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Predator-Two Preys Population Dynamics Model with Adaptive Management

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Abstract: In this paper, a model of predator-two preys system dynamics is considered. Within the framework of such model, it is assumed that preys are not competitors. The hypothesis that the simultaneous existence of three species excluding realization of regimes of mass propagation is tested. It is believed that elimination of one of prey populations leads to conditions when other population obtains the opportunity to escape from predator's control and to realize mass propagation. In this model, a nonlinear control law is taken into account. This variant of system management allows provision of effective control of population sizes. In particular, this control mechanism does not allow phytophagous escaping out of control of their enemies, and prey population sizes stabilize at lower values than it was observed before using control law. Thus, there is a principal possibility of organizing prey population sizes control by release of additional preys into the system and the control influence.

Keywords: Model of predator-two preys system dynamics, Routh-Hurwitz criterion, Escape-effect, Lyapunov function, Nonlinear control law

1 Introduction

One of basic problems of the evolution of insect populations is to explain how stabilization and adaptation to particular environmental conditions evolve through natural selection. Population can be presented as a group of single species which prefer to live together in the same location and has a unique physical adaptation and distribution in time and space [1]-[3]. The research and study of population dynamics focuses on these changes as why, how, and when they occur. In entomology, a clear understanding of population dynamics is useful for interpreting survey data, predicting pest outbreaks, and evaluating the effectiveness of control approaches [4]-[6].

Many researchers seek to explain the observation in nature fluctuations in the size of populations with the help of simple mathematical models of ecological system dynamics [23,5,6,19,2,15]. This is quite natural, since it is always advisable to construct possibly simpler models of real processes, greatly coarsening the phenomena studied, and only by eliminating the shortcomings of the coarsened models, to introduce their complication (Kostitzin, 1937 ; Maynard Smith, 1968, 1974; Lyapunov, Bagrinovskaya, 1975) [7]-[10].

This is one of aspects of the mathematical modeling development of ecological systems. On the other hand, there is a large amount of experimental data [11,12] on the most diverse biological populations, the nature of the variation in the numbers which the researchers seek to describe with the help of mathematical models (Berryman, 1992; Allen, 2010; Nedorezov, 2012). The basis for constructing theory of the dynamics of the population forest insects is the identification of regularities that emerges, which suggests a sharp increase in numbers, and the study of the driving mechanisms of this phenomenon [22]. A necessary stage in the formation of the theory should be the development of recommendations on the control of population sizes. They must be taken into account in different conditions and at different phases of the development of mass propagation.

In our publications [5,6,16] close attention was turn to the analysis of the trajectories behavior of the dynamical systems with an unexpected change in the initial values of the numbers in the model of phytophagous-entomophagous system dynamics

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[13]-[15]. It is observed when chemical and biological matters are used to reduce the number of phytophagous, and also when additional entomophagous are released into the system [23,24]. There is a number of papers in which the authors consider another method of controlling the number of pests, associated with the introduction of additional species of parasitoids into ecosystems (see, for example, [4,21,22]).

In this paper we compare two different models of ecosystem dynamics: one prey - two predator, and two preys - one predator. First of all, conditions of stability of systems are analyzed and basic features of observed stability are presented for considered cases. Obtained results demonstrate that addition of new population to ecosystem can be considered as potential regulator which leads to conditions of stabilization of levels of populations.

2 Model one prey – two predators

Let's consider dynamic model one prey - two predators:

$$\begin{aligned} \frac{dx}{dt} &= x\left(\alpha_1 - \beta_1 x - \frac{\gamma_1 z_1}{1+\pi_1 x^2} - \frac{\gamma_2 z_2}{1+\pi_2 x^2}\right), \\ \frac{dz_1}{dt} &= z_1\left(-\alpha_2 - \beta_2 z_1 + \frac{\gamma_3 x^2}{1+\pi_1 x^2} - \delta_1 z_1 z_2\right), \\ \frac{dz_2}{dt} &= z_2\left(-\alpha_3 - \beta_3 z_2 + \frac{\gamma_4 x^2}{1+\pi_2 x^2} - \delta_2 z_1 z_2\right), \end{aligned} \quad (1)$$

where $x(t)$, $z_1(t)$, $z_2(t)$ are numbers of preys, first and second predator populations respectively at time t;; α_j, β_j, γ_j, π_j, $\delta_j \equiv \text{const} > 0$ are parameters of the model (1). Within the framework of this model it is assumed that prey can escape of the control of every predator or both predators.

