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Gerber Shiu Function of a Risk Model with Two Classes of Claims, Random Incomes and Markov-Modulated System Parameters

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Abstract: In this paper we consider the Gerber-Shiu (G-S) discounted penalty function of a risk model with two classes of claims, random income and all system parameters controlled by independent Markov-Modulated (MM) environments. The model is motivated by the flexibility in modeling the two different claim arrival processes, for instance, fatal and nonfatal conditions in health insurance or from the four wheelers and two wheelers in vehicle insurance. Also the states of MM processes describe, for example, the epidemic types in health insurance or weather conditions in vehicle insurance. It is natural to consider that the controlling environment conditions are different for the discount rates, risk arrivals and income arrivals. We establish the system of integral equations satisfied by the G-S function, given the initial environment states for the risk model. Assuming that the random income size is exponentially distributed, explicit expressions for Laplace transform of the G-S function are derived. As an illustration, explicit results are obtained for the ruin probabilities when claim sizes are also exponentially distributed and some other numerical results are also presented.

Keywords: Gerber-Shiu discounted penalty function; Markov-Modulated risk model; two classes of claims; random incomes; Laplace transform

1 Introduction

The MM risk model is also termed as Markovian Regime Switching model in the finance and actuarial science literature. This model can very well capture the feature that insurance policies may need to change if economic or political environment changes. Recently, there have been resurgent interests in using regime switching models in finance and actuarial science. We concentrate on a model having two types of claims like, one originating from two wheeler accidents and the other from four wheeler accidents. Also, it can be considered as the first type to be from the insurance risk caused by traditional insurance claims, and the second type is the financial risk resulting from investments. Risk models which allow the insurance company to make investments to generate income from the available resources are of great interest. The risk model we discuss here is a fairly generalized model with all its parameters controlled by independent Markovian environments. Li and Garrido [2] considered a risk process with two classes of independent risks, derived a system of integro-differential equations for the non-ruin probabilities and obtained explicit results when the arrival processes are Poisson and generalized Erlang (2) through Laplace transforms. Note that the model in Gao[8] which is similar, derives the Gerber-Shiu function with zero initial surplus and the probability generating function for the Gerber-Shiu function. Also Hao et.al. [9] and Hang et.al. [4] discuss Gerber-Shiu discounted penalty function with zero initial surplus and deal with the Laplace transform of the Gerber-Shiu discounted penalty function. Some explicit expressions for the Gerber-Shiu discounted penalty functions with positive initial surplus are obtained when the claim size distributions belong to the rational family. Juan et.al. [6] considers a Markov-dependent risk model with constant dividend barrier. A system of integro-differential equations with boundary conditions satisfied by the expected discounted penalty function, with given initial environment state, is derived and solved. Ruin measures for the risk model involving two independent classes of risks is also studied by many researchers. We refer the reader to, Li et.al.[3] and Zhang et.al. [5] for some details. As an extension to these, Ji

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et.al.[7] investigated the renewal risk model with two classes of claims by assuming that both the claim number processes have phase type inter-claim times. Wuyuan et.al. [11] discussed the perturbed risk model with two independent classes of risks under multiple thresholds and derived the expected discounted penalty function for the model. Gerber-Shiu function of a compound Poisson risk model with constant interest force was extensively studied in Cai et.al. [1]. Some results of random income models are addressed in Gao [8], Hao et.al. [9] and Zhao et.al. [10].The Markov-modulated risk model with two types of claims can be seen as an extension of a classical Markov-Modulated risk model with frequent and in frequent claims. It is natural and of great use to extend these problems for risk models involving Markov-Modulated environments.

This paper is organized as follows: After introducing the model in detail with all its notations in Section 2, the system of integral equations for G-S function, given the initial surplus and the initial environmental states are developed in Section 3. In Section 4, we derive the Laplace transform of the G-S functions with the assumption of exponential random incomes. Further, when the claim sizes are also exponentially distributed, explicit expressions for the Laplace transform of the G-S functions are given in Section 5. For an illustration of the method, in Section 6, solution is presented when there are two states for the random income environment and only one environment each for all others. Ruin probabilities and deficit at ruin are computed for some specific cases. Then, the risk model having two environment states each for claim 1 and claim 2 arrivals and only one state each for others is solved. Finally, in Section 7, the effect of claim sizes on ruin probabilities is presented through a numerical example. Section 8 concludes the paper.

2 The Model and Assumptions

Let (Ω, A, P) be a probability space and all the random variables defined below are on this space. We introduce a risk model involving the system parameters controlled by different markovian environment processes. Let $E_1(t), t \geq 0$ be a homogenous continuous-time Markov chain taking values in a finite set $E_1 = 1,2,.....d_1$ with generator $\alpha_{ij}, \alpha_{ii} = -\alpha_i$. α_{ij} is assumed to be homogenous, irreducible and recurrent which governs the discounted force of interest δ in the G-S function of the risk model. When the state of economy is $E_1 = i$ then the discounted force of interest is δ_i.

