic fuzzy positive implicative filter of L if it satisfies

- (i) $\mu_A(1) \geq \mu_A(x)$ and $\nu_A(1) \leq \nu_A(x)$ for all $x \in L$,
- (ii) $\mu_A(y) \geq \min\{\mu_A(x \rightarrow ((y \rightarrow z) \rightarrow y)), \mu_A(x)\}$ for all $x, y, z \in L$
- (iii) $\nu_A(y) \leq \max\{\nu_A(x \rightarrow ((y \rightarrow z) \rightarrow y)), \nu_A(x)\}$ for all $x, y, z \in L$.

Definition 2.11.[4] Every intuitionistic fuzzy positive implicative filter of L is an intuitionistic fuzzy fantastic filter.

3 Intuitionistic fuzzy associative filter in lattice implication algebra

The concept of intuitionistic fuzzy associative filter is introduced in a lattice implication algebra. The equal conditions for an intuitionistic fuzzy filter to be an intuitionistic fuzzy associative filter are introduced.

Definition 3.1. Let x be a fixed component of a lattice implication algebra L . An IFS A of L is called an intuitionistic fuzzy associative filter of L with respect to x such that,

- (i) $\mu_A(1) \geq \mu_A(x)$ and $\nu_A(1) \leq \nu_A(x)$
- (ii) $\mu_A(z) \geq \min\{\mu_A(x \rightarrow (y \rightarrow z)), \mu_A(x \rightarrow y)\}$
- (iii) $\nu_A(z) \leq \max\{\nu_A(x \rightarrow (y \rightarrow z)), \nu_A(x \rightarrow y)\}$ for all $y, z \in L$.

An intuitionistic fuzzy associative filter of L with respect to all $x \neq 0$ is called an intuitionistic fuzzy associative filter of L . Obviously, an intuitionistic fuzzy associative filter with respect to 0 is constant.

Example 3.2. Let $L = \{0, a_1, b_1, c_1, d_1, 1\}$ be a poset. Define a unary operation " $'$ ", and a binary operation " \rightarrow " on L , and define \vee and \wedge operations on L as follow: $x \vee y = (x \rightarrow y) \rightarrow y$ and $x \wedge y = ((x' \rightarrow y') \rightarrow y')'$ respectively for all $x, y \in L$.

L is a lattice implication algebra defined as follows. An

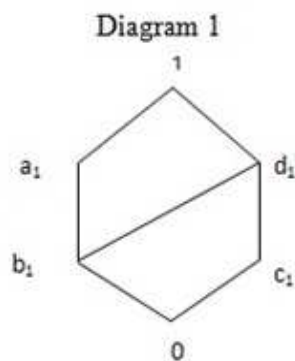


Table 1: Operator order reversing involution in L

x	x'
0	1
a_1	d_1
b_1	c_1
c_1	b_1
d_1	a_1
1	0

Table 2: Operator \rightarrow in L

\rightarrow	0	a_1	b_1	c_1	d_1	1
0	1	1	1	1	1	1
a_1	d_1	1	a_1	c_1	c_1	1
b_1	c_1	1	1	c_1	c_1	1
c_1	b_1	a_1	b_1	1	a_1	1
d_1	a_1	1	a_1	1	1	1
1	0	a_1	b_1	c_1	d_1	1

IFS A in L defined by

$$\mu_A(x) = \begin{cases} 0.8, & x \in \{1, a_1, b_1\} \\ 0.7, & x \in \{0, c_1, d_1\} \end{cases}$$

$$\nu_A(x) = \begin{cases} 0.6, & x \in \{0, c_1, d_1\} \\ 0.3, & x \in \{1, a_1, b_1\} \end{cases}$$

for all $x \in L$. Then A is an intuitionistic fuzzy associative filter of L as for a_1 and b_1 yet not regarding c_1 and d_1 .

Proposition 3.3. An IFS A of L is an intuitionistic fuzzy associative filter of a lattice implication algebra L with respect to $x \in L$, then $\mu_A(x) = \mu_A(1)$ and $\nu_A(x) = \nu_A(1)$.