This model can be considered as generalization of our previous model of one predator system dynamics model which was obtained [21] in a result of modification of the model of predator – prey system dynamics with saturation-effect [12]. Analysis of this one prey – one-predator system dynamics model is presented in our papers [16,17,18]. In figure 1, there is an example of behavior of model (1) trajectories for following values of parameters:
$\alpha_1 = 15$, $\alpha_2 = 0.02$,$\alpha_3 = 12.55$,$\beta_1 = 14.6$, $\beta_2 = 12.3$, $\beta_3 = 19$, $\gamma_1 = 15$, $\gamma_2 = 16.61$, $\gamma_3 = 110.7$, $\gamma_4 = 12.4$, $\delta_1 = 120$, $\delta_2 = 8.19$

Figure 1 depicts scenarios of stabilization for model (1). We have obtained regimes with asymptotic stabilization of population sizes at any fixed levels. As it is pointed out above, stability of ecosystem can be increased in a result of release of new phytophagous species into system. It can be considered as appearance of new kind of food for predators which allows supporting predator's size on higher level.

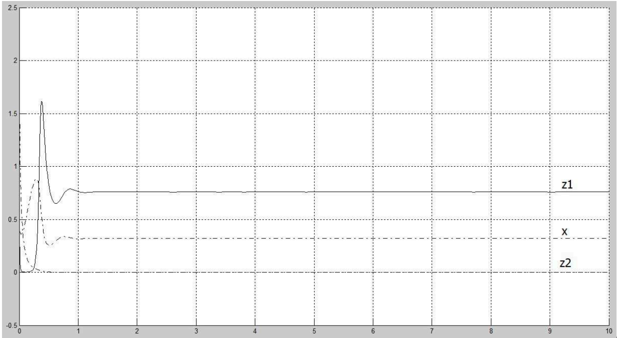

Fig. 1: Regime of asymptotic stabilization in model (1).

The study model shows that the prey eliminates from second predator Z2 in the system. Until the extinction of the second predator, the numbers of the prey and the first predator fluctuate.

Let's consider the following model of predator - two preys system dynamics with additional assumption that preys cannot compete inside:

$$\begin{aligned} \frac{dx}{dt} &= \lambda x\left(\alpha_1 - \beta_1 x - \frac{\gamma_1 z}{1+x^2}\right), \\ \frac{dy}{dt} &= \mu y\left(\alpha_2 - y - \gamma_2 z\right), \\ \frac{dz}{dt} &= z\left(-\alpha_3 - z + \frac{\gamma_3 x^2}{1+x^2} + \gamma_4 y\right). \end{aligned} \quad (2)$$

In this model $x(t)$, $y(t)$, $z(t)$ are numbers of first and second prey populations and predator respectively at time t;; α_j, β_j, $\gamma_j \equiv \text{const} > 0$ are parameters of the model (2).

3 Stationary states

System of nonlinear algebraic equations

$$z = \frac{(\alpha_1 - \beta_1 x)(1+x^2)}{\gamma_1} = \frac{\alpha_2 - y}{\gamma_2} = \frac{\gamma_3 x^2}{1+x^2} + \gamma_4 y - \alpha_3 \quad (3)$$

allows determination of coordinates of stationary points of the system (2) in positive part of phase space. System (3) can be transformed into equation

$$x^5 - \frac{\alpha_1}{\beta_1}x^4 + 2x^3 + x^2\left[\frac{\gamma_1\gamma_3 + \alpha_2\gamma_1\gamma_4 - \alpha_3\gamma_1 - 2\alpha_1 - 2\alpha_1\gamma_2\gamma_4}{\beta_1(1+\gamma_2\gamma_4)}\right] + \\ + x + \frac{\alpha_2\gamma_1\gamma_4 - \alpha_3\gamma_1 - \alpha_1 - \alpha_1\gamma_2\gamma_4}{\beta_1(1+\gamma_2\gamma_4)} = 0 \quad (4)$$

With the help of system (3) it is easy to obtain coordinates of stationary points in *int* $(X \cdot Z)$ where the escape-effect can be observed:

$$x^5 + \frac{\alpha_1}{\beta_1}x^4 + 2x^3 + x^2\frac{\gamma_1\gamma_3 - \gamma_1\alpha_3 - 2\alpha_1}{\beta_1} + x - \frac{\alpha_1 + \alpha_3\gamma_1}{\beta_1} = 0 \quad (5)$$

It is possible to find values of parameters (for polynomials (4) and (5)) when dynamic system (2) has three points in *int* $(X \cdot Z)$ and one point in int R_3^+. Let $\alpha_1 = 10,\ \gamma_1 = 15,\ \alpha_3 = 1,\ \gamma_3 = 10$. Then the number of points in *int* $(X \cdot Z)$ depends on the number of positive roots of the polynomial (5), which has the following form for pointed out parameters:

$$x^5 - 10x^4 + 2x^3 + 115x^2 + x - 25 = 0, \tag{6}$$

Polynomial (6) has three roots on the interval (0, 10). The number of points in int R_3^+ is determined by the polynomial (4):

$$\tilde{P}(x) = x^5 - 10x^4 + 2x^3 + x^2 \frac{115+15\alpha_2\gamma_4-20\gamma_2\gamma_4}{1+\gamma_2\gamma_4} + x - \\ - \frac{15\alpha_2\gamma_4-35-10\gamma_2\gamma_4}{1+\gamma_2\gamma_4} = 0. \tag{7}$$

