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# Production, Capacity and Workforce Planning: A Mathematical Model Approach 

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#### Abstract

In the era of globalization, integrated planning for production, workforce, and capacity is the key factor for attaining success in any industry. This paper provides a mathematical model to determine the best possible combination of required capacity, workforce, and lot size for a multi-product, multi-stage, and multi-model production system. The model provides a blueprint of a detailed workforce distribution plan to optimize the available capacity and determine the ideal lot size based on the ways (i.e. tray/trolley) of handling material to control the space required for setup and floor area. This model uses linear programming to reduce the manufacturing cost. A real-world numerical illustration proves the competency of this model, which - for an industry - serves as a practical guide to achieve the best combination of capacity, workforce, and lot size.


Keywords: Mathematical model, linear programming, one-piece flow, capacity planning, workforce planning.

## 1 Introduction

Since the beginning of the 21st century, many industries has only on the means to reduce the production cost. Later, instead of using a dedicated line, manufacturing industries used the concept of multi-product in multi-model production line to enhance other resources and the equipment level. The main objective of the multi-model line is to reduce both production and setup cost. In a multi-model production line, different products (i.e. A, B, C, etc.) pass through various process cells/stages/machines (i.e. main and sub-assembly) that have a mix of dedicated and shared resource facility (Figure 1). A majority of computational and practical concepts of production and capacity-planning factors are independent [1]. However, in real-world production planning, factors of workforce flexibility and machine flexibility impact capacity and utilization. Similarly, capacity-planning factor for forecast demand, overtime, and under time is significantly influenced by the production-planning elements of production cost, inventory-holding cost, back-order cost, lead time, and delivery time. Integrated planning to reduce cost has not been adopted in many real-world approaches. Thus for many manufacturers, integrating strategies of production
and other related operations appear to be a prerequisite for ensuring performance excellence.

The research scope of integration is still vast as many researchers have proposed different concepts for production capacity planning and workforce planning models separately for the objective function of minimizing the overall production cost in the Production Planning Control (PPC) framework. Most of the production capacity planning and workforce planning models are deterministic and use either Linear Programming (LP), Integer Programming (IP), or Mixed Integer LP (MILP) [2-12]. Optimization approaches based on pure simulation or simulation-based experimental designs propose and consider the uncertainties associated with production capacity planning in the PPC framework [13-16]. Capacity and workforce planning problems considered are dynamic in nature [17, 18] and heuristics [19].

Multi-criterion Production Capacity and Workforce Planning (PCWFP) models with objectives of cycle time minimization, the number of setup/setup time minimization, profit maximization, overtime minimization, and maximizing resource utilization, along with cost minimization have been also resolved [20,21]. Very few authors suggested that handling the capacity problems is oversimplified using the non-traditional

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Fig. 1: Multi-Product, Multi-Stage, and Multi-Model with Shared Resource Facility
search techniques such as genetic algorithm, Simulated Annealing (SA), tabu search, swarm intelligence, and pattern search [22]. These methods mostly use CPLEX, LINGO, LINDO, MATLAB (i.e. MATrix LABoratory), C/C++ programming language. The main purpose of search techniques is the multi-directional search to find the global optimum.

In this paper, we develop a PCWFP model to distribute the optimal workforce for performing a set of tasks with the main objective function of the manufacturing cost minimization in an automobile manufacturing industry with multi-period, multi-stage, and multi-product system. Many worthwhile thoughts have been developed on lean production. The remaining paper is organized as follows. Section 2 provides a detailed literature review on the model development and techniques based on capacity-workforce planning optimization. Section 3 deals with the problem description and assumptions made for the model development. The mathematical model and details of constraints are provided in Section 4. Section 5 describes practical implementation of the model and managerial insights in detail.

## 2 Literature Review

The workforce planning is one of the most complex managerial tasks in the PPC network when calculating the capacity and aggregate planning. It becomes more complex when capacity is the most sensitive factor to the product mix and process sequence, and new bottleneck tools would enter the system as the demand changes due to uncertainties. Hopp et al. [23] clearly explained the relationship between strategic PPC tasks of the forecast, capacity plan, and workforce plan, which formed the basis of the aggregate plan to achieve the determined
production. In real-world problem, there is a gap in the calculation of workforce planning factors of workforce assignment and workforce utilization with overtime and hiring/firing being the crucial elements in the performance of a manufacturing system. Therefore, the workforce planning is a productive pathway to continue further study. Many research articles have been published on the PCWFP; with various capacity and demand fluctuations [2, 4, 6, 11, 16, 19, 24-33]. Mathematical model and programming techniques have been used extensively for solving capacity-planning problems compared to static-, simulation-, and queuing-based analysis [26].

Few studies have solved capacity-workforce-type problem with the advanced mathematical tools such as goal programming, dynamic programming, non-LP, and various search techniques. Leung et al. [21] address the aggregate production planning problem with diverse functioning constraints, including workforce level, production capacity, factory locations, machine utilization, storage space, and other resource limitations, which are solved by a pre-emptive goal. Chen [8] mainly focused on the product mix problem and order selection based on the optimal set of work/customer to maximize the operational profit over a planning horizon with mixed IP (MIP) using CPLEX solver. Lot-sizing and sequencing of multiple products on capacity-constrained resources is the significant decision in the capacitated-lot-sizing problem [34] formulated by the MILP. Ryu [35] proposes an integrated planning framework for production capacity and supply chain capacity formulated by the MILP. Mula et al. [36] proposed a fuzzy mathematical programming model for the capacity and material requirement planning problem. Lee et al. [37] considered the decomposition approach for solving the capacity problem in the flexible assembly system. Kim and Uzsoy [19] represented models for capacity planning problem that clearly distinguish the relationship between capability levels and functional performance criteria such as work in progress and throughput. The problem is solved using two constructive heuristic and extent of the Lagrangian heuristic to resolve the multi-stage problem. Hsu and Le [38] studied mixed integer non-linear programming model based on the SA for the capacity optimization problem to enhance the supply chain network in a high-tech manufacturing industry. Mokhtari et al. [31] produced an integrated model between production capacity planning and operational scheduling, and then a two-phase genetic algorithm approach for solving the crashing and sequencing functions.

Most of the above-mentioned paper deals with planning approaches used over the years, which are relevant to the proposed production capacity workforce and still have one or more following limitations:

[^1]single-stage/assembly with multi-product, but rarely discussed multi-stage/assembly with multi-product and multi-period system. In addition, solving real-life problems involves using various production-associated variables (i.e. production capacity, product flow, volume, lot size, machine capacity, processing time, setup time, and line stoppage and maintenance downtime).
-However, many research models provide application potential, but research implementation in real-life problems is still lacking. Incorporation of production capacity, workforce, and its level of utilization and then lot-size determination based on the material handling size are rare decision variables for cost minimization.
We have developed the mathematical model for PCWFP for multi-model production line in various process types of an industry. Note that comparing the competence of different optimization technique is not in the scope of this paper.