Proof. If $x=0,1$ it is trivial. Let x be assumed as neither 0 nor 1. Then

$$\begin{aligned} \mu_A(x) &\geq \min\{\mu_A(x \rightarrow (1 \rightarrow x)), \mu_A(x \rightarrow 1)\} \\ &= \min\{\mu_A(x \rightarrow x), \mu_A(1)\} \\ &= \min\{\mu_A(1), \mu_A(1)\}. \end{aligned}$$

By the definition and

$$\mu_A(x) \geq \mu_A(1), \mu_A(x) = \mu_A(1)$$

Also

$$\begin{aligned} \nu_A(x) &\leq \max\{\nu_A(x \rightarrow (1 \rightarrow x)), \nu_A(x \rightarrow 1)\} \\ &= \max\{\nu_A(x \rightarrow x), \nu_A(1)\} \\ &= \max\{\nu_A(1), \nu_A(1)\}. \end{aligned}$$

By the definition and $\nu_A(x) \leq \nu_A(1), \nu_A(x) = \nu_A(1)$.

Theorem 3.4. Every intuitionistic fuzzy associative filter of lattice implication algebra L with respect to 1 is an intuitionistic fuzzy filter.

Proof. Consider A be an intuitionistic fuzzy associative filter of L with respect to 1. Then

$$\begin{aligned} \mu_A(z) &\geq \min\{\mu_A(1 \rightarrow (y \rightarrow z)), \mu_A(1 \rightarrow y)\} \\ &= \min\{\mu_A(y \rightarrow z), \mu_A(y)\} \end{aligned}$$

Table 3: Operator order reversing involution in L

x	x'
0	1
a_2	c_2
b_2	b_2
c_2	a_2
1	0

Table 4: Operator \rightarrow in L

\rightarrow	0	a_2	b_2	c_2	1
0	1	1	1	1	1
a_2	c_2	1	1	1	1
b_2	b_2	c_2	1	1	1
c_2	a_2	b_2	c_2	1	1
1	0	a_2	b_2	c_2	1

Therefore $\mu_A(z) \geq \min\{\mu_A(y \rightarrow z), \mu_A(y)\}$

$\nu_A(z) \leq \max\{\nu_A(1 \rightarrow (y \rightarrow z)), \nu_A(1 \rightarrow y)\}$

$= \max\{\nu_A(y \rightarrow z), \nu_A(y)\}$

Therefore $\nu_A(z) \leq \max\{\nu_A(y \rightarrow z), \nu_A(y)\}$

Hence, A is an intuitionistic fuzzy filter of L.

Corollary 3.5. Each intuitionistic fuzzy associative filter of lattice implication algebra is an intuitionistic fuzzy filter. The opposite of the above corollary may not be true, as shown by the following example.

Example 3.6. Let $L = \{0, a_2, b_2, c_2, 1\}$. The partial order relation can be defined on L as $0 < a_2 < b_2 < c_2 < 1$, and $x \wedge y = \min\{x, y\}, x \vee y = \max\{x, y\}$, for all $x, y \in L$ and " \rightarrow " and " \neg " as follows the Table 3 and Table 4: Then $(L, \wedge, \vee, \neg, \rightarrow)$ is a lattice implication algebra. An intuitionistic fuzzy set A can be defined in L by

$$\mu_A(x) = \begin{cases} 0.8, & x \in \{1\} \\ 0.7, & \text{Otherwise} \end{cases}$$

$$\nu_A(x) = \begin{cases} 0.2, & x \in \{1\} \\ 0.3, & x \in \{a_2, c_2, 0\} \\ 0.4, & x \in \{b_2\} \end{cases}$$

Hence A is an intuitionistic fuzzy filter, but not an intuitionistic fuzzy associative filter of L with respect to a_2 , for $c_2 \in L, \mu_A(c_2) \neq \min\{\mu_A(1), \mu_A(1)\}$ and $\nu_A(c_2) \neq \max\{\nu_A(1), \nu_A(1)\}$.