Let's denote by K_1 and K_2 the following values:

$$K_1 = \frac{115+15\alpha_2\gamma_4-20\gamma_2\gamma_4}{1+\gamma_2\gamma_4},\ \ K_2 = -\frac{15\alpha_2\gamma_4-35-10\gamma_2\gamma_4}{1+\gamma_2\gamma_4}.$$

Obviously, for any fixed positive values of the parameters $\gamma_2$and γ_4 one can choose the parameter α_2 such that the polynomial (7) has one root on the interval (0, 10). If parameter α_2 is rather big and inequality $K_1 \geq 10^3$ is truthful then $10x^4 < K_1x^2$ on the interval (0, 10) and, consequently, the polynomial $\widetilde{P}$ increases monotonically on this interval. Since $K_2 < 0$ for sufficient large value of parameter α_2, $\widetilde{P}(0) < 0$ and $\widetilde{P}(10) > 0$, then polynomial (6) has one root only.

4 Particular case

Thus, the system of ordinary differential equations (particular case of the model (2))

$$\begin{aligned} \frac{dx}{dt} &= \lambda x\left(10 - x - \frac{15z}{1+x^2}\right) = \lambda x Q_1, \\ \frac{dy}{dt} &= \mu y(\alpha_2 - y - \gamma_2 z) = \mu y Q_2, \\ \frac{dz}{dt} &= \varepsilon z\left(-1 - z + \frac{10x^2}{1+x^2} + \gamma_4 y\right) = \varepsilon z Q_3 \end{aligned} \tag{8}$$

for certain values of the parameters α_2, γ_2 and γ_4 have one equilibrium state in *int* R_3^+. Let's consider stationary states of the system of equations (8). Point (0, 0, 0) is a saddle, with a single outgoing separatrix (z axis). The Jacobian of system (8) has the following form:

$$J = \left\| \begin{matrix} \lambda Q_1 + \lambda x\left(-1 - \frac{30xz}{(1+x^2)^2}\right) & 0 & -\frac{15\lambda x}{1+x^2} \\ 0 & \mu Q_2 - \mu y & -\mu\gamma_2 y \\ \frac{20\varepsilon xz}{(1+x^2)^2} & \varepsilon\gamma_4 z & \varepsilon Q_3 - \varepsilon z \end{matrix} \right\|.$$

Let $(\bar{x}, \bar{y}, \bar{z})$ be a stationary state of the system in *int* R_3^+. The Jacobian at this point has the form:

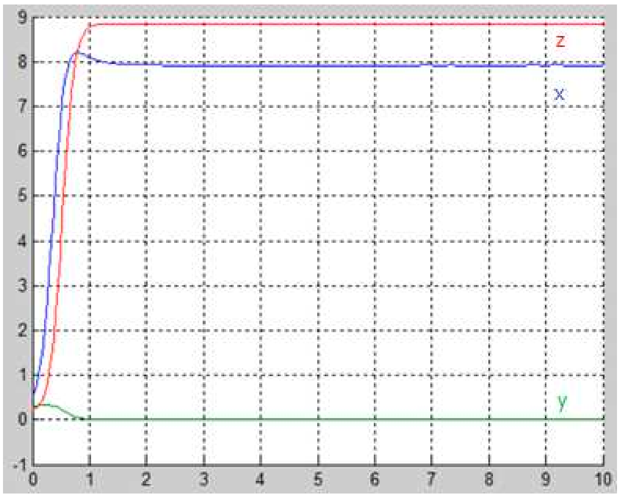

Fig. 2: Population dynamics for model (2) in case (2.a)

$$J = \left\| \begin{matrix} -\lambda\bar{x} - 30\lambda\frac{\bar{x}^2\bar{z}}{(1+\bar{x}^2)^2} & 0 & -\lambda\bar{x}\frac{15}{1+x^2} \\ 0 & -\mu\bar{y} & -\mu\gamma_2\bar{y} \\ 20\varepsilon\frac{\bar{x}\bar{z}}{(1+\bar{x}^2)^2} & \varepsilon\gamma_4\bar{z} & -\varepsilon\bar{z} \end{matrix} \right\|.$$