## 3 Problem description and assumptions

Our aim is to develop a mathematical model that completely integrates production in regular time, contingency production of the overtime/holiday working (i.e. lay-off/hiring workforce), inventory, back-order, and setup cost of multi-product, multi-period stages of a real-world manufacturing situation. Thus, the optimization models that consider the objectives such as the change in capacity level maximize the shared resource utilization, but other than total cost minimization these models rarely solve detailed setup cost based on lot-size optimization. Such models are also oversimplified (i.e. not comprehensive to study multi-period, multi-stage, and multi-product system). Therefore there has been the need to develop a flexible mathematical model dealing with production capacity-workforce planning problems for fluctuated demand scenarios. Most real-world problems are non-linear to solve [39] and linear relationship between all variables does not always supply the best results; but too much of non-linear relationships makes the problem very difficult to solve.

To solve the model, first we define the indices, input variables (i.e. parameter), and decision variables, then provide the detailed problem formulation in Section 4.

## Indices

| $i:$ | Index for time periods $(i=1, \ldots, I)$ |
| :--- | :--- |
| $c:$ | Index for manufacturing cells $(c=1, \ldots, C)$ |
| $m:$ | Index for stations/machines $(m=1, \ldots, M)$ |
| $v:$ | Index for product varieties $(v=1, \ldots, V)$ |
| $x:$ | Index for production channel $(x=1,2$, and 3$)$ |
| If $x=1 \Rightarrow$ | Normal production channel (i.e. consider |
|  | 1st and 2nd shift) |
| If $x=2 \Rightarrow$ | Contingency-overtime production channel |

(i.e. consider 3rd shift)

If $x=3 \Rightarrow$ Contingency-lay-off production channel (i.e. Holiday shifts)
$n$ : Index for number of material handling $(n=1, \ldots, N)$

## Input variables

$F D_{i v}$ : Forecast demand
$C_{c, \max }$ : Maximum available capacity of all three shifts in cell $c$ (i.e. $C_{i v}+C_{i v}^{\prime}$ )
$C_{i v}: \quad$ Available (i.e. working day) capacity in all shifts during the period $i$ for product $v$
$C_{i v}^{\prime}: \quad$ Additional (i.e. holiday) capacity in all shifts during the period $i$ for product $v$
$C_{i}: \quad$ Capacity during the period $i$

## For production cost

$T$ : Total working hours/day (i.e. total number of shifts per day $\times$ working hours per shift)
$h$ : Total working time per shift
$S: \quad$ Total number of shifts
TOCt $_{c v}$ : Total operator cycle time
$B N t_{c v}$ : Bottleneck time
$W D_{i}$ : $\quad$ Number of working days
$H D_{i}$ : Number of holidays
$\gamma_{c}: \quad$ Overall Equipment Efficiency (i.e. OEE)
$\rho_{c}$ : $\quad$ Number of additional facilities (i.e. parallel)
$\partial_{i}: \quad$ Absenteeism and other delays
$O P_{c v}$ : Average productivity per man per shift
$W_{\text {icv,min }}$ : Minimum number of workforce
$W_{i c v, \max }$ : Maximum number of workforce (i.e. including holidays)
$W_{i}$ : $\quad$ Required workforce based on demand
$W_{i \partial} \quad$ Required workforce based on demand with delay
$C P_{c v x}$ : Production cost per unit channel $\times$ for product $v$ in cell $c$

## For setup cost

$\alpha_{c}$ : $\quad$ Material handling (i.e. container) size in cell $c$
$P t_{c v}$ : Processing time in cell $c$ for product $v$
$S t_{c v}: \quad$ Average setup time in cell $c$ for product $v$
$L t_{\left(n, \alpha_{c}\right)}$ : Current $n$ number of material handling operation time in cell $c$
$\operatorname{Ltrd}_{c v}$ : Operation time ratio in cell $c$
$C S_{v}$ : Average setup cost/unit time of cells for product $v$

## For inventory cost

$P U_{i c v}$ : Customer priority and uptime decision factor during the period $i$ in cell $c$ for product $v$ (i.e. $P U_{i c v} \leq 1$ )
$H_{i}$ : Inventory carrying cost of semi-finished product per unit in period $i$
$C I_{i v}$ : Inventory carrying cost of finished product per unit in period $i$ for product $v$
$N P_{v}$ : $\quad$ Number of partitions for each product $v$
$N V_{p}: \quad$ Number of products in each partition $p$
$C P_{c v}$ : Production cost per unit for product $v$ in cell $c$

## For back-order cost

$C B_{i v}$ : Average penalty cost per unit of unfulfilled demand in period $i$ for product $v$

## Decision variables:

## For production cost

$W_{i c 1}$ : Total workforce for normal production during the period $i$ in cell $c$
$W_{i c 2}$ : Total workforce for contingency overtime production during the period $i$ in cell $c$
$W F_{i c}$ : Total workforce for contingency lay-off production during the period $i$ in cell $c$ (i.e. lay-off/hiring workforce)
$P_{i c v x}$ : Total units produced through channel $x$ during the period $i$ in cell $c$ for product $v$

## For setup cost

$N_{c v}$ : $\quad$ Total quantity of materials handling in cell $c$ for product $v$
$\beta_{c v}$ : Lot-size in cell $c$ for product $v$
$S N_{i c v}$ : Number of setup during the period $i$ in cell $c$ for product $v$

## For inventory cost

$B t_{c v}$ : Batch processing time in cell $c$ for product $v$
$P R_{c v}$ : Production rate in cell $c$ for product $v$
$A P R_{i c v}$ : Average production rate in cell $c$ for product $v$
$W t_{i c v}$ : Waiting time during the period $i$ in cell $c$ for product $v$
$I_{i c v}: \quad$ Total finished product inventory during the period $i$ in cell $c$ for product $v$ (i.e. $c=1$ always)
$P A_{i v}$ : Number finished unit dispatched to customer dispatch center (CDC) during the period $i$ for product $v$

## For back-order cost

$B_{i v}: \quad$ Total unfulfilled demand in period $i$ for product $v$

## 4 Problem formulation

The main objective of our mathematical model is to find the best possible capacity required and lot-sizes at each production stage by minimizing the Total Manufacturing Cost (TMC). TMC includes production, setup, inventory-holding (including main and sub-assembly), and back-order cost. The motive of the objective function is to balance the production rates at successive production stages of multi-product through a concurrent adjustment of the capacity, workforce, and lot-size, and considering the trade-off between the capacity cost and savings in setup, inventory and back-order costs. We've used two following main novel approaches: (1) Distributing the workforce regarding normal, overtime, and lay-off production channels based on available capacity and


Fig. 2: The Decision Framework Flow of the PCWFP Model
required demand and (2) Reducing the production and setup cost by determining lot-size based on the floor space and the number of material handled. Figure 2 shows the decision framework flow of the PCWFP model. On a receipt of an order, the available capacity and the workforce level within available resource facility are checked to know when the required workforce level $\left(W_{i}\right)$ is less than or equal to usual production workforce ( $W_{i} \leq W_{i c 1}$ ), and whether the usual production channel is activated. However, when the required workforce level $\left(W_{i}\right)$ is greater than the normal production workforce (i.e. $W_{i}>W_{i c 1}$ ), the contingency overtime production channel would be activated along with the ordinary channel $\left(W_{i c 1}+W_{i c 2}\right)$. When the need to satisfy the upcoming high/uncertain demand from more than two channels of workforce levels $\left(W_{i}>\left(W_{i c 1}+W_{i c 2}\right)\right)$ arises, the contingency lay-off production channel would be
activated along with the other two channels ( $W_{i c 1}+W_{i c 2}+W F_{i c}$ ). The details of the objective function and constraints are provided in the following subsections.