Consider a risk model in continuous time with random incomes, two types of claims type 1 and type 2 and let $U(t)$ denote the surplus process of an insurance company. Then we have

$$U(t) = u + \sum_{p=1}^{M(t)} X_p - \left(\sum_{q=1}^{N_1(t)} Y_q + \sum_{r=1}^{N_2} Z_r \right)$$

Where $u \geq 0$, is the initial capital, X_p denotes the pth random income amount with $M(t), t \geq 0$ denoting the number of random incomes arrival up to time t, Y_q denotes the amount of the qth claim of first type with $N_1(t), t \geq 0$ representing the number of claims of first type occurring in $(0,t]$. Similarly Z_r is the rth claim of second type and $N_2(t), t \geq 0$ is the number of claims of second type occurring in $(0,t]$. The rates and distributions of random incomes and claims are governed by different environments. i.e., all of these are controlled by an independent homogenous irreducible and recurrent markov processes $E_2(t), E_3(t)$ and $E_4(t)$ respectively with finite state space $E_2 = 1,2,...d_2$, $E_3 = 1,2,...d_3, E_4 = 1,2,...d_4$.The intensity matrices are defined by the elements, $\beta_{ij}, \nu_{ij}, \rho_{ij}$ respectively and $\beta_{ii} = -\beta_i, \nu_{ii} = -\nu_i, \rho_{ii} = -\rho_i$. The stationary vectors of these processes are given by π_{1i}, π_{2i} and π_{3i} respectively.

Furthur we assume that at time t the income arrivals is according to a Poisson process with parameter c_j given the corresponding environment $E_2(t) = j$ and the premium amount has the distribution $F_j(x)$, density f_j and finite mean μ_{X_j} .Similarly claim type 1 occurs according to Poisson process with intensity λ_k when $E_3(t) = k$ and the corresponding type 1 claim amount distribution $G_k(y)$ with density g_k and finite mean μ_{Y_k} . Type 2 claim occurs according to Poisson process with intensity η_l when $E_4(t) = l$ and corresponding claim amount distribution is $H_1(z)$ with density h_l and finite mean μ_{Z_l} . The time of ruin

$$T_u = inf\,(t \geq 0 : U(t) < 0)$$

is first instant where the surplus becomes negative. Our main interest in this article is the G-S function of the model. In order that ruin does not occur with certainty, one has to assume the safety loading condition given by,

$$\sum_{j=1}^{d_2} \pi_{1j} c_j \mu_j \geq \sum_{k=1}^{d_3} \pi_{2k} \lambda_k \mu_k + \sum_{l=1}^{d_4} \pi_{3l} \eta_l \mu_l$$

The G-S function with stochastic discounted interest force driven by the MM risk process is defined by

$$\phi_{ijkl}(u) = E[e^{-\delta_i T_u} w(U(T_u^-), |U(T_u)| I(T_u < \infty)) / U(0) = u, E_1(0) = i, E_2(0) = j, E_3(0) = k, E_4(0) = l]$$

where $I(.)$ is the indicator function ; T_u denotes the time of ruin,$U(T_u^-)$is the surplus immediately prior to ruin,

$|U(T_u)|$ is the deficit at ruin and $w(x,y)$is a nonnegative and bounded penalty function. We can interpret $e^{-\delta_i T_u}$ as the stochastic discount factor of the risk model.

3 Integral equation of the G-S function

In this section we derive the system integral equations satisfied by the G-S function. Consider $U(t)$ in an infinitesimal time interval $(t, t+h)$. These are the different situations which may arise for the risk model.

- No claims occurs, no change in the claim states k and$l$, no premium arrival, no change in premium state j and no change in the interest state i in the interval $(t,t+h)$ then the surplus process renews itself without any change in the initial surplus amount.
- There is a change in the state of interest i in $(t,t+h)$ but all others have no change, causes the renewal of the system with no change in the initial surplus amount.
- No claim arrivals, but change in claim 1 governing state k, renews the system with no change in the initial surplus amount.
- Claim 1 occurs in $(t,t+h)$ indicates change in the initial surplus amount with renewal of the risk process.
- Change in claim 2 governing state l in $(t,t+h)$, renews the risk process without change in the initial surplus amount.
- Claim 2 occurs in $(t,t+h)$ changes the initial surplus amount with renewal of the risk process.
- A random income arrival in $(t,t+h)$ renews the risk process with different initial surplus amount.
- Change in the random income state j in $(t,t+h)$ renews the risk process without change in the initial surplus amount.
- All other events occur with total probability o(h).