The eigenvalues of the Jacobian satisfy the following cubic equation:

$$-\xi^3 + A_1\xi^2 + A_2\xi + A_3 = 0, \tag{9}$$

where coefficients A_k depend on the parameters of the system:

$$\begin{aligned} A_1 &= -\lambda\bar{x} - 30\lambda\frac{\bar{x}^2\bar{z}}{(1+\bar{x}^2)^2} - \mu\bar{y} - \varepsilon\bar{z} < 0, \\ A_2 &= -\frac{300\lambda e\bar{x}^2\bar{z}}{(1+\bar{x}^2)^3} - \bar{y}\bar{z}\varepsilon\mu(1+\gamma_2\gamma_4) - \left(\lambda\bar{x} + 30\lambda\frac{\bar{x}^2\bar{z}}{(1+\bar{x}^2)^2}\right)(\mu\bar{y} + \varepsilon\bar{z}) < 0, \\ A_3 &= -\left(\lambda\bar{x} + 30\lambda\frac{\bar{x}^2\bar{z}}{(1+\bar{x}^2)^2}\right)(\varepsilon\mu\bar{z}\bar{y} + \varepsilon\mu\gamma_2\gamma_4\bar{z}\bar{y}) - 300\lambda\mu\varepsilon\frac{\bar{x}^2\bar{z}^2}{(1+\bar{x}^2)^3} < 0. \end{aligned}$$

Consequently, polynomial (9) has no positive roots, and by Descartes' theorem of signs it has either one or three negative real roots [19,20]. Numerical analysis allowed obtaining regimes with asymptotic stabilizations of population sizes at any fixed levels (these regimes are presented in Fig. 1).

In Fig. 2-3 we present an examples computed with these models with the analysis for following cases, blue, green and red colors of line means 1-3 questions:

(a) $l = 1; \alpha_1 = 10,\ \alpha_2 = 1, \alpha_3 = 1 \beta_1 = 1,$ $\gamma_1 = 15,\ \gamma_2 = 1,\ \gamma_3 = 10,\ \gamma_4 = 1$

(b) $l = 1; \alpha_1 = 17,\ \alpha_2 = 1.6, \alpha_3 = 5.5 \beta_1 = 1,$ $\gamma_1 = 45,\ \gamma_2 = 1.6,\ \gamma_3 = 10,\ \gamma_4 = 1$

Fig. 2-3 depict a population dynamics for model (2), the reflective properties of the structural stability on a large scale for various initial values are shown. In these figures we can analyze the optimal strategy for model (2). First, we observe an increase in the number of all populations, then the number of the second prey drops to zero, and the growth of the number of the first prey and the predator continues. After the extinction of the second

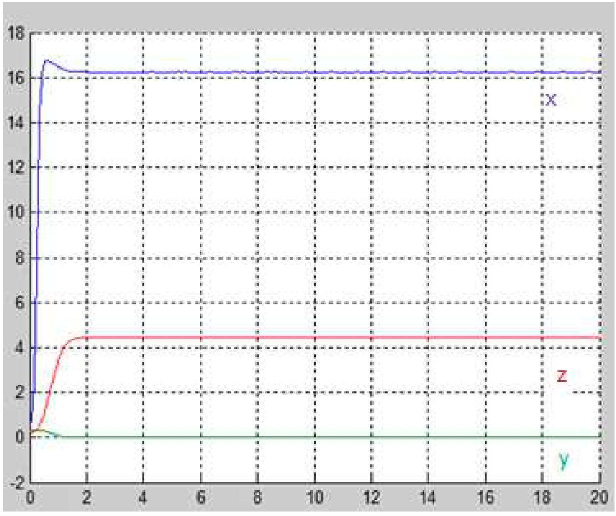

Fig. 3: Population dynamics for model (2) in case (2.b)

prey, the numbers of the first prey and the predator stabilize at a non-zero level.

Analysis of this system shows that the stabilization of the system is possible when the second phytophage (the prey) dies out.

Finally, we model experiments of various stability-related measures to compare the relative efficiency of these methods. Our results suggest that under most conditions, 3th population (presented as 2nd prey) as seem to be the optimal strategy. Thus, the simultaneous existence of three species can exclude regimes of mass multiplication. In this case, the degeneration of one population (y≡0) leads to the fact that the other gets the opportunity to escape from the control of enemies.

5 Analysis of model with nonlinear control law

Let's consider model of ecosystem dynamics with nonlinear control system which contains a control law in the form of one-parameter structurally stable mappings [24,25]:

$$\begin{aligned} \frac{dx}{dt} &= \lambda x \left(\alpha_1 - \beta_1 x - \frac{\gamma_1 z}{1+x^2}\right) - \left(x^3 + k_1 x\right), \\ \frac{dy}{dt} &= \mu y \left(\alpha_2 - y - \gamma_2 z\right) - \left(y^3 + k_1 y\right), \\ \frac{dz}{dt} &= z \left(-\alpha_3 - z + \frac{\gamma_3 x^2}{1+x^2} + \gamma_4 y\right) - \left(z^3 + k_1 z\right). \end{aligned} \tag{10}$$

