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Interference of a Single Photon in the Mach-Zehnder Interferometer

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Abstract: The problem of interference of a single particle in the two path experiment is considered by giving an example of the Mach-Zehnder interferometer with a single photon. Interference of the photon in the Mach-Zehnder interferometer is defined by the wave function of the interferometer, not by the wave function of the photon, while the photon travel one of the pathes at random.

Keywords: two path experiment, Mach-Zehnder interferometer

1 Introduction

In quantum mechanics [1], a single particle interferes with itself in the two path experiment. Several interpretations were developed to tackle the problem, e.g. [2,3,4] and references therein. According to the standard interpretation [5], two components of the particle travel two pathes. The superposition of the two components under recombination entails the interference of the particle. The statement that two components of a single particle travel two pathes is not verifiable. The measurement on one of the pathes always reveals the whole particle, not the component.

There are two options in the two path experiment, interference of two pathes or detection of the particle on one of the pathes. In the latter case, one gets which way information of the particle. In the former case, there is no which way information of the particle. The two options are incompatible in one and the same experiment. The problem can be described in terms of the Bohr's complementarity principle [6]. Which way information termed as distinguishability (D) and interference termed as visibility (V) obey the relationship [7]

$$D^2 + V^2 \leq 1. \tag{1}$$

Bohr's complementarity signifies impossibility to have both which way information, $D = 1$, and interference, $V = 1$.

We have the facts to be explained. The first is the interference of the particle due to the superposition of two pathes. The second is that the particle can be detected as a whole, not as two components traveling two pathes. An explanation was suggested in [8], assuming that the interference of the particle is caused by the superposition state of the apparatus, not by the state of the particle.

Motivated by the interpretation of the two path experiment [8], the scheme of the weak measurement within the Mach-Zehnder interferometer was suggested [9] in which the measurement after the interferometer gives which way information while the disturbance of the interferometer is negligible due to the weak measurement. Also, the scheme of the two-slit experiment with two screens was designed [10] to study the problem of interference of a single photon in the classical configuration.

In the present paper, we shall consider the problem and give some reasoning to substantiate the assumption suggested in [8]. We shall study the problem by giving an example of the Mach-Zehnder interferometer, being a variant of the two path experiment.

2 Mach-Zehnder interferometer with a single photon

Consider the two path experiment with a single photon. We shall take the Mach-Zehnder interferometer as a variant of the two path experiment. The scheme of the Mach-Zehnder interferometer is depicted in Fig. 1. Consider the work of the interferometer with a single

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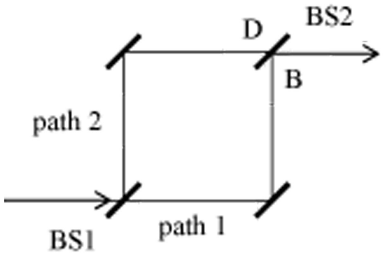

Fig. 1: Mach-Zehnder interferometer. BS1, BS2: beam splitter 1, 2; B: bright port; D: dark port.

photon. According to the standard interpretation [5], the incoming photon is split into two components at the first 50/50 beam splitter. The state of the photon is described by the superposition of two pathes

$$|\psi\rangle = (|1\rangle + |2\rangle)/\sqrt{2}. \quad (2)$$

The two components travel path 1 and 2, recombining at the second 50/50 beam splitter. When blocking the pathes of the interferometer, one can detect the photon at path 1 or 2 at random, with equal probability 50%.

Let the momentum of the photon be $p = \hbar k$ where $\hbar$ is the Planck constant, k is the wave vector. The wave functions of the two components of the photon traveling path 1 and 2 are $e^{ipr_1/\hbar}$ and $e^{ipr_2/\hbar}$ respectively. The interferometer is tuned such that the pathes 1 and 2 be of the same length. In this case, the phase difference of two pathes is equal zero, $r_2 - r_1 = 0$. Every reflection of the photon from the beam splitter or mirror adds a phase shift of $\pi/2$ to the wave function of the photon. The superposition of two pathes at the second beam splitter produces an interference of the photon. When the photon goes out of the second beam splitter in the right direction, the photon is reflected 2 times both on path 1 and 2. The phase difference of two pathes is equal zero thus giving the constructive interference. When the photon goes out of the second beam splitter in the up direction, the photon is reflected 1 time on path 1 and 3 times on path 2. The phase difference of two pathes in the up direction is equal π thus giving the destructive interference. Therefore, the photon leaves the second beam splitter through the bright port in the right direction with the unity probability and through the dark port in the up direction with the null probability.