Conditioning on the time of the first event (random income or claim type 1or claim type 2 or change in environment), we have

$$\phi_{ijkl}(u) = (1-[\alpha_i+\delta_i+c_j+\beta_j+\lambda_k+\nu_k+\eta_l+\rho_l]h+O(h))\phi_{ijkl}(u) + \sum_{s=1,s\neq i}^{d_1}\alpha_{is}\phi_{sjkl}(u)h + \sum_{t=1,t\neq k}^{d_3}\nu_{kt}\phi_{ijtl}(u)h$$
$$+\lambda_k h[\int_0^u \phi_{ijkl}(u-y)dG_k(y)+\int_u^\infty w(u,y-u)dG_k(y)]$$
$$+\sum_{v=1,v\neq l}^{d_4}\rho_{lv}\phi_{ijkv}(u)h+\eta_l h$$
$$[\int_0^u \phi_{ijkl}(u-z)dH_l(z)+\int_u^\infty w(u,z-u)dH_l(z)]$$
$$+ c_j h\int_0^\infty \phi_{ijkl}(u+x)dF_j(x) + \sum_{w=1,w\neq j}^{d_2}\beta_{jw}\phi_{iwkl}(u)h+O(h)$$

Cancelling the common terms and dividing by h and then taking limit, we obtain the system of integral equations for the G-S function as

$$(\alpha_i+\delta_i+c_j+\beta_j+\lambda_k+\nu_k+\eta_l+\rho_l)\phi_{ijkl}(u) =$$
$$\sum_{s=1,s\neq i}^{d_1}\alpha_{is}\phi_{sjkl}(u)+\sum_{t=1,t\neq k}^{d_3}\nu_{kt}\phi_{ijtl}(u)+$$
$$\sum_{v=1,v\neq l}^{d_4}\rho_{lv}\phi_{ijkv}(u)+\sum_{w=1,w\neq j}^{d_2}\beta_{jw}\phi_{iwkl}(u)+$$
$$\lambda_k[\int_0^u \phi_{ijkl}(u-y)dG_k(y)+\int_u^\infty w(u,y-u)dG_k(y)]$$
$$+\eta_l[\int_0^u \phi_{ijkl}(u-z)dH_l(z)+\int_u^\infty w(u,z-u)dH_l(z)]$$
$$+c_j\int_0^\infty \phi_{ijkl}(u+x)dF_j(x).$$

4 Laplace transform of G-S function for the exponential random incomes

In this section, we consider the case that the random incomes are exponentially distributed. The expected discounted penalty function can be explicitly obtained for some specific settings. Let $F_j(x)=1-e^{-\xi_j x}, x\geq 0, \xi_j\geq 0$ and
$A(u)=\int_0^\infty \phi_{ijkl}(u+x)dF_j(x)$

$$(\alpha_i+\delta_i+c_j+\beta_j+\lambda_k+\nu_k+\eta_l+\rho_l)\phi_{ijkl}(u) =$$
$$\sum_{s=1,s\neq i}^{d_1}\alpha_{is}\phi_{sjkl}(u)+\sum_{t=1,t\neq k}^{d_3}\nu_{kt}\phi_{ijtl}(u)+$$
$$\sum_{v=1,v\neq l}^{d_4}\rho_{lv}\phi_{ijkv}(u)+\sum_{w=1,w\neq j}^{d_2}\beta_{jw}\phi_{iwkl}(u)+$$
$$\lambda_k[\int_0^u \phi_{ijkl}(u-y)dG_k(y)+\int_u^\infty w(u,y-u)dG_k(y)]$$
$$+\eta_l[\int_0^u \phi_{ijkl}(u-z)dH_l(z)+\int_u^\infty w(u,z-u)dH_l(z)]$$
$$+c_j\int_0^\infty \phi_{ijkl}(u+x)\xi_j e^{-\xi_j}xdx$$

Let $C=\alpha_i+\delta_i+c_j+\beta_j+\lambda_k+\nu_k+\eta_l+\rho_l$,

$$w_1(u)=\int_u^\infty w(u,y-u)dG_k(y)$$

and $w_2(u)=\int_u^\infty w(u,z-u)dH_l(z)$.
Then

$$C\phi_{ijkl}(u) = \sum_{s=1,s\neq i}^{d_1}\alpha_{is}\phi_{sjkl}(u)+\sum_{t=1,t\neq k}^{d_3}\nu_{kt}\phi_{ijtl}(u)+$$
$$\sum_{v=1,v\neq l}^{d_4}\rho_{lv}\phi_{ijkv}(u)+\sum_{w=1,w\neq j}^{d_2}\beta_{jw}\phi_{iwkl}(u)+$$
$$\lambda_k[\int_0^u \phi_{ijkl}(u-y)dG_k(y)+w_1(u)]+\eta_l[\int_0^u \phi_{ijkl}(u-z)dH_l(z)+$$
$$w_2(u)]+c_j\xi_j\int_0^\infty \phi_{ijkl}(u+x)e^{-\xi_j}xdx$$

Denoting $\phi_i(s)$ as the Laplace transform of $\phi_i(u)$, i.e.; $\phi_{ijkl}(s)=\int_0^\infty e^{-su}\phi_{ijkl}(u)du$

$$A(s)=\frac{\phi_{ijkl}(s)-\phi_{ijkl}(\xi_j)}{1-\frac{s}{\xi_j}}$$
$$b_1(s)=\int_0^\infty e^{-sy}dG_k(y)$$
$$b_2(s)=\int_0^\infty e^{-sz}dH_l(z)$$