A system has four equilibrium points:

$$X_{s1} = (0,0,0), X_{s2} = \left(\frac{\alpha_1}{\beta_1}, 0, 0\right),$$

$$X_{s3} = (0,\alpha_2,0), X_{s4} = (0,0,-\alpha_3)$$

Let's rewrite the system in terms of the components of the Lyapunov's function:

$$\begin{aligned} \frac{\partial V}{\partial x_1} &= x_1^3 + k_1 x_1 - \lambda \alpha_1 x_1 + \lambda \beta_1 x_1^2 + \frac{\lambda \gamma_1 x_1 x_3}{1+x_1^2}, \\ \frac{\partial V}{\partial x_2} &= x_2^3 + k_1 x_2 - \mu \alpha_2 x_2 + \mu x_2^2 + \mu \gamma_2 x_2 x_3, \\ \frac{\partial V}{\partial x_3} &= .x_3^3 + k_1 x_3 + \alpha_3 x_3 - \gamma_4 x_2 x_3 + x_3^2 - \frac{\gamma_3 x_1^2 x_3}{1+x_1^2}. \end{aligned}$$

Complete derivative of Lyapunov's function is given as follows:

$$\begin{aligned} W(x_1,x_2,x_3) = &- \left[\lambda \alpha_1 x_1 - \lambda \beta_1 x_1^2 - \frac{\lambda \gamma_1 x_1 x_3}{1+x_1^2} - x_1^3 - k_1 x_1\right]^2 \\ &- \left[\mu \alpha_2 x_2 - \mu x_2^2 - \mu \gamma_2 x_2 x_3 - x_2^3 - k_1 x_2\right]^2 - \\ &- \left[-\alpha_3 x_3 + \gamma_4 x_2 x_3 - x_3^2 + \frac{\gamma_3 x_1^2 x_3}{1+x_1^2} - x_3^3 - k_1 x_3\right]^2 \end{aligned}$$

A function $W(x_1,x_2,\ldots,x_n) \leq 0$. The Hessian has the following form:

For full derivative of Hessian is presented as follows:

$$H_{x_s=0} = \begin{Vmatrix} -2(\lambda\alpha_1 - k_1)^2 & 0 & 2(\lambda\alpha_1 - k_1) \\ 0 & -2(\mu\alpha_2 - k_1)^2 & 0 \\ 2(\lambda\alpha_1 - k_1) & 0 & -2(\alpha_3 - k_1)^2 \end{Vmatrix}$$

Thus, for the stationary point $x_s = 0$ the total derivate by time is performed as:

$$W(x_1,x_2,x_3) = -4x_1^2(\lambda\alpha_1 - k_1)^2 - 4x_2^2(\mu\alpha_2 - k_1)^2$$

$$-4x_3^2(\alpha_3 - k_1)^2 + 8x_1x_3(\lambda\alpha_1 - k_1)$$

For determination of positive-definite condition for the Lyapunov's function, let's calculate the Hessian function:

$$\begin{aligned} &\frac{\partial^2 V}{\partial x_1^2} = 3x_1^2 + k_1 - \lambda\alpha_1 + 2\lambda\beta_1 x_1 + \frac{\lambda\gamma_1 x_3\left(1+x_1^2\right) - 2\lambda\gamma_1 x_1^2 x_3}{\left(1+x_1^2\right)^2}; \frac{\partial^2 V}{\partial x_1 \partial x_2} = 0; \frac{\partial^2 V}{\partial x_1 \partial x_3} = \frac{\lambda\gamma_1 x_1}{1+x_1^2}; \\ &\frac{\partial^2 V}{\partial x_2^2} = 3x_2^2 + k_1 - \mu\alpha_1 + 2\mu x_2 + \mu\gamma_2 x_3; \frac{\partial^2 V}{\partial x_2 \partial x_1} = 0; \frac{\partial^2 V}{\partial x_2 \partial x_3} = \mu\gamma_2 x_2; \\ &\frac{\partial^2 V}{\partial x_3^2} = 3x_3^2 + k_1 + \alpha_3 - \gamma_4 x_2 + 2x_3 - \frac{\gamma_3 x_1^2}{1+x_1^2}; \frac{\partial^2 V}{\partial x_3 \partial x_1} = -\frac{2\gamma_3 x_1 x_3 - 2\gamma_3 x_1^3 x_3}{\left(1+x_1^2\right)^2}; \frac{\partial^2 V}{\partial x_3 \partial x_2} = 0 \end{aligned}$$

For the first point $x_s = 0$:

$$H_{x_s=0} = \begin{Vmatrix} k_1 - \lambda\alpha_1 & 0 & 0 \\ 0 & k_1 - \mu\alpha_1 & 0 \\ 0 & 0 & k_1 + \alpha_3 \end{Vmatrix}$$

