Applied Mathematics & Information Sciences
An International Journal

Ideal theory in graded semirings

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Received: 12 Jun 2012; Revised 2 Sep. 2012 ; Accepted 15 Sep. 2012
Published online: 1 Jan. 2013

Abstract: An A-semiring has commutative multiplication and the property that every proper ideal B is contained in a prime ideal P, with $\sqrt{B}$, the intersection of all such prime ideals. In this paper, we define homogeneous ideals and their radicals in a graded semiring R. When B is a proper homogeneous ideal in an A-semiring R, we show that $\sqrt{B}$ is homogeneous whenever $\sqrt{B}$ is a k-ideal. We also give necessary and sufficient conditions that a homogeneous k-ideal P be completely prime (i.e., $F \notin P, G \notin P$ implies $FG \notin P$) in any graded semiring. Indeed, we may restrict F and G to be homogeneous elements of R.

Keywords: semiring, (k-) ideal, homogeneous (ideal), graded.

1. Introduction

The notion of semiring was first introduced by H. S. Vandiver in 1934, and since then many other researchers also developed the theory of semirings as a generalization of rings. Semirings occur in different mathematical fields, e.g., as ideals of a ring, as positive cones of partially ordered rings and fields, in the context of topological considerations, and in the foundations of arithmetic, including questions raised by school education ([6]). In the 1980's the theory of semirings contributed to computer science, since the rapid development of computer science needed additional theoretical mathematical background. The semiring structure does not contain an additive inverse, and this point is very helpful in developing the theoretical structure of computer science. For example, hemirings, as a semiring with zero and commutative addition, appeared in a natural manner in some applications to the theory of automata and formal languages. Recently, J. S. Han and et al. ([5]) discussed semiring orders in semirings. We refer to J. S. Golan's remarkable book for general reference ([4]).

2. Preliminaries

There are many different definitions of a semiring appearing in the literature. Throughout this paper, a semiring will be defined as follows:

A set R together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$, respectively) will be called a *semiring* provided:

(i)addition is a commutative operation;
(ii)there exist $0 \in R$ such that $x+0 = x$ and $x0 = 0x = 0$ for each $x \in R$, and
(iii)multiplication distributes over addition both from the left and from the right.

The element 0 in item (ii) is called the zero of the semiring R. A subset S of the semiring R will be called a *subsemiring* of R provided (1) $x + y \in S$ and $xy \in S$ whenever $x, y \in S$ and (2) $0 \in S$. By an *additive subsemigroup*, we mean a subset of R that contains the zero of R and is closed under addition. A subset I of a semiring R will be called an *ideal* if $a, b \in I$ and $r \in R$ implies $a + b \in I, ra \in I$ and $ar \in I$.

3. Graded semirings

Whenever R is a commutative semiring, we let $R[X]$ denote the semiring of polynomials with coefficients in R over the transcendental element X. Then each non-zero element $f(X)$ in $R[X]$ can be represented by a unique sum $\sum_{i=1}^{k} a_{n_i} X^{n_i}$ of non-zero monomials $a_{n_i} X^{n_i}$ of unique degrees. An ideal B in $R[X]$ is said to be *homogeneous*

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if $f(X) = \sum_{i=1}^{k} a_{n_i} X^{n_i} \in B$ implies $a_{n_i} X^{n_i} \in B$, for each i. In this article, a number of properties of homogeneous ideals will be derived. However, it will not be necessary to restrict ourselves to polynomial semirings since homogeneous ideals can be studied in semirings more general than polynomial semirings, namely in graded semirings. The first problem encounted will be the presentation of a definition of a graded semiring.

Definition 3.1. Let $\{R_q\}_{q\in Z}$ be a collection of additive subsemigroups of a semiring R. We say that R is the *internal direct sum* of the collection $\{R_q\}_{q\in Z}$ provided:

(1)Each non-zero element x in R has a unique representation of the form $\sum_{i=1}^{k} r_{q_i}$ where $q_i \neq q_j$ if $i \neq j$ and r_{q_i} is a non-zero element in R_{q_i} for each i; and
(2)If $r_{q_1}, r_{q_2}, \cdots, r_{q_k}$ are non-zero elements in R where $q_i \neq q_j$ if $i \neq j$, and each $r_{q_i} \in R_{q_i}$, then $\sum_{i=1}^{k} r_{q_i}$ is a non-zero element in R.

The notation "$R = \sum_{q\in Z} R_q$ " will be used to indicate that the semiring R is the internal direct sum of the collection $\{R_q\}_{q\in Z}$. Although the collection $\{R_q\}_{q\in Z}$ in the above definition could be taken with Z any non-empty set, we will always use Z to denote the ring of integers.