Theorem 3.7. Let A be an intuitionistic fuzzy filter of a lattice implication algebra L. Then A is an intuitionistic fuzzy associative filter of if and only if it satisfies

$$\begin{aligned} \mu_A((x \rightarrow y) \rightarrow z) &\geq \mu_A(x \rightarrow (y \rightarrow z)), \\ \nu_A((x \rightarrow y) \rightarrow z) &\leq \nu_A(x \rightarrow (y \rightarrow z)) \text{ for all } x, y, z \in L. \end{aligned} \quad (1)$$

Proof. Let A satisfy $\mu_A((x \rightarrow y) \rightarrow z) \geq \mu_A(x \rightarrow (y \rightarrow z))$,

$v_A((x \rightarrow y) \rightarrow z) \leq v_A(x \rightarrow (y \rightarrow z))$ for all $x, y, z \in L$

Hence

$$\mu_A(z) \geq \min\{\mu_A(x \rightarrow y) \rightarrow z, \mu_A(x \rightarrow y)\}$$

$$\geq \min\{\mu_A(x \rightarrow (y \rightarrow z)), \mu_A(x \rightarrow y)\}$$

$$\text{and } v_A(z) \leq \max\{v_A(x \rightarrow y) \rightarrow z, v_A(x \rightarrow y)\}$$

$$\leq \max\{v_A(x \rightarrow (y \rightarrow z)), v_A(x \rightarrow y)\} \text{ for all } x, y, z \in L$$

Therefore, A is an intuitionistic fuzzy associative filter of L.

Conversely A can be assumed as an intuitionistic fuzzy associative filter of L and $x, y, z \in L$. The following can be considered.

$$x \rightarrow ((y \rightarrow z) \rightarrow ((x \rightarrow y) \rightarrow z)) = (y \rightarrow z) \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow z))$$

$$= (y \rightarrow z) \rightarrow ((x \rightarrow y) \rightarrow (x \rightarrow z))$$

$$= (x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z))$$

$$= 1 \in F$$

$$\text{Further } \mu_A((x \rightarrow y) \rightarrow z) \geq \min\{\mu_A(x \rightarrow ((y \rightarrow z) \rightarrow ((x \rightarrow y) \rightarrow z))), \mu_A(x \rightarrow (y \rightarrow z))\}$$

$$= \min\{\mu_A(1), \mu_A(x \rightarrow (y \rightarrow z))\}$$

$$= \mu_A(x \rightarrow (y \rightarrow z))$$

$$\text{Therefore } \mu_A((x \rightarrow y) \rightarrow z) \geq \mu_A(x \rightarrow (y \rightarrow z))$$

$$v_A((x \rightarrow y) \rightarrow z) \leq \max\{v_A(x \rightarrow ((y \rightarrow z) \rightarrow ((x \rightarrow y) \rightarrow z))), v_A(x \rightarrow (y \rightarrow z))\}$$

$$= \max\{v_A(1), v_A(x \rightarrow (y \rightarrow z))\}$$

$$= v_A(x \rightarrow (y \rightarrow z))$$

$$\text{Therefore } v_A((x \rightarrow y) \rightarrow z) \leq v_A(x \rightarrow (y \rightarrow z))$$

Another comparable condition for an intuitionistic fuzzy filter to be an intuitionistic fuzzy associative filter is given in the following theorem.

Theorem 3.8. Let A be an intuitionistic fuzzy filter of lattice implication algebra L. Then A is an intuitionistic fuzzy associative filter of L if and only if it satisfies,

$$\mu_A(y) \geq \mu_A(x \rightarrow (x \rightarrow y)), v_A(y) \leq v_A(x \rightarrow (x \rightarrow y)) \quad (2) \text{ for all } x, y \in L.$$

Proof. Let A be an intuitionistic fuzzy filter of a lattice implication algebra L.