3 Interpretation of the two path experiment

An explanation of the two path experiment was suggested [8] in which the particle is described by the pure state and the superposition state is attributed to the apparatus. According to the interpretation [8], interference of the particle is caused by the superposition state of the apparatus, not by the state of the particle. In what follows, we shall give some reasoning to substantiate the treatment of the two path experiment suggested in [8] by giving an example of the Mach-Zehnder interferometer.

According to the Bohr's complementarity principle, interference at the second beam splitter is conditioned on the indistinguishability of the pathes 1 and 2. The momenta of the photons traveling path 1 and 2 are of the different directions that is in conflict with the indistinguishability of the pathes 1 and 2. From this one can infer that the interferometer is not able to distinguish the momenta of the photons traveling path 1 and 2.

In the standard quantum mechanics [5], interaction of the particle and the apparatus can be described by

$$e^{-iH\tau/\hbar}(|\psi_p(r,t)\rangle|\psi_a(r,t)\rangle) \quad (3)$$

where H is the Hamiltonian of the interaction, τ is the time of the interaction, $|\psi_p(r,t)\rangle$ is the state of the particle, $|\psi_a(r,t)\rangle$ is the state of the apparatus. The interaction of the particle and the apparatus puts the state of the apparatus into correspondence to the state of the particle. In the standard quantum mechanics, time is a classical parameter, e.g. [11] and references therein. The problem of quantization of time was discussed in several works, e.g. [12]. An approach to the quantization of space and time in the model of the photon was considered in [13].

Following [13] consider interaction of the photon and the apparatus in the model of the photon with quantum space and time. Consider the photon of the momentum $p = \hbar k$ and the energy $E = \hbar\omega$ where ω is the frequency of the photon. Energy and momentum are orthogonal variables, as well as time and space coordinates. Therefore, interaction of the photon and the apparatus in the energy-time and momentum-space domains should be described by the orthogonal operators. Let the photon be originally in some state in space $|\psi_{p0}(r)\rangle$ and time $|\psi_{p0}(t)\rangle$, and the apparatus in some state in space $|\psi_{a0}(r)\rangle$ and time $|\psi_{a0}(t)\rangle$. Interaction of the photon and the apparatus in the momentum-space domain can be described by

$$e^{-ipr/\hbar}(|\psi_{p0}(r)\rangle|\psi_{a0}(r)\rangle) = |\psi_p(r)\rangle|\psi_a(r)\rangle. \quad (4)$$

Interaction of the photon and the apparatus in the energy-time domain can be described by

$$e^{-iE\tau/\hbar}(|\psi_{p0}(t)\rangle|\psi_{a0}(t)\rangle) = |\psi_p(t)\rangle|\psi_a(t)\rangle. \quad (5)$$

After the measurement, the apparatus goes over into the states in space and time correlating with the corresponding states of the photon in space and time. The correlations between the states of the photon and apparatus in space and time signify that the apparatus handles the photon of the momentum p and energy E.

Heisenberg uncertainty relations in the momentum-space domain

$$\Delta r \Delta p \geq \frac{\hbar}{2} \tag{6}$$

and in the energy-time domain

$$\Delta t \Delta E \geq \frac{\hbar}{2} \tag{7}$$

restrict the measurement of the photon by the apparatus. The Heisenberg uncertainty relation eq. (6) restricts the measurement of the photon of the momentum p at the time t. For the uncertainty in momentum $\Delta p = 0$, the uncertainty in the space coordinate is $\Delta r = \infty$. This means that the space coordinate is uncertain, and the wave functions in space can be defined as $e^{ipr/\hbar}$. The Heisenberg uncertainty relation eq. (7) restricts the measurement of the photon of the energy E at the space coordinate r. For the uncertainty in energy $\Delta E = 0$, the uncertainty in the time coordinate is $\Delta t = \infty$. This means that the time coordinate is uncertain, and the wave functions of the photon and apparatus can be defined in time as $e^{iEt/\hbar}$. The wave functions in space and time are limited only by the experimental conditions.