### 4.1 The objective function

The objective function of total cost minimization is provided in Equation (1);
Total Cost $(Z)$ Minimization $=\{[$ Production Cost of (Normal channel + Contingency-overtime channel + Contingency-lay-off channel) ${ }^{a}+$ [Setup costt $]^{b}+$ [Inventory-Holding Cost (Main assembly ${ }^{c 1}$ + Sub-assembly $\left.\left.{ }^{c 2}\right)\right]+[$ Back-order Cost $\left.] d\right\}$

## $\operatorname{Min}_{Z}=$

$$
\left\{\begin{array}{l}
\left(\left[\sum_{x=1}^{3} \sum_{c=1}^{C} \sum_{v=1}^{V} \sum_{i=1}^{I} P_{i c v x} \times C P_{c v x}\right]\right)^{a}  \tag{1}\\
+\left(\left[\sum_{v=1}^{V} \sum_{c=1}^{C} \sum_{i=1}^{I} S N_{i c v} \times S t_{c v} \times C S_{c v}\right]\right)^{b} \\
+\left(\left[\sum_{v=1}^{V} \sum_{c=2}^{C-1} \sum_{i=1}^{I} S N_{i c v} \times W t_{i c v} \times \beta_{c v} \times C P_{c v} \times H_{i}\right]^{c_{1}}\right. \\
+\left[\sum_{v=1}^{\left.\left.c_{c=1} \sum_{c=1} \sum_{i=1}^{I} I_{i c v} \times C I_{i c v}\right]^{2}\right)}\right. \\
+\left(\left[\sum_{v=1}^{V} \sum_{c=1}^{I} \sum_{i=1}^{I} B_{i v} \times C B_{i v}\right]\right)^{d}
\end{array}\right\}
$$

where
$a=$ production cost of (normal channel + contingency-overtime channel + contingency-lay-off channel)
$b=$ setup cost
$c_{1}=$ inventory-holding cost (main assembly)
$c_{2}=$ inventory-holding cost (sub-assembly)
$d=$ Back-order Cost
The required product quantities produced during normal, overtime, and lay-off production are calculated using equations (2)-(4), respectively. Equation (2) calculates the number of normal production workforce, equation (3) calculates the number of overtime workforce in case of upcoming high demand/irregular order/priority changes, setting off the overtime channel. Equation (4) calculates the lay-off workforce whenever any delay such as machine breakdown, material delay, capacity shortage due to uncertain demand occurs in the process flow, activating the lay-off channel.

If $x=1$ (normal channel) $P_{i c v 1}=O P_{c v} \times W_{i c 1} \forall i, c$, and $v$

$$
\begin{align*}
& \text { If } x=2(\text { contingency-overtime channel }) \\
& \qquad P_{i c v 2}=O P_{c v} \times W_{i c 2} \forall i, c, \text { and } v \tag{3}
\end{align*}
$$

$$
\begin{align*}
& \text { If } x=3 \text { (contingency-lay-off channel) } \\
& \qquad P_{i c v 3}=O P_{c v} \times W F_{i c} \forall i, c, \text { and } v \tag{4}
\end{align*}
$$

Workforce planning: The maximum level of workforce design in three shifts, including of working on a holiday, required demand workforce, and additional delay during all the demand periods is calculated using equations (5)(8), respectively, based on efficient workforce scheduling [40].

$$
\begin{gather*}
W_{i c v, \text { min }}=\left[\left(\text { TOCt }_{c v} / B N t_{c v}\right) \times W D_{i} \times S \times \gamma_{c} \times \rho_{c}\right] \\
\forall i, c, \text { and } v  \tag{5}\\
W_{i c v, \max }=\left[\left(\text { TOCt }_{c v} / B N t_{c v}\right) \times\left(W D_{i}+H D_{i}\right)\right. \\
 \tag{6}\\
\left.\times S \times \gamma_{c} \times \rho_{c}\right] \forall i, c, \text { and } v  \tag{7}\\
W_{i}=F D_{i v} / O P_{c v} \forall i, c, \text { and } v  \tag{8}\\
W_{\partial i}=\left(W_{i}+\partial_{i} \times W_{i}\right) \forall i, c, \text { and } v
\end{gather*}
$$

The novel approach from equations (5) to (11) is a detailed workforce distribution plan based on the maximum available capacity and the expected demand. In general, demand increase results in immediate increases in the number of workforce to achieve the essential result; but after a certain level/point, even increasing the number of workforces does not achieve the desired output, and that level/point is called Workforce Break Even Point (WBEP). Figure 3 provides a detailed understanding of the WBEP. The chart represents increasing the number of workforce to reach the maximum level of cell/machines capacity of 1859 parts/day with 11 workforce/day; if the deployed workforce is more than the design workforce (i.e. 11 workforces/day), the output level (1859 parts/day) will be the same, that point is WBEP. Based on this strategy, a detailed workforce distribution plan is calculated from equations (9) to (11). Equation (9) calculates the normal production workforce and Equation (10) calculates the overtime production workforce from the total number of available shifts per day (i.e. decision-making process). Equation (11) relates to selecting contingency lay-off production workforce when the capacity and another shortfall occur as evident from equation (11), which shows that lay-off workforce is permitted only when the normal workforce is absent or
when workforce requirement is more than the maximum workforce for each cell and other reasons for delays. Nowadays industries, more particularly the automobile manufacturing industry, maintain a contingent workforce based on the hiring/firing/lay-off concept instead of permanent workforce [41] when the demand is uncertain. Workforce distribution is one of the challenging processes in a shared workforce and in an uncertain workforce environment [42, 43].