We finally arrive at,

$$C\phi_{ijkl}(s)=\sum_{s=1,s\neq i}^{d_1}\alpha_{is}\phi_{sjkl}(s)+\sum_{t=1,t\neq k}^{d_3}\nu_{kt}\phi_{ijtl}(s)+$$
$$\sum_{v=1,v\neq l}^{d_4}\rho_{lv}\phi_{ijkv}(s) + \sum_{w=1,w\neq j}^{d_2}\beta_{jw}\phi_{iwkl}(s) + \lambda_k[\phi_{ijkl}(s)b_1(s)+$$
$$w_1(s)]+\eta_l[\phi_{ijkl}(s)b_2(s)+w_2(s)]+c_jA(s). \text{ .... (1)}$$

5 Explicit expression for Laplace Transform of G-S function with exponential claims

In this section we assume that the markov processes associated with discounting environment and the random income have two states each and the claim 1 and claim 2 environments have only one state each. i.e.; $d_1 = 2, d_2 = 2, d_3 = 1, d_4 = 1$. Then the Laplace transform of G-S function $\phi_{ijkl}(s)$ reduces to $\phi_{ij}(s)$. Also we assume that both claims follow exponential distributions with respective parameters a_k and b_l i.e;.$G_k(y) = 1 - e^{-a_k y}, y \geq 0, a_k \geq 0$,

$H_l(z) = 1 - e^{-b_l z}, z \geq 0, b_l \geq 0$ and

let $C_{ij} = \alpha_i + \delta_i + c_j + \beta_j + \lambda_1 + \nu_1 + \eta_1 + \rho_1$.

Then system (1) is

$$\phi_{ij}(s) = \frac{1}{C_{ij}} \sum_{s=1,s\neq i}^{2} \alpha_{is}\phi_{sj}(s) + \frac{1}{C_{ij}} \sum_{w=1,w\neq j}^{2} \beta_{jw}\phi_{iw}(s) + \frac{\lambda_1}{C_{ij}}[\phi_{ij}(s)b_1(s) + w_1(s)] + \frac{\eta_1}{C_{ij}}[\phi_{ij}(s)b_2(s) + w_2(s)] + \frac{c_j}{C_{ij}}A(s)$$

So we have the following four equations,

$$\phi_{11}(s) = \frac{\alpha_{12}}{C_{11}}\phi_{21}(s) + \frac{\beta_{12}}{C_{11}}\phi_{12}(s) + \frac{\lambda_1}{C_{11}}[\phi_{11}(s)b_1(s) + w_1(s)] + \frac{\eta_1}{C_{11}}[\phi_{11}(s)b_2(s) + w_2(s)] + \frac{c_1}{C_{11}}\left(\frac{\phi_{11}(s) - \phi_{11}(\xi_1)}{1 - \frac{s}{\xi_1}}\right)$$

$$\phi_{12}(s) = \frac{\alpha_{12}}{C_{12}}\phi_{22}(s) + \frac{\beta_{21}}{C_{12}}\phi_{11}(s) + \frac{\lambda_1}{C_{12}}[\phi_{12}(s)b_1(s) + w_1(s)] + \frac{\eta_1}{C_{12}}[\phi_{12}(s)b_2(s) + w_2(s)] + \frac{c_2}{C_{12}}\left(\frac{\phi_{12}(s) - \phi_{12}(\xi_2)}{1 - \frac{s}{\xi_2}}\right)$$

$$\phi_{21}(s) = \frac{\alpha_{21}}{C_{21}}\phi_{11}(s) + \frac{\beta_{12}}{C_{21}}\phi_{22}(s) + \frac{\lambda_1}{C_{21}}[\phi_{21}(s)b_1(s) + w_1(s)] + \frac{\eta_1}{C_{21}}[\phi_{21}(s)b_2(s) + w_2(s)] + \frac{c_1}{C_{21}}\left(\frac{\phi_{21}(s) - \phi_{21}(\xi_1)}{1 - \frac{s}{\xi_1}}\right)$$

$$\phi_{22}(s) = \frac{\alpha_{21}}{C_{22}}\phi_{12}(s) + \frac{\beta_{21}}{C_{22}}\phi_{21}(s) + \frac{\lambda_1}{C_{22}}[\phi_{22}(s)b_1(s) + w_1(s)] + \frac{\eta_1}{C_{22}}[\phi_{22}(s)b_2(s) + w_2(s)] + \frac{c_2}{C_{22}}\left(\frac{\phi_{22}(s) - \phi_{22}(\xi_2)}{1 - \frac{s}{\xi_2}}\right)$$