To prove that A is an intuitionistic fuzzy associative filter of L, it is enough to prove that (1) and (2) are equivalent.

setting $x=y$ in (1)

$$\mu_A((y \rightarrow y) \rightarrow z) \geq \mu_A(y \rightarrow (y \rightarrow z)),$$

$$\mu_A(1 \rightarrow z) \geq \mu_A(y \rightarrow (y \rightarrow z)),$$

$$\mu_A(z) \geq \mu_A(y \rightarrow (y \rightarrow z)),$$

and

$$v_A((y \rightarrow y) \rightarrow z) \leq v_A(y \rightarrow (y \rightarrow z)),$$

$$v_A(1 \rightarrow z) \leq v_A(y \rightarrow (y \rightarrow z)),$$

$$v_A(z) \leq v_A(y \rightarrow (y \rightarrow z)),$$

Hence, (1) \Rightarrow (2)

Conversely (2) can be assumed to be satisfied. The following can be considered

$$(x \rightarrow (y \rightarrow z)) \rightarrow (x \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow z))) = 1 \rightarrow ((x \rightarrow (y \rightarrow z)) \rightarrow (x \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow z))))$$

$$= ((y \rightarrow z) \rightarrow (x \rightarrow (x \rightarrow y) \rightarrow z)) \rightarrow ((x \rightarrow (y \rightarrow z)) \rightarrow (x \rightarrow (x \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow z))))$$

$$= (x \rightarrow (y \rightarrow z)) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow (x \rightarrow y) \rightarrow z))) \rightarrow (x \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow z)))$$

$$\geq (x \rightarrow (y \rightarrow z)) \rightarrow (x \rightarrow (y \rightarrow z))$$

$$= 1$$

As $(x \rightarrow (y \rightarrow z)) \rightarrow (x \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow z))) \in F$

Let $\mu_A((x \rightarrow y) \rightarrow z) \geq \{\mu_A(x \rightarrow ((x \rightarrow y) \rightarrow z))\}$

$$\geq \min\{\mu_A(x \rightarrow (y \rightarrow z)) \rightarrow (x \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow z))), \mu_A(x \rightarrow (y \rightarrow z))\}$$

$$= \min\{\mu_A(1), \mu_A(x \rightarrow (y \rightarrow z))\}$$

$$= \mu_A(x \rightarrow (y \rightarrow z))$$

$$\mu_A((x \rightarrow y) \rightarrow z) \geq \mu_A(x \rightarrow (y \rightarrow z))$$

$$\text{Similarly } v_A((x \rightarrow y) \rightarrow z) \leq v_A(x \rightarrow (y \rightarrow z))$$

Hence (2) \Rightarrow (1), as (1) and (2) are equivalent by theorem 2.7, A is an intuitionistic fuzzy associative filter.

Theorem 3.9. Every intuitionistic fuzzy associative filter of lattice implication algebra is an intuitionistic fuzzy fantastic filter.

Proof. Let A be an intuitionistic fuzzy associative filter of L. By Corollary 3.5, A is an intuitionistic fuzzy filter of L.

$$(y \rightarrow x) \rightarrow (x \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow y) \rightarrow x)) = (y \rightarrow x) \rightarrow (x \rightarrow (((x \rightarrow y) \rightarrow y) \rightarrow (x \rightarrow x)))$$

$$= (y \rightarrow x) \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow y) \rightarrow 1)$$

$$= (y \rightarrow x) \rightarrow (x \rightarrow 1)$$

$$= (y \rightarrow x) \rightarrow 1$$

$$= 1$$

which implies that

$$(y \rightarrow x) \rightarrow (x \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow y) \rightarrow x)) \in F$$

$$\text{Therefore, } \mu_A(((x \rightarrow y) \rightarrow y) \rightarrow x) \geq \mu_A(x \rightarrow (x \rightarrow ((x \rightarrow y) \rightarrow y) \rightarrow x))$$

$$= \min\{\mu_A((y \rightarrow x) \rightarrow (x \rightarrow (x \rightarrow (x \rightarrow y) \rightarrow y) \rightarrow x)), \mu_A(y \rightarrow x)\}$$