Fig. 3: Workforce Break Even Point (WBEP)

## For selecting normal production workforce:

If
$\operatorname{Max}\left(\left[2 / 3 \times W_{i c v, m i n}\right],\left[2 / 3 \times\left(W_{i}+\partial_{i} \times W_{i}\right)\right]\right)$
$\leq\left(W_{i}+\partial_{i} \times W_{i}\right)$
Select
$W_{i c 1}=\operatorname{Min}\left(\left[2 / 3 \times W_{i c v, \text { min }}\right],\left[2 / 3 \times\left(W_{i}+\partial_{i} \times W_{i}\right)\right]\right)$
Else If
$\operatorname{Max}\left(\left[2 / 3 \times W_{\text {icv,min }}\right],\left[2 / 3 \times\left(W_{i}+\partial_{i} \times W_{i}\right)\right]\right)$
$\leq 2 / 3 \times\left(W_{i}+\partial_{i} \times W_{i}\right)$
Select
$W_{i c 1}=\operatorname{Max}\left(\left[2 / 3 \times W_{i c v, m i n}\right],\left[2 / 3 \times\left(W_{i}+\partial_{i} \times W_{i}\right)\right]\right)$
Else

$$
\begin{equation*}
\left.\left.W_{i c 1}=\left(W_{i}+\partial_{i} \times W_{i}\right)\right]\right) \forall i, c, \text { and } v \tag{9}
\end{equation*}
$$

For selecting contingency overtime production workforce

$$
\begin{align*}
& W_{i c 2}= \operatorname{Min}( \\
& {\left.\left[1 / 3 \times W_{i c v, m i n}\right],\left[\left(W_{i}+\partial_{i} \times W_{i}\right)-W_{i c 1}\right]\right) }  \tag{10}\\
& \forall i, c, \text { and } v
\end{align*}
$$

For selecting contingency lay-off production workforce

$$
\begin{equation*}
W F_{i c} \leq \min \left(W C_{i} W B_{i}\right) \forall i, c, \text { and } v \tag{11}
\end{equation*}
$$

where

$$
\begin{aligned}
& W C_{i}=H D_{i} \times\left(\left(W_{i}+\partial_{i} \times W_{i}\right) / W D_{i}\right) \\
& W B_{i}=\left(W_{i}+\partial_{i} \times W_{i}\right)-W C_{i c 1}-W C_{i c 2}
\end{aligned}
$$

An illustrative example is presented for better understanding the workforce distribution (equations 9-11), decision variables, and condition. Here the primary underlying assumption is that $2 / 3$ of shifts/day considered normal production, $1 / 3$ of shifts/day is considered overtime production, and lay-off work is found on an available holiday time. Select the normal production workforce ( $W_{i c 1}$ ) value; if the maximum of $2 / 3$ design workforce or $2 / 3$ required demand workforce is less than or equal to the required workforce. Select value $W_{i c 1}$ if it is equal to minimum of $2 / 3$ design workforce or $2 / 3$ required demand workforce. Else, maximum of $2 / 3$ design workforce or $2 / 3$ required demand workforce should be less than or equal to the $2 / 3$ required demand workforce. Select the value $W_{i c 1}$ if it is equal to maximum of $2 / 3$ design workforce or $2 / 3$ required demand workforce, otherwise select the required demand workforce. The main objective is to select a workforce limit, within the available capacity and not beyond the WBEP. Similarly, we decide the level of the contingent overtime production workforce ( $W_{i c 2}$ ) and contingent lay-off production workforce $\left(W F_{i c}\right)$ from the available capacity and not beyond the WBEP.

Setup and lot-size: The need for setup time and setup reduction becomes increasingly important in production lines with an extensive product variant [44]. The benefit of controlling the setup time process is not only the increased production capacity without buying the new facility, but other significant positive impacts on quality, scrap and rework, inventory along with the system flexibility and responsiveness to the customer.


Fig. 4: Lot-Size $\left(B_{c v}\right)$ Yield Variability Based on Operation Time $\left(L t_{(n, \alpha c)}\right)$

Equations (12)-(16) employ a novel approach for estimating the number of setup from the lot-size. Lot-sizes are derived from equation (15); when we increase the lot-size, the operation time ratio differences
are significantly reduced (i.e. production-operation time). After attaining the optimal value (i.e. breakeven point) even increasing the lot-size has no/small impact on production-operation time [45]. From Figure 4 it can be observed that after 1.33 sec there is no greater impact on operation time. Thus, the optimal value of lot-size is 200 . Based on this strategy, the number of material-handling containers has been calculated by applying equation (14), the operation time ratio selection depends on production supervisor.

$$
\begin{equation*}
L t_{\left(n, \alpha_{c}\right)}=P t_{c v}+\left(S t_{c v} /\left(n \times \alpha_{c}\right)\right) \forall i, c, \text { and } v \tag{12}
\end{equation*}
$$

$$
\begin{gather*}
\operatorname{Ltrd}_{c v}=\left(L t_{\left(n, \alpha_{c}\right)}-L t_{\left(n+1, \alpha_{c}\right)}\right) / L t_{\left(n, \alpha_{c}\right)} \forall i, c, \text { and } v  \tag{13}\\
N_{c v}=\left\{\begin{array}{ll}
(n+1)_{c v} & \text { if } L t r d_{c v} \geq 4 \% \\
n_{c v} & \text { otherwise }
\end{array} \forall i, c, \text { and } v\right.  \tag{14}\\
\beta_{c v}=N_{c v} \times \alpha_{c} \forall i, c, \text { and } v  \tag{15}\\
S N_{i c v}=\sum_{v=1}^{V} \sum_{c=1}^{C} \sum_{i=1}^{I} F D_{i v} / \beta_{c v} \forall i, c, \text { and } v \tag{16}
\end{gather*}
$$

Adjustment of the workforce plan and output rate maximization based on bottleneck/non-bottleneck process plan is conceived while controlling and estimating the capacity planning [46]. Here the capacity planning of available capacity is computed from the bottleneck of each cell using equation (17). Additional capacity includes working on holiday and the maximum available capacity calculated, respectively, from equations (18) and (19).

$$
\begin{align*}
& C_{i v}=\min \left\{\left(\left[W D_{i} \times T \times \rho_{c}\right] / B N t_{c v}\right)_{c e l l s}\right\} \forall i, c, \text { and } v  \tag{17}\\
& C_{i v}^{\prime}=\min \left\{\left(\left[H D_{i} \times T \times \rho_{c}\right] / B N t_{c v}\right)_{c e l l s}\right\} \forall i, c, \text { and } v  \tag{18}\\
& \left.C_{c, \max }=\min \left\{\left(\left[W D_{i}+H D_{i}\right) \times T \times \rho_{c}\right] / B N t_{c v}\right)_{c e l l s}\right\} \\
& \forall i, c, \text { and } v \tag{19}
\end{align*}
$$

Inventory and unmet demand: Equations (20)-(25) are used for calculating the inventory and unmet demand for main and sub-assembly in each production stage during all the periods. Equation (20) calculates the processing time for a batch size. The production rate is inversely proportional to the processing time (21). In equation (22), average production rate and uptime decision factor mainly depend on the customer priority and product variety, respectively. Equation (23) is applied
for estimating the mean waiting time [47] for the production rate, the quantity of lot-size, and waiting time mainly used for semi-finished product/sub-assembly inventory value calculation. This type of problem is rarely solved and is especially meant for cellular-type manufacturing industry. Equation (24) deals with the total CDC capacity, while equation (25) is used for calculating the finished units dispatched to CDC based on available/customer-defined CDC capacity. In equation (26), the inventory units equal the divergence between the initial stock, a total number of finished goods (i.e. main assembly), and the total number of goods sent to CDC from the main assembly during each period. Here we assume that the initial inventory equals the available capacity $\left(C_{i v}\right)$. Equation (27) calculates the unmet demand (i.e. back-order) found by subtracting the initial back-order, total number of goods transported from the main assembly to CDC, and forecast demand.