After some rearrangement and simplifications, we get

$$\phi_{11}(s)[1 - \frac{\lambda_1}{C_{11}}b_1(s) - \frac{\eta_1}{C_{11}}b_2(s) - \frac{c_1\xi_1}{C_{11}(\xi_1 - s)}] = \frac{\alpha_{12}}{C_{11}}\phi_{21}(s) + \frac{\beta_{12}}{C_{11}}\phi_{12}(s) + \frac{\lambda_1}{C_{11}}w_1(s) + \frac{\eta_1}{C_{11}}w_2(s) - \frac{c_1\xi_1\phi_{11}(\xi_1)}{C_{11}(\xi_1 - s)}$$

$$\phi_{12}(s)[1 - \frac{\lambda_1}{C_{12}}b_1(s) - \frac{\eta_1}{C_{12}}b_2(s) - \frac{c_2\xi_2}{C_{12}(\xi_2 - s)}] = \frac{\alpha_{12}}{C_{12}}\phi_{22}(s) + \frac{\beta_{21}}{C_{12}}\phi_{11}(s) + \frac{\lambda_1}{C_{12}}w_1(s) + \frac{\eta_1}{C_{12}}w_2(s) - \frac{c_2\xi_2\phi_{12}(\xi_2)}{C_{12}(\xi_2 - s)}$$

$$\phi_{21}(s)[1 - \frac{\lambda_1}{C_{21}}b_1(s) - \frac{\eta_1}{C_{21}}b_2(s) - \frac{c_1\xi_1}{C_{21}(\xi_1 - s)}] = \frac{\alpha_{21}}{C_{21}}\phi_{11}(s) + \frac{\beta_{12}}{C_{21}}\phi_{22}(s) + \frac{\lambda_1}{C_{21}}w_1(s) + \frac{\eta_1}{C_{21}}w_2(s) - \frac{c_1\xi_1\phi_{21}(\xi_1)}{C_{21}(\xi_1 - s)}$$

$$\phi_{22}(s)[1 - \frac{\lambda_1}{C_{22}}b_1(s) - \frac{\eta_1}{C_{22}}b_2(s) - \frac{c_2\xi_2}{C_{22}(\xi_2 - s)}] = \frac{\alpha_{21}}{C_{22}}\phi_{12}(s) + \frac{\beta_{21}}{C_{22}}\phi_{21}(s) + \frac{\lambda_1}{C_{22}}w_1(s) + \frac{\eta_1}{C_{22}}w_2(s) - \frac{c_2\xi_2\phi_{22}(\xi_2)}{C_{22}(\xi_2 - s)}$$

So we get the equations,

$$\phi_{11}(s) = \frac{\frac{\alpha_{12}}{C_{11}}\phi_{21}(s) + \frac{\beta_{12}}{C_{11}}\phi_{12}(s) + \frac{\lambda_1}{C_{11}}w_1(s) + \frac{\eta_1}{C_{11}}w_2(s) - \frac{c_1\xi_1\phi_{11}(\xi_1)}{C_{11}(\xi_1 - s)}}{[1 - \frac{\lambda_1}{C_{11}}b_1(s) - \frac{\eta_1}{C_{11}}b_2(s) - \frac{c_1\xi_1}{C_{11}(\xi_1 - s)}]} \quad \ldots\ldots \quad (2)$$

$$\phi_{12}(s) = \frac{\frac{\alpha_{12}}{C_{12}}\phi_{22}(s) + \frac{\beta_{21}}{C_{12}}\phi_{11}(s) + \frac{\lambda_1}{C_{12}}w_1(s) + \frac{\eta_1}{C_{12}}w_2(s) - \frac{c_2\xi_2\phi_{12}(\xi_2)}{C_{12}(\xi_2 - s)}}{[1 - \frac{\lambda_1}{C_{12}}b_1(s) - \frac{\eta_1}{C_{12}}b_2(s) - \frac{c_2\xi_2}{C_{12}(\xi_2 - s)}]} \quad \ldots\ldots \quad (3)$$

$$\phi_{21}(s) = \frac{\frac{\alpha_{21}}{C_{21}}\phi_{11}(s) + \frac{\beta_{12}}{C_{21}}\phi_{22}(s) + \frac{\lambda_1}{C_{21}}w_1(s) + \frac{\eta_1}{C_{21}}w_2(s) - \frac{c_1\xi_1\phi_{21}(\xi_1)}{C_{21}(\xi_1 - s)}}{[1 - \frac{\lambda_1}{C_{21}}b_1(s) - \frac{\eta_1}{C_{21}}b_2(s) - \frac{c_1\xi_1}{C_{21}(\xi_1 - s)}]} \quad \ldots\ldots \quad (4)$$

$$\phi_{22}(s) = \frac{\frac{\alpha_{21}}{C_{22}}\phi_{12}(s) + \frac{\beta_{21}}{C_{22}}\phi_{21}(s) + \frac{\lambda_1}{C_{22}}w_1(s) + \frac{\eta_1}{C_{22}}w_2(s) - \frac{c_2\xi_2\phi_{22}(\xi_2)}{C_{22}(\xi_2 - s)}}{[1 - \frac{\lambda_1}{C_{22}}b_1(s) - \frac{\eta_1}{C_{22}}b_2(s) - \frac{c_2\xi_2}{C_{22}(\xi_2 - s)}]} \quad \ldots\ldots \quad (5)$$

To solve the above system of equations, we first consider equations (2) and (4). The denominators of $\phi_{11}(s)$and $\phi_{21}(s)$ are the same and so we shall obtain the zeros of these denominators. The denominators has exactly two positive real roots $r_1$and r_2 . We assume that these roots are distinct. Since $\phi_{ij}(s)$ is finite for all s, we know that $r_i, i = 1,2$ should be zeros of corresponding numerators also.

