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## Mathematical Modeling and Quantitative Feedback design for Industrial Nonlinear Storage Tank Level Control (NSTLC) Process

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**Abstract:** Industrial Nonlinear control (NSTLC) process is one of the most important and challenging tasks in all industrial processes, where the water or liquids takes place. Due to the cross sectional area of spherical tank, the automatic level control is difficult with respect to its various desired range of operating points. In this paper, the mathematical model of spherical tank has been obtained by conducting open loop experimental test. Based on the transfer function and the state space model of the tank, the step response analysis has been taken into account. Model validation has been analyzed with respect to process dynamics. Then after the proposed Fuzzy-Quantitative Feedback Theory (F-QFT) tuning of P+I+D scheme has been introduced to control the level of the spherical tank effectively using satisfied time domain specifications (zero offset, minimum ISE, minimum peak overshoot, quick settling etc...) The simulation and experimental results have been obtained based on servo (set point change) and regulatory (load change) operating conditions.

Keywords: Spherical tank,NSTLC,Fuzzy,F-QFT, PID,System Identification, ISE and IAE

## **1** Introduction

In process industries, the multi stage-drying and evaporation processes done with nonlinear spherical tanks because of maintaining differential pressure inside the process tank. The control system design should maximize the product quality, minimize the power consumption. The model of the spherical tank has been obtained using either mass and energy balance equations or system identification methods (black box, white box and grey box). The final expression of plant model may be in the form of transfer function model with transportation delay. Optimizing model parameters (gain, delay time and time constant) yields better output response under servo and regulatory operating conditions. The design criteria for choosing model parameters are error performance indices.

## 1.1 Conventional control schemes

Ziegler Nichols (1942) proposed the tuning methodology [3] for identifying tuning parameters of P, PI and PID

controllers based on open loop-time domain and continuous cycling method (closed loop-frequency domain). Cohen-Coon (1953) introduced empirical formula for calculating tuning parameters of P, PI and PID controllers for controlling First Order with time delay (FODT) processes. This method provides, the step response of all the process exhibit sigmoidal ('S'-shape) curves. Later, Astrom-Hagglund (1984) proposed closed loop tuning of PID controller. In this method, the ultimate gain and time period was calculated from sustained oscillatory response of the process output. R.Anandanatarajan and M.chidamparam et al. (2006) [2] introduced automatic online gain scheduling of PID controller. This work clearly indicates the effectiveness of gain scheduling while process variables are under processing. Vijayan-Panda (2012) proposed double (inner and outer) feedback loop structure for obtaining better stability and output performance. In this method the outer loop for set point tracking purpose and inner loop for stabilizing the process output. Then S.P.Selvaraj and A.Nirmalkumar (2015) introduced GA for online tuning

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of PI controller parameters. This method tunes the controller parameters with various operating conditions.

### 2 Mathematical Modelling of Spherical Tank

The mathematical model represents the physical systems and its dynamics with respect to time and frequency domain[9] [10] [16]. In this paper, the spherical tank model was obtained using first principle and transfer function model. This implies complete information about all inputs and outputs of process plant.

2.1 Modelling based on Mass Balance Equation



Fig. 1: Spherical Tank outline view with Specifications

Fig.1 shows outline view of spherical tank. Based on mass balance theory, the total accumulation of tank is equal to the difference between input and output flow rate of the process tank[1]. From figure.1, following mass balance equations are obtained.

$$F_{in} - F_{out} = AH = V = \frac{4}{3}\pi R^3 \tag{1}$$

where,  $A = 4\pi R^2$  and R = h

$$\frac{H}{h} = \frac{R}{r} = \gg R = \frac{rh}{H} \tag{2}$$

$$\frac{dV}{dt} = A\frac{dH}{dt} = F_{in} - F_{out} \tag{3}$$

where,  $F_{out} = \frac{h}{R_v} = C\sqrt{h} and F_{in} = A\frac{dH}{dt} + \frac{h}{R_v}$ 

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$$F_{in}(s) = AsH(s) + \frac{H(s)}{R_{\nu}}$$
(4)

$$\frac{H(s)}{F_{in}(s)} = \frac{R_v}{1 + AR_v s} = \frac{K}{1 + \tau s}$$
(5)

where,  $K = R_v, \tau = AR_v, R_v = \frac{\sqrt{h}}{C} = valve \ constant$ . H=actual level of liquid. The equation (5) shows the

H=actual level of liquid. The equation (5) shows the transfer function of spherical tank system with two varying factor K and  $\tau$ .