$$= \min\{\mu_A(1), \mu_A(y \rightarrow x)\}$$

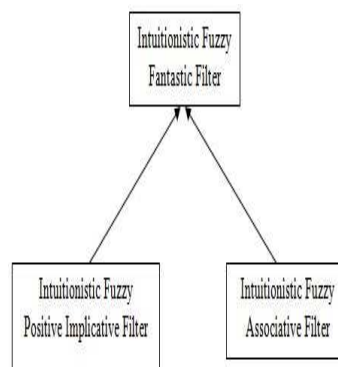
$$= \mu_A(y \rightarrow x)$$

$$\text{Therefore } \mu_A(((x \rightarrow y) \rightarrow y) \rightarrow x) \geq \mu_A(y \rightarrow x)$$

$$\text{Similarly } v_A(((x \rightarrow y) \rightarrow y) \rightarrow x) \leq v_A(y \rightarrow x)$$

Hence A is an intuitionistic fuzzy fantastic filter of L.

The following implication gives the relation among intuitionistic fuzzy fantastic filter, intuitionistic fuzzy positive implicative filter and intuitionistic fuzzy associative filter.



The converse implications of the above relation are not true as shown by the following example.

Example 3.10. Consider $L = \{0, a_3, b_3, 1\}$ be a set with partial ordering. An unary operation " $'$ " and a binary

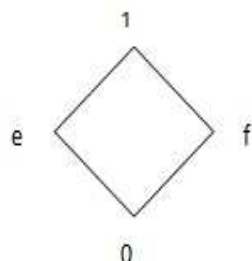
Table 5: Operator order reversing involution in L

x	x'
0	1
e	f
f	e
1	0

Table 6: Operator \rightarrow in L

\rightarrow	0	e	f	1
0	1	1	1	1
e	f	1	1	1
f	e	f	1	1
1	0	e	f	1

operation " \rightarrow " on L can be defined as follow:



\vee and \wedge operation can be defined on L as follow:

$$x \vee y = (x \rightarrow y) \rightarrow y, x \wedge y = ((x' \rightarrow y') \rightarrow y')$$

At that point L is a lattice implication algebra. Characterize an intuitionistic fuzzy set A can be defined in L by

$$\mu_A(x) = \begin{cases} 0.7, & x \in \{0, e, f\} \\ 0.8, & x \in \{1\} \end{cases} \quad \nu_A(x) = \begin{cases} 0.3, & x \in \{0, e, f\} \\ 0.1, & x \in \{1\} \end{cases}$$

Obviously, A is an intuitionistic fuzzy fantastic filter, but not an intuitionistic fuzzy associative filter as for $e, f, 0 \in A, \mu_A(0) \neq \min\{\mu_A(1), \mu_A(1)\}$ and $\nu_A(0) \neq \max\{\nu_A(1), \nu_A(1)\}$. Similarly it is not an intuitionistic fuzzy positive implicative filter as $\mu_A(f) \neq \min\{\mu_A(1), \mu_A(1)\}$ and $\nu_A(f) \neq \max\{\nu_A(1), \nu_A(1)\}$.

4 Conclusion

In this paper, the concept of intuitionistic fuzzy associative filter in lattice implication algebra is introduced. The properties and equivalent conditions of

intuitionistic fuzzy associative filters are discussed. The relations between intuitionistic fuzzy associative filter, intuitionistic fuzzy fantastic filter and intuitionistic fuzzy positive implicative filter are shown. The present study would serve as a foundation for further study of the structure of minimal filters and enrich corresponding many-valued logical system. Future studies can include theoretical extensions driven by practical applications. In particular, on the one hand, regarding theory, extensions can be made in the mathematical lattice of fuzzy numbers based on interval α -cuts. On the other hand, regarding practical applications, machine intelligence might be pursued; in mobile/humanoid robot applications, planning by reasoning that accommodates uncertainty can be done.

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