Similarly the denominators of $\phi_{12}(s)$ and $\phi_{22}(s)$ in (3), (5) are the same and we obtain the roots of the denominator as $r_3$and r_4.

Hence using the roots on the numerators we can write 4 linear equations with 4 unknowns $\phi_{11}(\xi_1), \phi_{12}(\xi_2), \phi_{21}(\xi_1)$, $\phi_{22}(\xi_2)$. We solve these to obtain its values. Finally using these values in the above equations, we shall obtain $\phi_{11}(s), \phi_{12}(s), \phi_{21}(s), \phi_{22}(s)$.

Now we shall give some numerical examples to illustrate the procedure.

6 Numerical illustrations

In this section, we shall present some numerical examples to illustrate applications of the method discussed above. For simplicity of the model, we suppose that the random income environment process has two states and all others have only one environment each. We set $d_1 = 1, d_2 = 2, d_3 = 1, d_4 = 1$ and $\alpha_i = 0; \beta_{11} = \beta_{22} = -1; \beta_{12} = \beta_{21} = 1; c_1 = 2; c_2 = 3; \xi_1 = 4; \xi_2 = 5; \lambda_1 = 0.5; \eta_1 = 0.25$. So claim 1 and claim 2 amounts are exponentially distributed with $G_k(y) = 1 - e^{-y}, H_l(z) = 1 - e^{-2z}$.

It is easy to check that the positive loading condition holds in this case.

Case: 1

In order to obtain the probability of ruin let us assume $\delta = 0, w(x,y) = 1$. Then the G-S functions $\phi_j(u), j = 1,2$ reduce to the ruin probabilities $\psi_1(u), \psi_2(u)$. For the values we set above, we obtain,

$$\psi_1(u) = 0.00144e^{-0.3594u} + 0.00015e^{-1.8777u}$$

$$\psi_2(u) = 0.00036e^{-0.3594u} + 0.00004e^{-1.8777u}$$

We depict the ruin probabilities versus initial surplus, $u \in [0,16]$ in the Figure:1.

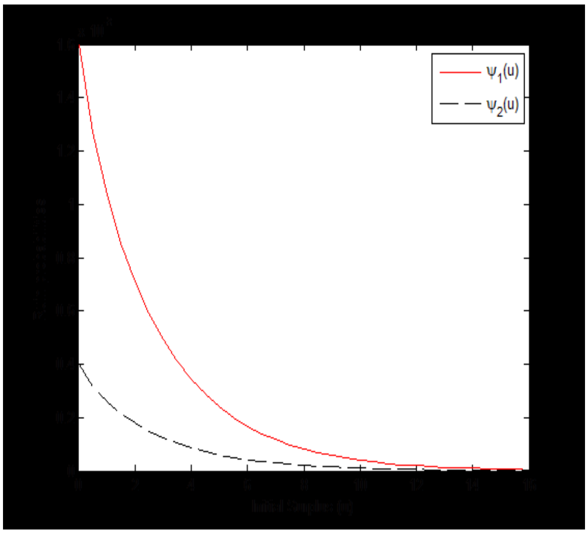

Fig. 1: Ruin Probabilities of two state random income risk model.

Case:2

With the above settings and $\delta = 0, w(x,y) = y$, we reduce the G-S function to the expected deficit at ruin, $\phi_1(u), \phi_2(u)$ in the two state models. Figure:2 shows the trends of the deficit at ruin with the initial surplus, $u \in [0,20]$ for this risk model. Also Figure:3 shows the G-S function for the risk model, $\phi_1(u), \phi_2(u)$ when the interest force $\delta = 1$ and $w(x,y) = y$ with the initial surplus, $u \in [0,20]$.

Case:3

In this case we fix the model with constant interest force and study the nature using the above same settings for $\delta = 1$ and $(x,y) = 1$. In Figure:4 we show the G-S function, $\phi_1(u), \phi_2(u)$ for the constant interest force $\delta = 1$ in the two states for the initial surplus, $u \in [0,8]$.

Case:4

In this we suppose that the claim 1 and claim 2 environmental processes has two states and the other parameters have no environment. We set

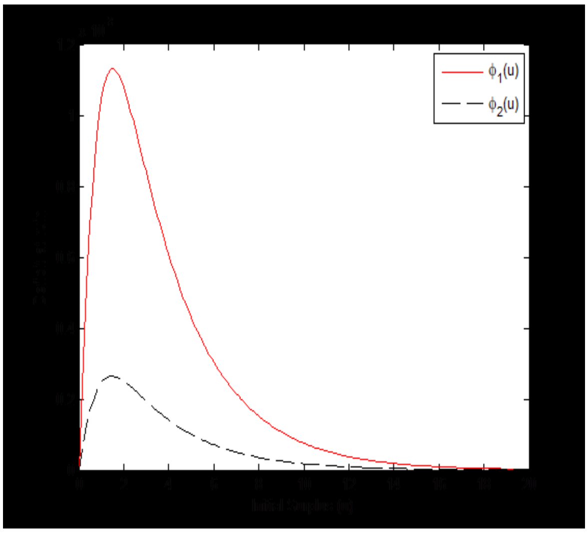

Fig. 2: Deficit at ruin for random income environment risk model.