The Hessian of Lyapunov's function classified as the following:

$$W(x_1,x_2,x_3) = (k_1 - \lambda\alpha_1)x_1^2 + (k_1 - \mu\alpha_1)x_2^2 + (k_1 + \alpha_3)x_3^2$$

The positive-definite of Lyapunov's function is determined by the following conditions:

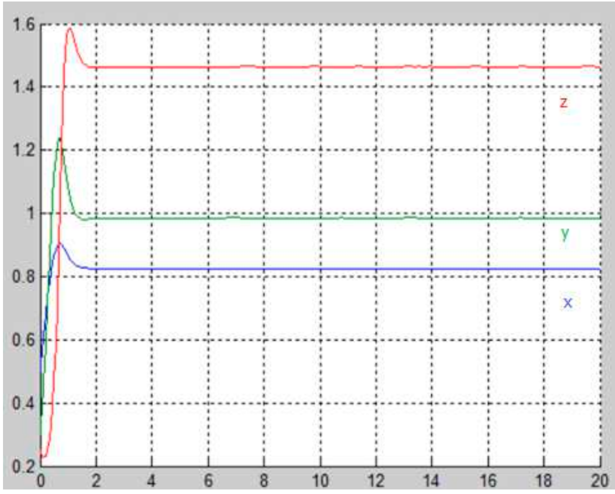

Fig. 4: Population dynamics for model (10) in case (a)

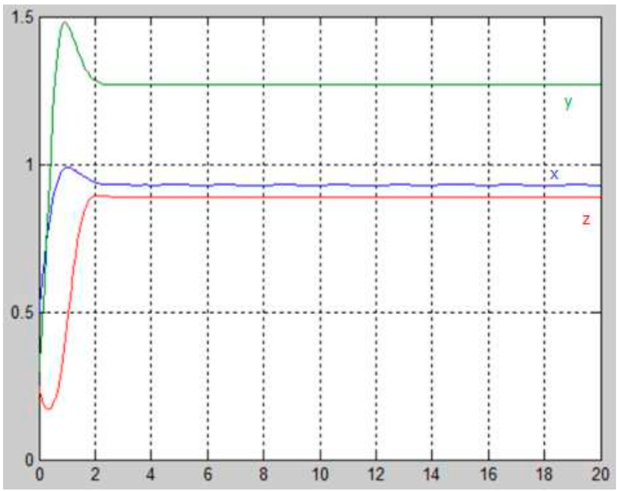

Fig. 5: Population dynamics for model (10) in case (b)

$$\begin{cases} k_1 - \lambda \alpha_1 \succ 0 \\ k_1 - \mu \alpha_1 \succ 0 \\ k_1 + \alpha_3 \succ 0 \end{cases}$$

Fig. 4-9 depict examples computed through such models with the analysis for following cases, blue, green and red colors of line means 1-3 (population) questions:

(a) $l = 2; \alpha_1 = 0.1,\ \alpha_2 = 1.3, \alpha_3 = 6 \beta_1 = 1,\ \gamma_1 = 0.5,\ \gamma_2 = 1.3,\ \gamma_3 = 0.5,\ \gamma_4 = 6.5, \mathrm{k} = 3, \mu = 1$

(b) $l = 2; \alpha_1 = 0.1,\ \alpha_2 = 1.3, \alpha_3 = 6 \beta_1 = 1,\ \gamma_1 = 0.5,\ \gamma_2 = 1.6,\ \gamma_3 = 0.5,\ \gamma_4 = 3.5, \mathrm{k} = 3,\ \mu = 1$

(c) $l = 2; \alpha_1 = 0.1,\ \alpha_2 = 1.3, \alpha_3 = 6 \beta_1 = 1,\ \gamma_1 = 0.5,\ \gamma_2 = 1.6,\ \gamma_3 = 0.5,\ \gamma_4 = 2.5, \mathrm{k} = 3, \mu = 1$

(d) $l = 1; \alpha_1 = 2,\ \alpha_2 = 1.3, \alpha_3 = 0.5, \beta_1 = 1,\ \gamma_1 = 0.5,\ \gamma_2 = 1.6,\ \gamma_3 = 0.5,\ \gamma_4 = 2.5, \mathrm{k} = 4, \mu = 1$

(e) $l = 1; \alpha_1 = 2,\ \alpha_2 = 5.3, \alpha_3 = 0.5, \beta_1 = 1,\ \gamma_1 = 1.5,\ \gamma_2 = 1.6,\ \gamma_3 = 0.05,\ \gamma_4 = 0.05, \mathrm{k} = 4, \mu = 0.2$

(f) $l = 1; \alpha_1 = 2,\ \alpha_2 = 2.3, \alpha_3 = 0.5, \beta_1 = 1,\ \gamma_1 = 1.5,\ \gamma_2 = 0.06,\ \gamma_3 = 0.05,\ \gamma_4 = 1.05, \mathrm{k} = 5, \mu = 0.2$

The figures show that in the initial phase there is an increase in the number of all three populations, then they stabilize at the non-fool level. Thus, analysis of the model with nonlinear control law shows that nonlinear control allows us to ensure the stability of the system while preserving all species, that is, maintaining species diversity.