Let Z be denote the ring of integers and let $\{R_q\}_{q\in Z}$ be a collection of additive subsemigroups of a semiring R. The symbol $\sum_{q\in Z} R_q$ will denote the set consisting of 0 together with all x in R for which there exist integers $q_1, q_2, \cdots, q_k$, ($q_i \neq q_j$ if $i \neq j$), and non-zero elements $r_{q_1}, r_{q_2}, \cdots, r_{q_k}$ such that $r_{q_i} \in R_{q_i}$ and $x = \sum_{i=1}^{k} r_{q_i}$.

Definition 3.2. A semiring R is said to be *graded* if there exists a collection $\{R_q\}_{q\in Z}$ of additive subsemigroups of R satisfying the following conditions:

(1)$R = \sum_{q\in Z} R_q$;
(2)$R_q R_{q'} \subset R_{q+q'}$ for each $q, q' \in Z$;
(3)If $r_{q_1}, r_{q_2}, \cdots, r_{q_k}$ are non-zero elements in R where $q_i \neq q_j$ if $i \neq j$, and each $r_{q_i} \in R_{q_i}$, then $\sum_{i=1}^{k} r_{q_i}$ is a non-zero element in R.

Whenever R is a graded semiring, the notation "$R = \sum_{q\in Z} R_q$" will be used to indicate that $\{R_q\}_{q\in Z}$ is the given collection of additive subsemigroups satisfying the above definition.

Theorem 3.3. *If $R = \sum_{q\in Z} R_q$ is a graded semiring, and if n and m are distinct integers, then $R_n \cap R_m = \emptyset$ or $R_n \cap R_m = \{0\}$, where 0 denotes the zero in R.*

Proof. Assume there exists a non-zero element x in R such that $x \in R_n \cap R_m$. Thus, $x \in R_n$ and $x \in R_m$ and it follows that x does not have a unique representation of the form $\sum_{i=1}^{k} r_{q_i}$. Consequently, R is not the internal direct sum of the collection $\{R_q\}_{q\in Z}$, a contradiction. Note that it may happen that $R_n \cap R_m = \emptyset$. □

Definition 3.4. Let $R = \sum_{q\in Z} R_q$ be a graded semiring. An element of R is said to be *homogeneous* if it belongs to an R_q, and it is said to be *homogeneous of degree* q if it belongs to R_q and is different from zero.

Since the graded semiring R is the internal direct sum of the collection $\{R_q\}_{q\in Z}$, it is clear that every non-zero element x in R can be written, in a unique way, as a finite sum of non-zero homogeneous elements of distinct degrees.

Definition 3.5. Let $R = \sum_{q\in Z} R_q$ be a graded semiring and let x be a non-zero element in R. If $\sum_{i=1}^{k} r_{q_i}$ is the decomposition of x into non-zero homogeneous elements of distinct degree, the homogeneous elements $r_{q_1}, r_{q_2}, \cdots, r_{q_k}$ will be called the *homogeneous components* of x, and the homogeneous component of x of least degree will be called the *initial component* of x.

Let R be a semiring and let $R[X]$ be the semiring of polynomials over R. If q is a non-negative integer, then R_q will denote the set consisting of the zero polynomial $O(X)$ together with all basic polynomials of degree q. If q is a negative integer, R_q will denote the set $\{O(X)\}$. It is clear that $\{R_q\}_{q\in Z}$ is a collection of additive subsemigroups of the semiring $R[X]$. With the aid of this notation, it can be shown that every polynomial semiring is a graded semiring.

Theorem 3.6. *If R is a semiring , then $R[X] = \sum_{q\in Z} R_q$ is a graded semiring, where the collection $\{R_q\}_{q\in Z}$ is defined as above.*

Proof. It is easy to observe that $R[X]$ is the internal direct sum of the collection $\{R_q\}_{q\in Z}$. If $a_q X^q$ and $b_{q'} X^{q'}$ are basic polynomials of degree q and q', respectively, then $a_q X^q \cdot b_{q'} X^{q'}$ is either $O(X)$ or a basic polynomial of degree $q+q'$. Thus, $R_q R_{q'} \subset R_{q+q'}$ for each $q, q' \in Z$. An inspection of the definition of addition in $R[X]$ shows that condition (3) in Definition 3.2 is satisfied, and the proof is complete. □

The following example will show that there exist graded semirings other than polynomial semirings, and it will be clear that the notion of a graded semiring is a generalization of the notion of a polynomial semiring.

Example 3.7. Let R be a semiring. Let $R_0 = R$ and let $R_q = \{0\}$ if q is a non-zero element in Z. It is clear that $\{R_q\}_{q\in Z}$ is a collection of additive subsemigroups of the semiring R. An inspection shows that $R = \sum_{q\in Z} R_q$ is a graded semiring. If R has only a finite number of elements, then the finite, graded semiring $R = \sum_{q\in Z} R_q$ cannot be a polynomial semiring, since polynomial semirings contain an infinite number of elements.

4. Homogeneous ideals in graded semirings

Definition 4.1. Let B be an ideal in the graded semiring $R = \sum_{q\in Z} R_q$. If the homogeneous components of each non-zero element in B belong to B, then B will be called a *homogeneous ideal*.