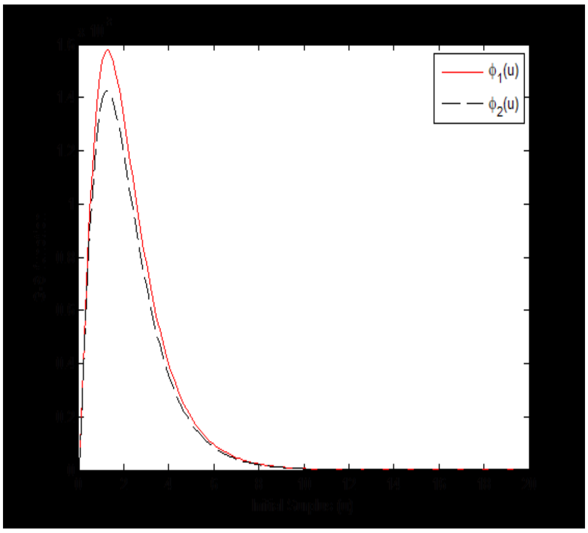

Fig. 3: G-S function for constant interest force $\delta = 1$ and $w(x,y) = y$ in a random income environment risk model.

$d_1 = 1, d_2 = 1, d_3 = 2, d_4 = 2$ implies $\alpha_i = 0; \beta_j = 0; \nu_{11} = \nu_{22} = -1; \nu_{12} = \nu_{21} = 1; \rho_{11} = \rho_{22} = -1; \rho_{12} = \rho_{21} = 1; c_1 = 3; \xi_1 = 1; \lambda_1 = 0.5; \lambda_2 = 1; \eta_1 = 0.25; \eta_2 = 0.75$, claim 1 and claim 2 are exponentially distributed with $G_k(y) = 1 - e^{-y}, H_l(z) = 1 - e^{-2z}$. Figure.5 depicts the nature of the ruin probabilities, $\psi_{11}(u), \psi_{12}(u), \psi_{21}(u), \psi_{22}(u)$ with respect to the initial surplus $u \in [0,10]$.

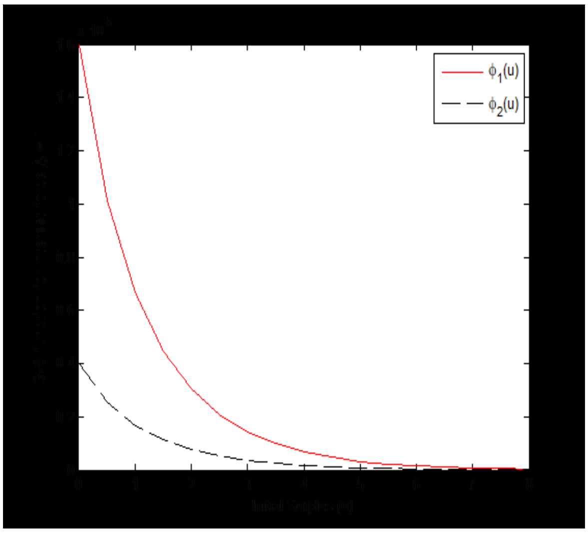

Fig. 4: G-S function for constant interest force $\delta = 1$.

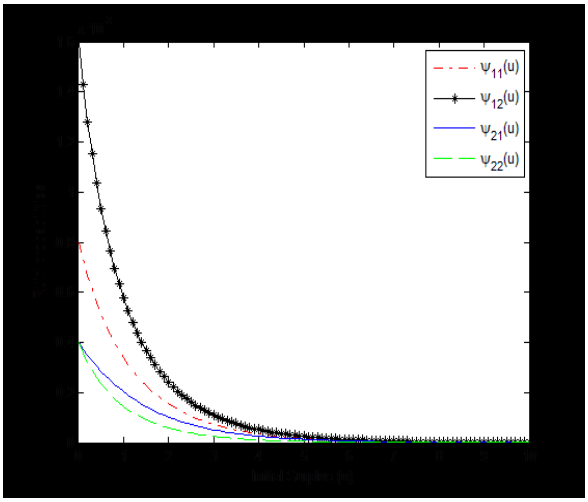

Fig. 5: Ruin Probabilities of two state claim 1 and claim 2 risk model.