Figures depict six different scenarios of stabilization in a data set.

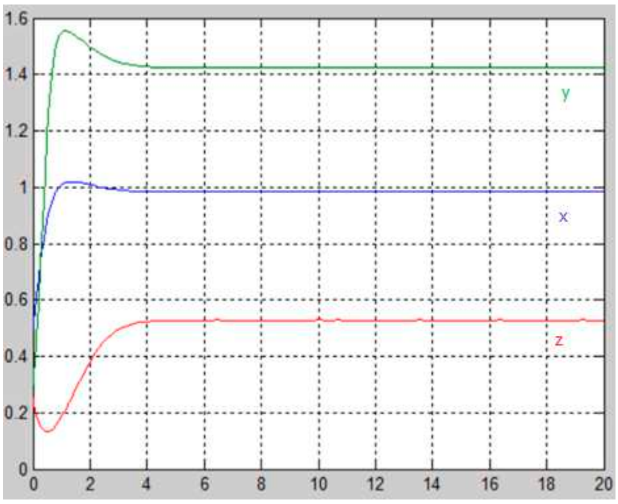

Fig. 6: Population dynamics for model (10) in case (c)

6 Conclusion

This work is devoted to problem of analysis of two models of one prey – two-predator system and two preys – one predator system. In the second case, it is assumed that preys are not competitors. Moreover, in both models it was assumed that populations are under permanent control. Provided numerical and analytical investigations allow showing that outbreak regimes of populations can be excluded when we use respective control law.

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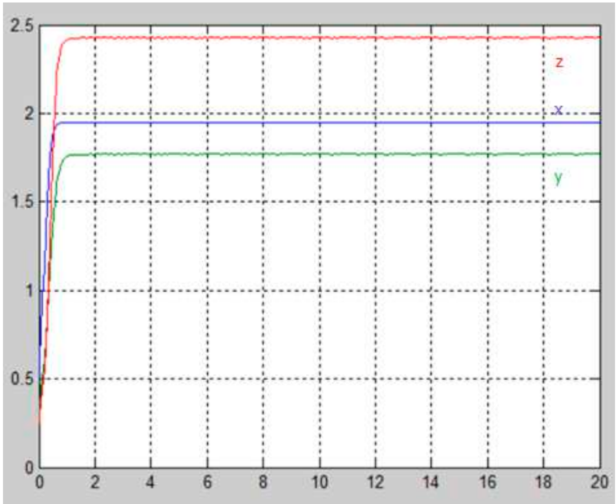

Fig. 7: Population dynamics for model (10) in case (d)

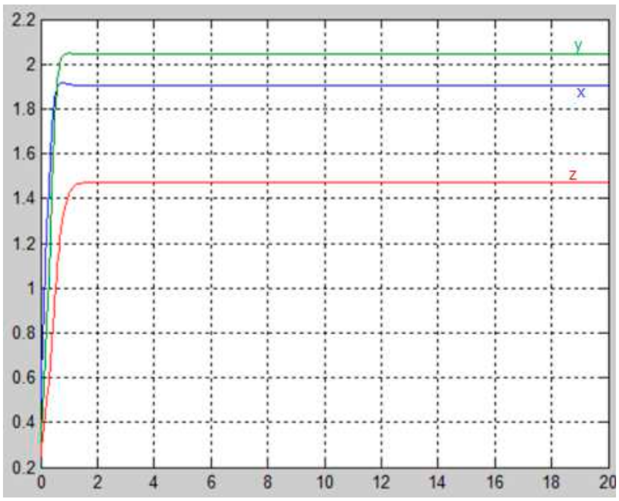

Fig. 8: Population dynamics for model (10) in case (e)

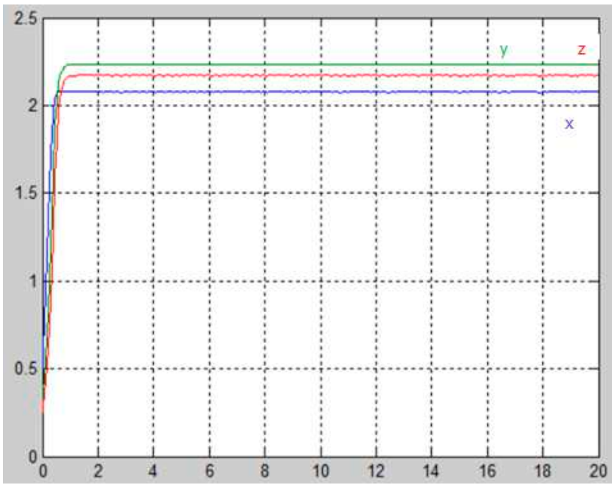

Fig. 9: Population dynamics for model (10) in case (f)