$$
\begin{equation*}
B t_{c v}=\beta_{c v} \times T O C t_{c v} \forall i, c, \text { and } v \tag{20}
\end{equation*}
$$

$$
\begin{equation*}
P R_{c v}=1 / B t_{c v} \forall i, c, \text { and } v \tag{21}
\end{equation*}
$$

$$
\begin{equation*}
A P R_{i c v}=P R_{c v} \times P U_{i c v} \forall i, c, \text { and } v \tag{22}
\end{equation*}
$$

$$
\begin{gather*}
\sum_{v=1}^{V} \sum_{c=2}^{C-1} \sum_{i=1}^{I} W t_{i c v}=\frac{A P R_{i c v}}{A P R_{i(c+1) v} \times\left(A P R_{i(c+1) v}-A P R_{i c v}\right)} \\
\forall i, c, \text { and } v  \tag{23}\\
C D C_{-} i n v=N P_{v} \times N V_{p} \forall i, c, \text { and } v  \tag{24}\\
P A_{i v}=C D C \_i n v \times W D_{i} \forall i, c, \text { and } v  \tag{25}\\
\sum_{v=1}^{V} \sum_{c=1} \sum_{i=1}^{I} I_{i c v}=\left[I_{i-1 v}+\left[P_{i v 1}+P_{i v 2}+P_{i v 3}\right]\right. \\
\left.-\left[\sum_{v=1}^{V} \sum_{i=1}^{I} P A_{i v} \times W D_{i}\right]\right] \forall i, c, \text { and } v  \tag{26}\\
\sum_{v=1}^{V} \sum_{c=1} \sum_{i=1}^{I} B_{i v}=\left[B_{i-1 v}+F D_{i v}-\left(\sum_{v=1}^{V} \sum_{i=1}^{I} P A_{i v} \times W D_{i}\right)\right] \\
\forall i, c, \text { and } v \tag{27}
\end{gather*}
$$

### 4.2 The constraints

In the real-world scenario, production capacity-workforce planning problems face several constraints related to production or recourses shortfall (e.g. cost, labour,
production capacity, and floor constraint), so this section presents the formulations for some real-world constraints.

Linear equality/inequality constraints: Equation (28) provides means to control the deployed normal, contingency overtime, and lay-off production workforce level that should be within the maximum design working day workforce level. The number of materials handled is controlled by the factory floor space constraint. It depends on the required demand and management decision on the lot-size determination as expressed in equation (29). The setup process optimization constraints are added for controlling the setup time and lot-size (equation 30). Each lot-size operation time is greater than the setup time of each lot-size for reducing/eliminating the machine idle time arising out of large setup time. This constraint is optimal when setup time is more than normal hours [48]. Equations (31) and (32) are used for determining inventory control (i.e. production quantity should be less than the forecast demand) and waiting time between the cells, respectively

$$
\begin{gather*}
W_{i c 1}+W_{i c 2}+W F_{i c} \leq W_{i c v, m i n} \forall i, c, \text { and } v  \tag{28}\\
N_{i c v} \leq 7 \forall i, c, \text { and } v  \tag{29}\\
\beta_{i c v} * T O C t_{c v}>S t_{i c v} \forall i, c, \text { and } v  \tag{30}\\
\sum_{v=1}^{V} \sum_{c=1}^{C} \sum_{i=1}^{I} S N_{i c v} \times \beta_{i c v} \leq F D_{i c v} \forall i, c, \text { and } v  \tag{31}\\
\sum_{v=1}^{V} \sum_{c=1}^{C} \sum_{i=1}^{I} W t_{i c v} \leq \sum_{v=1}^{V} \sum_{i=1}^{I} W t_{i v} \forall i, c, \text { and } v \tag{32}
\end{gather*}
$$

Bound constraints and non-negative constraints: These constraints are used for obtaining faster and more reliable solutions. Equations (33)-(36) are bound constraints used for restricting the decision variables. For example, equation (33) is used for controlling the normal, overtime, and lay-off workforce within the minimum and maximum level of design workforce.

$$
\begin{equation*}
W_{i c v, \min } \leq W_{i c 1}+W_{i c 2}+W F_{i c} \leq W_{i c v, \max } \forall i, c, \text { and } v \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
0 \leq S N_{i c v} \leq F D_{i c v} / \beta_{i c v} \forall i, c, \text { and } v \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
W t_{i c, \text { min }} \leq W t_{i v} \leq W t_{i c, \text { max }} \forall i, c, \text { and } v \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
W_{i c 1}, W_{i c 2}, W F_{i c}, S N_{i c v} \geq 0 \forall i, c \text {, and } v \tag{36}
\end{equation*}
$$

## 5 Practical implementation and managerial insights

The previous section indicated the importance of the integrated PCWFP model, problem formulation, and constraints. In this section, the problem is solved by the LP methodology and coded in the MATLAB. The result focuses on minimizing the overall manufacturing cost. Therefore, the proposed PCWFP model provides a balance between normal production and contingency production. Inventory and back-order level consider both the assembly and sub-assembly of the production line. The lot-size depends on the number of setup and the number of material handling size (i.e. space constraint). A great deal of workforce level is required for all manufacturing stages. One of the main approaches is representing the bound constraints for capacity expansion processes when the demand is more than available capacity. The most significant performance of the recommended model is to evaluate and minimize the yield variability of lot-size (Figure 4), batch size, processing time, setup time, line stoppage time, and maintenance downtime in order to maximize the organization operations, leading to minimization in the manufacturing price.

Real-world data have been collected on auto-electrical parts manufacturing industry, accommodating multi-model, multi-product, and multi-stage Cellular Manufacturing System (CMS) with JIT (i.e. Just In Time) environment under volatile market conditions. The proposed mathematical model aims to bring down the manufacturing cost and helps the decision maker of the company in efficient allotment of workforce in various sub-assembly levels for several customer requirements considering available facilities. The model concurrently generates the production capacity-workforce plan to oversee the resources available at all the levels and effectively controls the lot-size.

### 5.1 Input Variables

The demand pattern for all the product varieties, the number of planned production days for 12 periods (months), input variables (i.e. parameter), and average setup cost/unit time are shown in Tables 1-4, respectively. It should be noted that the value of zero for any input variable means that the concerned process is not suitable for a particular variety of product. For example, Table 3 shows that variety 1 does not pass through processes 4 and 19.