7 Effect of claim sizes on ruin probability

The claim size parameters play an important role in the ruin probabilities of any model. In this section we discuss in details about the relationship between the claim sizes and the ruin probabilities of the risk model. The Laplace transform of the aggregate claim amount at ruin can be obtained as a special case of the generalized Gerber-Shiu function as studied by Yu (2013), but the procedures to invert such transforms can be complicated. To further generalize the MM risk model, we assume the random income rates vary according to a Markovian environment process. For this model, through the generalized Gerber-Shiu function we study the effect of claim 1 sizes on the ruin probability. Expressions for our generalized Gerber-Shiu function can be found by applying the same approach as in Hao and Yang [8]. For illustration purpose we set $d_1 = 1, d_2 = 2, d_3 = 1, d_4 = 1$ implies $\alpha_i = 0; \beta_{11} = \beta_{22} = -1; \beta_{12} = \beta_{21} = 1; c_1 = 2; c_2 = 3; \xi_1 = 4; \xi_2 = 5; \lambda_1 = 0.5; \eta_1 = 0.25$, claim 2 are exponentially distributed with $H_l(z) = 1 - e^{-2z}$. For varying claim 1 size parameters the ruin probabilities, $\psi_1(u), \psi_2(u)$ for the risk model is depicted in Figure:6.

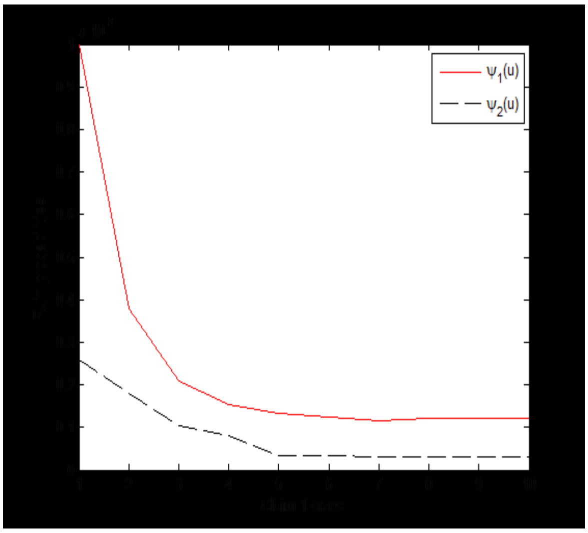

Fig. 6: Ruin Probabilities for varying claim 1 size parameters for the two state model

8 Conclusion

This paper investigates a risk model with two types of claims, random income and with Markov Modulated system parameters, which is an extension of the classical risk model with one type of claims having markov modulated arrival rates. The two types of claims considered here may be different in origin and with different environments. The integral equation resembles that of Hao et.al. [9] but with many additional terms. Though the solution of such Markov Modulated risk model is not easy, the method in Hao et.al. [9] can be replicated to obtain the solution of the risk model with exponential random incomes. Furthermore in the case of a model with limited states, the solution to the G-S function is obtained. Finally, the effect of the initial surplus on different measures like ruin probability, deficit at ruin etc. is illustrated by numerical examples. The effect of claim size parameters on the ruin probability is also estimated and represented using numerical examples.

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References

[1] J.Cai and D.C.M.Dickson, On the expected discounted penalty function at ruin of a surplus process with interest.Insurance Math. Econom. Vol.30,pp. 389-404 (2002).

[2] S.Li and J. Garrido, Ruin probabilities for two classes of risk processes, ASTIN Bulletin Vol. 35 , No. 1, pp. 61-77 (2005).

[3] S.Li & Y. Lu, On the expected discounted penalty functions for two classes of risk processes, Insurance Math. And Econom., Vol. 36, pp. 179-193 (2005).

[4] Zhang, Z., Li, S., & Yang, H. The Gerber-Shiu discounted penalty functions for a risk model with two classes of claims. Journal of Computational and Applied Mathematics, Vol. 230 , No.2, pp. 643-655 (2009). http://doi.org/10.1016/j.cam.2009.01.002.

[5] Z. M. Zhang, S. Li & H.Yang, The Gerber-Shiu discounted penalty functions for a risk model with two classes of claims, J.Comput. Appl.Math., Vol.230, pp.643-655 (2009).

[6] Juan, L., Jiancheng, X., & Yijun, H. On the expected discounted penalty function in a Markov-dependent risk model with constant dividend barrier, Acta Mathematica Scientia, Vol. 30 No.5, pp. 1481-1491 (2010). http://doi.org/10.1016/S0252-9602(10)60140-3.

[7] L.Ji & C. Zhang, The Gerber-Shiu functions for two classes of renewal risk processes, J. Comput. Appl. Math., Vol. 233, pp. 2575-2589 (2010).

[8] Shan Gao, The Gerber-Shiu function of a risk model with two classes of claims and random income. World Academy of Science, Engineering and Technology, Vol.5, No.12, pp. 911-916 (2011).

[9] Y. Hao, H. Yang, On a compound poisson risk model with delayed claims and random incomes. Journal of Computational and Applied Mathematics, Vol. 230, No.2, pp. 643-655 (2011).

[10] Y.X.Zhao, C.C.Yin, The expected discounted penalty function under avrenewal risk model with stochastic, Appl.Math. Comput. Vol.218, pp.6144-6154 (2012).

[11] Wuyuan, Jiang., Zhaojun, Yang. The Expected Discounted Penalty function for two classes of risk processes perturbed by diffusion with multiple thresholds. Indian J.Pure Appl. Math., Vol. 45, No.4, pp. 479-495 (2014).

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