References

[1] Allen L.J.S., An introduction to stochastic processes with applications to biology. Boca Raton, Fl. CRC Press. 2010.
[2] Bazykin A.D., Nonlinear dynamics of interacting populations. World Scientific. New Jersey. 1998.
[3] Berryman A.A., On choosing models for describing and analyzing ecological time series, Ecology, Vol. 73, pp. 694-698 (1992).
[4] Gennaro V., Entomophagous species in integrated pest control, Italian J. Zoology, Vol. 47, No. 3, pp. 409-417 (1980).
[5] Isaev A.S., Khlebopros R.G., Nedorezov L.V., Kondakov Yu.P., Kiselev V.V., Forest Insect Population Dynamics. Nauka, Novosibirsk, 1984.
[6] Isaev A.S., Khlebopros R.G., Nedorezov L.V., Kondakov Yu.P., Kiselev V.V., Soukhovolsky V.G., Population Dynamics of Forest Insects, Nauka, Moscow, 2001.
[7] Kostitzin V.A., La Biologie Mathematique. Paris: A. Colin.1937.
[8] Lyapunov A.A., Bagrinovskaya G.P., On methodological problems of mathematical biology. In: Mathematical modeling in biology, Nauka, Moscow, 1975.
[9] May R.M., Biological populations obeying difference equations: Stable points, stable cycles, and chaos, J. of Theor. Biol., Vol. 51, No.2, pp. 511-524 (1975).
[10] Maynard Smith J., Mathematical Ideas in Biology. Cambridge Univ. Press. N.Y., 1968.
[11] Maynard Smith J., Models in Ecology. Cambridge Univ.Press. N.Y., 1974.
[12] Nedorezov L.V., The Generalized Models of Predator-Prey System with Satiation Effect. J. of General Biology, Vol. 45, No. 2, pp. 273-275 (1984).
[13] Nedorezov L.V., Modeling of Forest Insect Outbreaks, Nauka, Novosibirsk, 1986.
[14] Nedorezov L.V., Chaos and Order in Population Dynamics: Modeling, Analysis, Forecast. Saarbrucken: LAP Lambert Academic Publishing, 2012.
[15] Nedorezov L.V., Omarov A.N.. Determination of Phase Portraits Structures of Population Dynamics. Institute of Forest and Wood, Krasnoyarsk, 1986.
[16] Nedorezov L.V., Omarov A.N., Stability and diversity in predator-prey systems, Izvestiya SB AS USSR, Vol. 2, pp. 63-67 (1990).
[17] Nedorezov L.V., Ut'yupin Yu. V., Ut'yupina S.P., Satiation Effect in a Model of Predator-Prey System. Siberian J. of Industrial Mathematics, Vol. 4, No. 1, pp. 150-164 (2001).
[18] Nedorezov L.V., Utyupin Yu.V., Continuous-Discrete Models of Population Dynamics: An Analytical Overview, State Public Scientific-Technical Library, Novosibirsk, 2011.
[19] Svirezhev Yu.M., Logofet D.O., Stability of biological communities. Nauka, Moscow. 1978.
[20] Tonnang H., Nedorezov L.V., Owino J., Ochanda H., Lohr B., Evaluation of discrete host -parasitoid models for

diamondback moth and Diadegma semiclausum field time population density series, Ecological Modelling, Vol. 220, pp. 1735-1744 (2009).

[21] Tonnang H., Nedorezov L.V., Ochanda H., Owino J., Lohr B. Assessing the impact of biological control of Plutella xylostella through the application of Lotka - Volterra model, Ecological Modelling, Vol. 220, pp. 60-70 (2009).

[22] Tonnang H., Nedorezov L.V., Owino J., Ochanda H., Lohr B. Host-parasitoid population density prediction using artificial neural networks: diamondback moth and its natural enemies. Agricultural and Forest Entomology, Vol. 12, No. 3, pp. 233-242 (2010).

[23] Varley C.C., Gradwell G.R., Hassell M.P., Insect population ecology: an analytical approach. Blackwell Scientific Publications, 1973.

[24] Yermekbayeva J.J., Omarov A.N., Beisenbi M.A., Abitova G.A., The Control of Population Tumor Cells via Compensatory Effect, J. AMIS, Vol. 8, No. 4, pp. 1559-1565 (2014).

[25] Yermekbayeva J.J., Omarov A.N., Nedorezov L.V., The design of adaptive control system with reference model for population dynamics, J. AMIS, Vol.12, No.1, pp. 55-61 (2018).

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