# 2.2 System Identification from Experimental Data

From equation (5), based on open loop test, the unknown value of gain K and time constant  $\tau$  are identified in order to get the transfer function of spherical tank [17] based on the open loop experimental data, the process reaction curve-based system identification procedure has been followed and the values of K and  $\tau$  are calculated [7]. The time domain data has been observed with each 500 milliseconds periodic intervals by means of increasing measurement accuracy. The specifications of process plant has been shown in Table.2.



Fig. 2: Process and Instrumentation layout for Spherical Tank level control process station

Fig.2 shows the proposed system "Process and Instrumentation layout" If the open loop test has been conducted for 0 to 43cm then the control performance of the entire range of system has been improved. But it is difficult to conduct such large number of experimental tests. There are several experimental tests carried out and the two different types of transfer function derived (for 0 to 27.5cm, 10 to 16.3cm and 0 to 43cm). The suitable



models are validated for entire operating range of spherical tank by interfacing with PC through LabVIEW DAQ. The experimental data are given in results and discussion section.

## **3 Fuzzy-Quantitative Feedback PID** Controller Design

Figure 3 shows the process flow chart of Quantitative feedback control for automatic gain scheduling PID controller[2] [4] [5]. The classical feedback theory is a frequency synthesis engineering approach to design effective robust controllers for both linear and nonlinear processes. This approach reduces the effect of process model uncertainty and explicitly uses the closed loop feedback control strategy. In this paper, fuzzy and Quantitative-feedback controller has been introduced to control the level of spherical tank effectively with minimized offset error [4][17][16]. The controller design of the system has considered frequency constraints [17] of the Nicholas chart and the controller parameters.[8]. The automatic loop shaping procedure has been introduced to tune the system parameters effectively with respect to its internal and external disturbances are being considered. The conventional PID controller[11] [13] output expression is given as,

$$C(S) = K_C \left( 1 + \frac{1}{\tau_i S} + \tau_D S \right) \tag{6}$$

The QFT design typically involves three basic steps and it is given below. 1.QFT bounds computation and fuzzy controller design for automatic loop shaping 2.Controller design with appropriate pre-filter 3. Design analysis. In QFT, the continuous plant is described as,

$$P = \left\{ P(S) = \frac{ka}{s(s+a)} : k \in [1, 10], a \in [1, 10] \right\}$$
(7)

The specification with 50-degree phase margin is given as,

$$\left|\frac{PG}{1+PG}(j\omega)\right| \le W_s = 1.2 forall \ P \in P\omega \in [0,\infty] \quad (8)$$

The plant with parametric and non-parametric[15] representation is given as,

$$P = \left\{ P\left(S\right) = \frac{ka}{s\left(s+a\right)} \left(1 + \Delta_{m}s\right) \right\}$$
(9)

where,

 $k \in [1, 10], a \in [1, 10], |\Delta_m(j\omega)| < R_m(\omega), \Delta_m(s)$ stable

#### 3.1 Automatic Loop shaping Methodology

The fuzzy logic controller includes four fundamental stages: inference mechanism with fuzzy logic rules and

de-fuzzification[1][5]. There are two contributions to the fuzzy controllers: absolute error (e(t)) and change in error (de(t)). The scopes of these sources of input are from 0 to 1, which are from the total estimations of system error and its subordinates based on scale factors. The triangle membership-functions (MFs) are used for each input variable. Fuzzification is the process of associating crisp, or numerical, input values with the linguistic terms of the corresponding input linguistic variables. For example, a fuzzy controller might associate the level reading from a Differential Pressure Transmitter (DPT) with the linguistic terms cold, moderate, and hot for the current temperature linguistic variable. Depending on the membership functions for the linguistic terms, the temperature value might correspond to one or more of the linguistic terms. The fuzzy logic controller[18] uses the following equation to calculate the geometric centre of the full area under the scaled membership functions.

$$mCOA = \frac{\int f(x) . x dx}{\int f(x) dx}$$
(10)

Where, mCOA is the modified centre of area. The interval of integration is between the minimum membership function value and the maximum membership function value<sup>[12]</sup>. The membership editing is done for every input and output variables. Figure. 5.2 represents membership function for the input variable, error (e). The membership editing for the second input variable, change in error (ce). In figure 3. The manual loop shaping is replaced by fuzzyfied automatic loop shaping for designing robust PID tuning mechanism[19]. Fig.3 shows the flow chart of proposed fuzzy rules which describes the relationships between input and output linguistic variables based on their linguistic terms. For example, you might define the following rule: IF current level is high AND desired level is low, THEN control valve setting is slightly closed position. The clauses current temperature is cold and desired temperature is moderate are the antecedents of this rule. The AND connective specifies how the fuzzy logic controller relates the two antecedents to determine the truth value for the aggregated rule antecedent. The control valve setting is close is the consequent of this rule. A rule base is the set of rules for a fuzzy system.