Table 1: Demand Pattern

| Period, $i$ | Product Variety, $v$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 1 | 19712 | 7414 | 6336 | 22572 | 6952 | 4994 | 7414 | 12166 | 1540 | 1892 | 6094 | 7106 | 154 | 418 |
| 2 | 20608 | 7751 | 6624 | 23598 | 7268 | 5221 | 7751 | 12719 | 1610 | 1978 | 6371 | 7429 | 161 | 437 |
| 3 | 23296 | 8762 | 7488 | 26676 | 8216 | 5902 | 8762 | 14378 | 1820 | 2236 | 7202 | 8398 | 182 | 494 |
| 4 | 22400 | 8425 | 7200 | 25650 | 7900 | 5675 | 8425 | 13825 | 1750 | 2150 | 6925 | 8075 | 175 | 475 |
| 5 | 21504 | 8088 | 6912 | 24624 | 7584 | 5448 | 8088 | 13272 | 1680 | 2064 | 6648 | 7752 | 168 | 456 |
| 6 | 22598 | 8500 | 7264 | 25876 | 7970 | 5725 | 8500 | 13947 | 1766 | 2169 | 6986 | 8147 | 177 | 480 |
| 7 | 16800 | 6319 | 5400 | 19238 | 5925 | 4257 | 6319 | 10369 | 1313 | 1613 | 5194 | 6057 | 132 | 357 |
| 8 | 17472 | 6572 | 5616 | 20007 | 6162 | 4427 | 6572 | 10784 | 1365 | 1677 | 5402 | 6299 | 137 | 371 |
| 9 | 21728 | 8173 | 6984 | 24881 | 7663 | 5505 | 8173 | 13411 | 1698 | 2086 | 6718 | 7833 | 170 | 461 |
| 10 | 16948 | 6375 | 5448 | 19407 | 5978 | 4295 | 6375 | 10461 | 1325 | 1627 | 5240 | 6111 | 133 | 360 |
| 11 | 19990 | 7519 | 6426 | 22891 | 7050 | 5065 | 7519 | 12338 | 1562 | 1919 | 6180 | 7207 | 157 | 424 |
| 12 | 20268 | 7623 | 6515 | 23209 | 7148 | 5135 | 7623 | 12509 | 1584 | 1946 | 6266 | 7307 | 159 | 430 |

Table 2: Number of Planned Production Days

| Planning Period, $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $W D_{i}$ | 22 | 23 | 26 | 25 | 26 | 24 | 25 | 26 | 25 | 24 | 24 | 25 |
| $H D_{i}$ | 8 | 8 | 4 | 6 | 5 | 6 | 6 | 4 | 6 | 7 | 4 | 6 |

Table 3: Input Variables (i.e. parameter) to the Model

| Cell, $c$ | Product variety, $v=1$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $O P_{c v}$ | $\gamma_{c}$ | $\rho_{c}$ | $T O C t_{c v}$ | $B N t_{c v}$ | $S t_{c v}$ | $\alpha_{c}$ | $C P \_c v 1$ | $C P_{\_} c v 2$ | $C P \_c v 3$ |
| 1 | 35 | 0.75 | 1.88 | 12.29 | 1.12 | 20 | 36 | 138.2 | 161.7 | 183.8 |
| 2 | 373 | 0.70 | 0.75 | 1.15 | 0.45 | 90 | 120 | 13.0 | 15.2 | 17.3 |
| 3 | 919 | 0.70 | 0.56 | 0.47 | 0.40 | 22 | 120 | 5.3 | 6.2 | 7.0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0 | 0.0 | 0.0 |
| 5 | 1470 | 0.70 | 0.38 | 0.29 | 0.25 | 15 | 120 | 3.3 | 3.9 | 4.4 |
| 6 | 1050 | 0.70 | 0.75 | 0.41 | 0.35 | 30 | 120 | 4.6 | 5.4 | 6.1 |
| 7 | 290 | 0.70 | 0.40 | 1.48 | 0.33 | 23 | 120 | 16.7 | 19.5 | 22.2 |
| 8 | 2205 | 0.70 | 0.38 | 0.20 | 0.17 | 32 | 120 | 2.2 | 2.6 | 2.9 |
| 9 | 551 | 0.70 | 0.94 | 0.78 | 0.67 | 15 | 48 | 8.8 | 10.3 | 11.7 |
| 10 | 401 | 0.70 | 1.00 | 1.07 | 0.92 | 25 | 48 | 12.1 | 14.1 | 16.0 |
| 11 | 298 | 0.70 | 0.94 | 1.44 | 1.23 | 20 | 48 | 16.2 | 19.0 | 21.6 |
| 12 | 848 | 0.70 | 0.94 | 0.51 | 0.43 | 20 | 48 | 5.7 | 6.7 | 7.6 |
| 13 | 67 | 0.70 | 1.13 | 6.42 | 0.72 | 15 | 48 | 72.2 | 84.5 | 96.0 |
| 14 | 490 | 0.70 | 1.29 | 0.88 | 0.87 | 10 | 48 | 9.9 | 11.6 | 13.1 |
| 15 | 538 | 0.70 | 0.75 | 0.80 | 0.68 | 10 | 48 | 9.0 | 10.5 | 12.0 |
| 16 | 441 | 0.70 | 1.32 | 0.98 | 0.83 | 30 | 48 | 11.0 | 12.8 | 14.6 |
| 17 | 2005 | 0.80 | 0.38 | 0.21 | 0.35 | 15 | 120 | 2.4 | 2.8 | 3.2 |
| 18 | 1696 | 0.80 | 0.38 | 0.25 | 0.22 | 15 | 120 | 2.9 | 3.3 | 3.8 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0 | 0.0 | 0.0 |
| 20 | 171 | 0.80 | 0.94 | 2.51 | 0.85 | 13 | 40 | 28.3 | 33.1 | 37.6 |
| 21 | 1161 | 0.80 | 0.38 | 0.37 | 0.32 | 15 | 40 | 4.2 | 4.9 | 5.5 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0 | 0.0 | 0.0 |
| 23 | 52 | 0.80 | 1.13 | 8.27 | 1.18 | 35 | 40 | 93.0 | 108.9 | 123.7 |

### 5.2 Computational results and discussion

The problem using MATLAB-R2013a is iterated using Intel(R) Core (TM) i3-5005U CPU $2.00 \mathrm{GHz}, 4 \mathrm{~GB}$ RAM
computer. The detailed production-workforce distributed plan (i.e. $W_{i c 1}, W_{i c 2}$, and $W F_{i c}$ ) and the number of setup based on lot-size (i.e. $S N_{c v}$ ) for the best output solution generated using LP for the 14th cell are shown in Tables

Table 4: Average Setup Cost/Unit Time/Variety

| Product <br> Variety, $v$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C S_{v}$ | 11.3 | 9.8 | 17.6 | 13.5 | 28.5 | 10.4 | 11.7 | 18.2 | 35.7 | 16.7 | 22.5 | 15.2 | 31.8 | 15.0 |