Consider interaction of the photon and the second beam splitter of the Mach-Zehnder interferometer described by eqs. (4,5). Due to the constraints imposed by the Heisenberg uncertainty relations eqs. (6,7), one cannot reveal the way of the photon within the interferometer. The second beam splitter is unaware of the way the photon took, path 1 or path 2. One can localize the state of the photon through the coordinates in space and time associated with the second beam splitter. It can be described by means of the δ-functions in space and time as $\psi_p(r_a) = \langle \delta(r_a)|\psi_p(r)\rangle$ and $\psi_p(t_a) = \langle \delta(t_a)|\psi_p(t)\rangle$ respectively. When handling the photon of the momentum p, the space coordinate is uncertain thus the state of the interferometer (second beam splitter) in space can be taken as a superposition of path 1 and 2. In this case, the state of the interferometer in space describes the possibilities provided to the photon by the interferometer. The total state of the system of the photon and interferometer in space is given by

$$|\Psi(r)\rangle = |\psi_p(r)\rangle(|\psi_{a1}(r)\rangle + |\psi_{a2}(r)\rangle)/\sqrt{2}. \tag{8}$$

Assume that the second beam splitter is controlled by the wave function of the interferometer in space. The superposition of two possible pathes of the photon makes the second beam splitter open the bright port and close the dark port. The photon thereby goes out of the second beam splitter through the open port. The way the photon take after the second beam splitter is defined by the wave function of the interferometer, not by the wave function of the photon. The superposition state of a single photon appears to be redundant. It is reasonable to think that the photon travel path 1 or 2 at random.

As follows from the foregoing reasoning, the photon after the first beam splitter can travel path 1 or 2 at random that is described by the state of the photon in space, $|\psi_{p1}(r)\rangle$ or $|\psi_{p2}(r)\rangle$. Interaction of the photon with the mirrors at pathes 1 and 2 described by eqs. (4,5) establishes the correlation between the states of the photon and mirrors in space and time. The mirror at one of the pathes say path 1 reflects the photon of the momentum p and energy E, and the other mirror at path 2 does not. The states of the mirror in space and time at path 1 correlates with the states of the photon in space and time, and the states of the mirror in space and time at path 2 do not. The total state of the system of the photon and interferometer in space is given by

$$|\Psi(r)\rangle = |\psi_{p1}(r)\rangle|\psi_{a1}(r)\rangle. \tag{9}$$

If there is no transfer of the momentum and energy of the photon to the mirror, one cannot reveal the way of the photon. If an absorber makes the projective measurement at one of the pathes say path 1, the momentum and energy of the photon is transferred to the absorber, and one can reveal the way of the photon.

One cannot use the projective measurement within the interferometer to have both interference and which way information as follows from the Bohr's complementarity principle [6]. A way to circumvent the limitation due to the Bohr's complementarity is to use the weak measurement [14]. The scheme on the basis of the Mach-Zehnder interferometer was suggested [9] in which the weak measurement occurs within the interferometer, and the projective measurement after the interferometer. In the weak measurement, the photon acquires a small transverse momentum which does not noticeably affect the interference at the second beam splitter. In the projective measurement at a distance from the interferometer, on can reveal the deviation of the photon's trajectory due to the transverse momentum of the photon. Thus, one has interference at the second beam splitter and which way information after the interferometer. By applying the momentum conservation, one can restore the photon's trajectory within the interferometer. The scheme allows to test the foregoing interpretation of the two path experiment in indirect way.

4 Conclusion

We have considered the problem of interference of a single particle in the two path experiment and given some reasoning to substantiate the treatment of the two path experiment suggested in [8]. We have studied the problem for the photon by giving an example of the Mach-Zehnder interferometer, being a variant of the two path experiment. According to the Bohr's complementarity principle, interference of a single photon in the Mach-Zehnder interferometer is conditioned on the indistinguishability of the pathes 1 and 2 while the momenta of the photons traveling path 1 and 2 are of the different directions. A way to resolve the contradiction is to assume that the second beam splitter of the interferometer is not able to distinguish the momenta of the photons. Due to the constraints imposed by the Heisenberg uncertainty relations, the second beam splitter is unaware of the photon's way within the interferometer. The wave function of the interferometer is formed as a superposition of two possible pathes of the photon. The superposition state of the interferometer gives the command to open the bright port of the second beam splitter and close the dark port. Interference of the photon is defined by the wave function of the interferometer, not by the wave function of the photon, while the photon travel path 1 or 2 at random.

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