## **4 Results and Discussions**

From figure.4, the model parameters of time constant(tau), delay time(td) and process gain(K)has been obtained.The First Order Process with Time Delay (FOPTD) transfer function model[12] given as,

$$G(s) = \frac{9.6e^{-7s}}{1291s + 1} \tag{11}$$

Equation 13 has been obtained for level of 0 to 27.5cm range. From figure.5, the transfer function for 10 to 16 cm



Fig. 3: Process flow chart for FQFT based Robust controller design



Fig. 4: Open loop test of spherical tank for 0 to 27.5cm range

level range is given as,

$$G(s) = \frac{8.15e^{-6.1s}}{1065s + 1} \tag{12}$$



Fig. 5: Open loop test of spherical tank for 10 to 16cm range

Figure.6 shows the Ziegler Nichols PID controller output



Fig. 6: ZN-PID output Response for set point 12cm

response for the set point of 12cm. It is observed that there is large damping occurred while controlling the level of the process tank. Figure.7 shows the fuzzy



**Fig. 7:** Fuzzy Quantitative Feedback PID controller output Response for set point 12cm

quantitative feedback PID controller output response for the set point of 12cm. It is observed that there is lower damping occurred while controlling the process variable as well as quick settling time and closely zero offset error





**Fig. 8:** Software and Hardware implementation using LabVIEW DAQ (Block Diagram view)



Fig. 9: Comparisons of ZN-PID and FQFT-PID with Servo and Regulatory

when compared to ZN PID controller. Figure.8 shows the interface with LabVIEW for spherical tank level control and Fig.9 shows the comparison output response for ZN and FQFT PID tuning. From the experimental results it is evident that the FQFT based PID controller output response provides optimum set-point tracking capabilities and closely null offset error with minimum ISE. In Annexure, Table.1 indicates the quantitative comparison values.

## **5** Conclusion

From the experimental results, it is evident that the proposed Fuzzy-Quantitative feedback on-line tuning of PID controller parameter provides better regulatory and servo response with minimum ISE and IAE when compared to conventional Ziegler Nichols tuning techniques. In conventional method, it is difficult to conduct 43 open-loop tests for controlling level at 0 to 43cm but this proposed methodology provides the optimum way to select the model of the process for entire operating ranges. The proposed tuning method gives the exact and optimum parameter values of the PI controller with minimum value of IAE and ISE for various load and set point changes when compared to ZN tuning techniques. With good set-point tracking ability, the

proposed controller has been adapted for non-linear tank level control schemes in order to improve product quality and safety in process industries with 25 percentage improved accuracy when compared to conventional PID controller. The experimental analysis of controller has been validated. [20].

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## Annexure

Table 1: Comparison of ZN-PID and FQFT-PID with ISE					
Controller	Gain value (Kc)	Integral time(min)	ISE in Servo response	ISE in Regulatory response	
F-QFT-PID	14.151	0.321	4.57	5.66	
ZN PID	22.121	0.251	6.05	10.78	

Table 2: Devices and Field Instruments Description

Devices/Field Instruments	Details		
Nonlinear Spherical Tank	Material: Stainless Steel, Diameter: 43 cm, (LRV= 436 mmH2O, URV=866 mmH2O, Volume: 42 litres		
VFD and Pump	VFD: ABB-ACS350, 3phase 4- 20 mA to 0 to 50 Hz. Pump: Grundfos-JP5 centrifugal pump, 3phase.		
DPT for level and flow	6200T		
measurement	Series, Range:0 to 6500 mmH2O, Output: 4 to 20mA+HART		
Control valves types	Linear, Air to open, Body:1", Trim1/2"		
Rotameter	150 to 1500 lph		
I/P converter	Input: - 20 mA, 25 psi Output: 3 to 15 psi		
LabVIEW NI USB 6211 DAQ	Analog input: 8, Analog output: 2, Resolution: 16 bits, Sampling rate: 250kS/s & input and output voltage: -10V to +10V		