Table 5: Normal Production - Workforce Distributed Plan $\left(W_{i c 1}\right)$

| Period, $i$ | Product Variety, $v$ for cell $c=14$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 1 | 27.62 | 0 | 10.26 | 46.99 | 10.82 | 0 | 11.01 | 17.05 | 2.4 | 3.06 | 0 | 0 | 0 | 0 |
| 2 | 28.88 | 0 | 10.73 | 49.12 | 11.32 | 0 | 11.51 | 17.82 | 2.51 | 3.2 | 10.96 | 13.9 | 0.3 | 0.75 |
| 3 | 32.65 | 0 | 12.13 | 55.53 | 12.79 | 0 | 13.01 | 20.15 | 2.83 | 3.62 | 12.38 | 15.71 | 0.34 | 0.85 |
| 4 | 31.39 | 0 | 11.66 | 53.39 | 12.3 | 0 | 12.51 | 19.37 | 2.72 | 3.48 | 11.91 | 15.11 | 0.32 | 0.81 |
| 5 | 44.76 | 0 | 11.19 | 51.26 | 11.81 | 0 | 13.01 | 18.6 | 2.62 | 3.34 | 12.38 | 14.5 | 0.31 | 0.85 |
| 6 | 31.67 | 0 | 11.76 | 36.26 | 12.41 | 0 | 12.01 | 18.66 | 2.63 | 3.51 | 11.43 | 15.24 | 0.31 | 0.78 |
| 7 | 34.97 | 0 | 8.75 | 40.05 | 13.7 | 0 | 10.23 | 14.53 | 2.04 | 2.61 | 9.72 | 11.33 | 0.25 | 0.67 |
| 8 | 36.37 | 0 | 9.10 | 41.65 | 14.25 | 0 | 10.64 | 15.11 | 2.13 | 2.72 | 10.11 | 11.79 | 0.26 | 0.69 |
| 9 | 45.23 | 0 | 11.31 | 51.79 | 11.93 | 0 | 12.51 | 18.79 | 2.64 | 3.38 | 11.91 | 14.66 | 0.32 | 0.81 |
| 10 | 35.28 | 0 | 8.82 | 40.4 | 9.31 | 0 | 10.32 | 14.66 | 2.06 | 2.63 | 9.80 | 11.43 | 0.25 | 0.67 |
| 11 | 41.61 | 0 | 10.41 | 47.65 | 10.98 | 0 | 12.01 | 17.29 | 2.43 | 3.11 | 11.43 | 13.48 | 0.29 | 0.78 |
| 12 | 42.19 | 0 | 10.55 | 48.31 | 11.13 | 0 | 12.35 | 17.53 | 2.47 | 3.15 | 11.72 | 13.67 | 0.30 | 0.8 |

Table 6: Contingency Overtime Production - Workforce Distributed Plan ( $W_{i c 2}$ )

| Period, $i$ | Product Variety, $v$ for cell $c=14$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 1 | 13.41 | 0 | 4.98 | 0 | 5.25 | 0 | 5.5 | 8.28 | 1.16 | 1.49 | 0 | 0 | 0 | 0 |
| 2 | 14.02 | 0 | 5.21 | 0 | 5.49 | 0 | 5.75 | 8.65 | 1.22 | 1.56 | 5.48 | 6.75 | 0.15 | 0.37 |
| 3 | 15.85 | 0 | 5.89 | 0 | 6.21 | 0 | 6.5 | 9.78 | 1.38 | 1.76 | 6.19 | 7.63 | 0.17 | 0.42 |
| 4 | 15.24 | 0 | 5.66 | 0 | 5.97 | 0 | 6.25 | 9.4 | 1.32 | 1.69 | 5.95 | 7.33 | 0.16 | 0.41 |
| 5 | 0 | 0 | 5.43 | 0 | 5.73 | 0 | 6.45 | 9.03 | 1.27 | 1.62 | 6.09 | 7.04 | 0.15 | 0.42 |
| 6 | 15.37 | 0 | 5.71 | 17.6 | 6.02 | 0 | 6 | 9.33 | 1.32 | 1.71 | 5.72 | 7.4 | 0.16 | 0.39 |
| 7 | 0 | 0 | 4.25 | 0 | 0 | 0 | 4.97 | 7.05 | 0.99 | 1.27 | 4.72 | 5.5 | 0.12 | 0.32 |
| 8 | 0 | 0 | 4.42 | 0 | 0 | 0 | 5.17 | 7.34 | 1.03 | 1.32 | 4.91 | 5.72 | 0.12 | 0.34 |
| 9 | 0 | 0 | 5.49 | 0 | 5.79 | 0 | 6.25 | 9.12 | 1.28 | 1.64 | 5.95 | 7.11 | 0.15 | 0.41 |
| 10 | 0 | 0 | 4.28 | 0 | 4.52 | 0 | 5.01 | 7.12 | 1 | 1.28 | 4.76 | 5.55 | 0.12 | 0.33 |
| 11 | 0 | 0 | 5.05 | 0 | 5.33 | 0 | 6 | 8.39 | 1.18 | 1.51 | 5.72 | 6.55 | 0.14 | 0.39 |
| 12 | 0 | 0 | 5.12 | 0 | 5.4 | 0 | 5.99 | 8.51 | 1.2 | 1.53 | 5.69 | 6.64 | 0.14 | 0.39 |

5-7, respectively. Similarly, we've got the final decision variables for all the cells (i.e. 23 cells), from that values we've derived the main function of TMC. Here the normal, overtime and lay-off workforce distribution plan are derived from the WBEP, such that workforce breakeven point is considered as maximum limit of the workforce allocation.

The PCWFP model tends to prefer the normal production workforce $\left(W_{i c 1}\right)$ and then contingency overtime production $\left(W_{i c 1}\right)$ when the required workforce $\left(W_{i}\right)$ is more than both channels $\left(W_{i c 1}+W_{i c 1}\right)$, and when only the contingency lay-off production channel is activated (i.e. Tables 5-7). Due to this, the laying-off workers reduce expenses. The number of setup is (i.e. Table 8) also controlled based on the required demand,
according to the available floor space and material handling size (equations 12-16). Table 9 shows the level of the main assembly (i.e. finished product) inventory and unmet demand (i.e. back-order) across all the periods. The level of unmet demand is zero in the 7th and 8th period. Even in the other period the unmet demand is very less. Therefore, the penalty cost and the loss are reduced to the maximum extent due to the unmet demand.

As mentioned, a problem has 12 periods, 14 product varieties, and 23 sub-assembly cells, thus each decision factor handles the $12 \times 14 \times 23$ variables for every iteration. The integrity constraints of the mathematical model are mentioned in subsection 4.2; constraints are relaxed and the problem makes some of the assumptions referred in Section 3 to find the globally optimal solution.

Table 7: Contingency Lay-off Production - Workforce Distributed Plan $\left(W F_{i c}\right)$

| Period, $i$ | Product Variety, $v$ for cell $c=14$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1.33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1.39 | 0 | 0 | 0 | 1.27 | 0 | 0 | 0.09 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1.57 | 0 | 0 | 0 | 1.44 | 0 | 0 | 0.1 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 1.51 | 0 | 0 | 0 | 1.38 | 0 | 0 | 0.1 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 2.44 | 1.04 | 0.14 | 0 | 2.27 | 0 | 0.02 | 0.16 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0.9 | 0 | 0 | 0 | 0.81 | 0 | 0 | 0.06 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0.08 | 0 | 0 | 0 | 0.03 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table 8: Number of Setup Based on Lot-Size (i.e. $S N_{c v}$ )

| Period, $i$ | Product Variety, $v$ for cell $c=14$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 1 | 58.67 | 0 | 18.86 | 67.18 | 20.69 | 0 | 22.07 | 36.21 | 4.58 | 5.63 | 0 | 0 | 0 | 0 |
| 2 | 61.33 | 0 | 19.71 | 70.23 | 21.63 | 0 | 23.07 | 37.85 | 4.79 | 5.89 | 18.96 | 22.11 | 0.48 | 1.3 |
| 3 | 69.33 | 0 | 22.29 | 79.39 | 24.45 | 0 | 26.08 | 42.79 | 5.42 | 6.65 | 21.43 | 24.99 | 0.54 | 1.47 |
| 4 | 66.67 | 0 | 21.43 | 76.34 | 23.51 | 0 | 25.07 | 41.15 | 5.21 | 6.4 | 20.61 | 24.03 | 0.52 | 1.41 |
| 5 | 64 | 0 | 20.57 | 73.29 | 22.57 | 0 | 24.07 | 39.5 | 5 | 6.14 | 19.79 | 23.07 | 0.5 | 1.36 |
| 6 | 67.26 | 0 | 21.62 | 77.01 | 23.72 | 0 | 25.3 | 41.51 | 5.26 | 6.46 | 20.79 | 24.25 | 0.53 | 1.43 |
| 7 | 50 | 0 | 16.07 | 57.26 | 17.63 | 0 | 18.81 | 30.86 | 3.91 | 4.8 | 15.46 | 18.03 | 0.39 | 1.06 |
| 8 | 52 | 0 | 16.71 | 59.54 | 18.34 | 0 | 19.56 | 32.1 | 4.06 | 4.99 | 16.08 | 18.75 | 0.41 | 1.1 |
| 9 | 64.67 | 0 | 20.79 | 74.05 | 22.81 | 0 | 24.32 | 39.91 | 5.05 | 6.21 | 19.99 | 23.31 | 0.51 | 1.37 |
| 10 | 50.44 | 0 | 16.21 | 57.76 | 17.79 | 0 | 18.97 | 31.13 | 3.94 | 4.84 | 15.6 | 18.19 | 0.4 | 1.07 |
| 11 | 59.49 | 0 | 19.13 | 68.13 | 20.98 | 0 | 22.38 | 36.72 | 4.65 | 5.71 | 18.39 | 21.45 | 0.47 | 1.26 |
| 12 | 60.32 | 0 | 19.39 | 69.07 | 21.27 | 0 | 22.69 | 37.23 | 4.71 | 5.79 | 18.65 | 21.75 | 0.47 | 1.28 |

Table 9: Total Inventory, Unmet Demand for Main Assembly from the LP Solutions

| Period, $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $I_{i}$ | 14268 | 32927 | 30646 | 25362 | 34582 | 33766 | 18482 | 35493 | 34582 | 32241 | 35636 | 35617 |
| $B_{i}$ | 2226 | 2328 | 2631 | 2530 | 1679 | 3011 | 0 | 0 | 2173 | 8 | 1625 | 1397 |

The problem has non-integer decision variables by default. The problem is also converged to a global optimum solution of TMC for all the cells, INR1592965258.6833. After adding the bound constraints, the TMC reduced from an initial solution and the final optimal solution is INR1570726175.2189, detailed manufacturing cost breakup for all the cells is shown in Table 10. Having added the bound constraints and one of the primary notifications of the solution, when the required capacity is beyond the design capacity (i.e. capacity expansion), the setup constraint (equation 31) gives the near-optimal global solution comparable to workforce constraint (equation 28); since the setup and setup cost control the other cost factors of inventory and waiting for time variables [49].

## 6 Conclusion and future research

LP-based PCWFP distribution model has evolved and considered three different types of production costs (regular, overtime, and lay-off production cost) along with inventory-holding cost, unmet demand cost, and detailed setup cost. This statement is true when the temporary workforce is used instead of the permanent workforce, especially in the automobile manufacturing sector. The PCWFP model is a better way of handling all constraints, especially the setup and inventory constraints, to find the global optimal solution. The mathematical model is the most appropriate tool to solve the capacity planning-type problem. In addition WFBP is one of the best practical strategies for controlling the workforce

Table 10: Total Manufacturing Cost (TMC) - Cellwise From the LP Solutions

| Cell | Manufacturing Cost <br> (No bound constraints) | Manufacturing Cost <br> (With bound constraints) | Remarks |
| :---: | :---: | :---: | :---: |
| 1 | 307177980.0012 | 307177980.0012 | No change |
| 2 | 91529307.9333 | 80409766.2011 | The required capacity beyond the design capacity |
| 3 | 49788327.7843 | 49788327.7843 | No change |
| 4 | 74086680.7309 | 74086680.7309 | No change |
| 5 | 37295760.0927 | 37295760.0927 | No change |
| 6 | 8900220.8589 | 8900220.8589 | No change |
| 7 | 9961272.0959 | 9961272.0959 | No change |
| 8 | 6123388.3232 | 6123388.3232 | No change |
| 9 | 17036626.7647 | 17036626.7647 | No change |
| 10 | 3363123.8120 | 3363123.8120 | No change |
| 11 | 28156168.4969 | 17036626.7647 | The required capacity beyond the design capacity |
| 12 | 68229566.4577 | 68229566.4577 | No change |
| 13 | 130063719.9663 | 130063719.9663 | No change |
| 14 | 22549253.7934 | 22549253.7934 | No change |
| 15 | 87221325.1559 | 87221325.1559 | No change |
| 16 | 38568003.2724 | 38568003.2724 | No change |
| 17 | 7511956.5212 | 75111956.5212 | No change |
| 18 | 5426322.7832 | 5426322.7832 | No change |
| 19 | 295154081.7937 | 295154081.7937 | No change |
| 20 | 91779256.6446 | 91779256.6446 | No change |
| 21 | 8346477.3271 | 8346477.3271 | No change |
| 22 | 10388080.5918 | 10388080.5918 | No change |
| 23 | 121556472.5355 | 121556472.5355 | No change |
| Cost(I,B) | 5151884.9465 | 5151884.9465 | No change |
| TMC | 1592965258.6833 | 1570726175.2189 | Up to $1.40 \%$ Total manufacturing cost reduced |

distribution in a practical manner. Future studies may extend the proposed model by altering or adding to the following points. Some of the input variables ( $\gamma_{c}, \rho_{c}$, and $\alpha_{c}$ ) should be considered deterministic and known factors so that the model extension may be stochastic in nature [50]. Unfortunately, the significant key decision variables such as $W_{i c 1}, W_{i c 2}, W F_{i c}$, and $S N_{c v}$ assume decimal values, rounding off those variables leads to an infeasible solution and requires total recalculation of the variables. Thus currently, we have restricted the integer value for the significant key decision variables. Compared to linear value, a mixed integer value may provide the ideal solution for the PCWFP model. Further extension of this research to develop a meta-heuristic solution approach to solve the model with the focus on solving large-scale problems is under progress by an author. Here we have considered only linear relations as constraints, but most of the problems seen in practice are non-linear in nature [39]. The non-linear relationship must be added (i.e. inventory and setup cost constraints) among the constraints, maybe it would play a role for a strong PCWFP model.

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[^1]:    -Most of the suggested mathematical models consider single-stage/assembly with single-product, multi-stage/assembly with single-product, or