Theorem 4.2. *If B is an ideal in the commutative semiring R, then $B[X]$ is a homogeneous ideal in the graded semiring $R[X] = \sum_{q\in Z} R_q$.*

Proof. Let $f(X) = \sum_{n=0}^{\infty} a_n X^n$ be a non-zero polynomial in $B[X]$ and let $\sum_{i=1}^{k} a_{q_i} X^{q_i}$ be the decomposition

of $f(X)$ into its homogeneous components. Clearly, $f(X) \in B[X]$ implies $a_n \in B \cup \{0\}$ for each non-negative integer n. Consequently, $a_{q_i} X^{q_i} \in B[X]$ $i = 1, 2, \cdots, k$, and it follows that $B[X]$ is a homogeneous ideal in $R[X]$. □

Further examples of homogeneous ideals can be constructed as follows:

Example 4.3. Let B be an ideal in the semiring R. In view of Example 3.7, $R = \sum_{q \in Z} R_q$ is a graded semiring, and it is clear that B is a homogeneous ideal in R.

Definition 4.4. Let $R = \sum_{q \in Z} R_q$ and $R' = \sum_{q \in Z} {R_q}'$ be two graded semirings. A homomorphism η of R into R' is said to be *homogeneous of degree* s if $\eta(R_q) \subset R'_{q+s}$ for each $q \in Z$.

Theorem 4.5. *Let η be a homogeneous homomorphism of the graded semiring $R = \sum_{q \in Z} R_q$ into the graded semiring $R' = \sum_{q \in Z} {R_q}'$. If $\ker(\eta) \neq \emptyset$, then $\ker(\eta)$ is a homogeneous ideal in R.*

Proof. Suppose η is a homogeneous homomorphism of degree s. Let p be a non-zero element in $\ker(\eta)$ and let $\sum_{i=1}^{k} r_{q_i}$ be the decomposition of p into homogeneous elements of distinct degrees. Since $p \in \ker(\eta)$, it is clear that

$$\sum_{i=1}^{k} r_{q_i}\eta = (\sum_{i=1}^{k} r_{q_i})\eta = p\eta = 0.$$

Since η is a homogeneous homomorphism of degree s, it follows that $r_{q_i}\eta \in R_{q_i+s}, i = 1, 2, \cdots, k$. Condition (3) of Definition 2 implies $r_{q_i}\eta = 0, i = 1, 2, \cdots, k$. Thus, the homogeneous components of p belong to $\ker(\eta)$ and the proof is complete. □

With the aid of the following definitions, a necessary and sufficient condition can be given for an ideal to be homogeneous in a graded semiring.

Definition 4.6. Let Φ be a set of non-zero elements in the semiring R. A *linear combination* in Φ is a sum $\sum_{i=1}^{k} r_i b_i$ where $r_1, r_2, \cdots, r_k$ are elements in R and $b_1, b_2, \cdots, b_k$ are elements in Φ.

Definition 4.7. Let B be an ideal in the semiring R. A subset Φ of B will be called a *basis* for B if B is the set of all linear combinations in Φ.

Theorem 4.8. *Let R be a graded semiring and let B be an ideal in R. If B has a basis, then B is homogeneous if and only if B has a basis consisting of homogeneous elements.*

Proof. Suppose B is a homogeneous ideal and let Φ be a basis for B. It is clear that $\Phi \subset B$. If $x \in \Phi$, let Φ_x denote the set of homogeneous component of x, and let $\Phi^* = \cup_{x \in \Phi} \Phi_x$. Since B is homogeneous, it follows that $\Phi_x \subset B$ for each $x \in \Phi$. Thus, Φ^* is a set of homogeneous elements and $\Phi^* \subset B$. Since B is an ideal, any linear combination in Φ^* is an element in B. Let p be an element in B. Since Φ is a basis for B, there exist elements $r_1, r_2, \cdots, r_n$ in R and there exist elements $b_1, b_2, \cdots, b_n$ in Φ such that $p = \sum_{j=1}^{n} r_j b_j$. Let $\sum_{i=1}^{m_j} {b_i}^j$ be the decomposition of b_j into homogeneous components. It is clear that ${b_i}^j \in \Phi^*$, for each i and j. Moreover,

$$p = \sum_{j=1}^{n} r_j b_j = \sum_{j=1}^{n} r_j (\sum_{i=1}^{m_j} {b_i}^j)$$
$$= \sum_{j=1}^{n} (\sum_{i=1}^{m_j} r_j {b_i}^j).$$

Consequently, p is a linear combination in Φ^*. Thus, if B has a basis, then the fact that B is homogeneous implies that B has a basis consisting of homogeneous elements.

Conversely, suppose that B has a basis Φ consisting of homogeneous elements. Let p be a non-zero element in B and let $\sum_{i=1}^{k} p_{q_i}$ be the decomposition of p into its homogeneous components. It will be shown that $p_{q_i} \in B$ for each i, and it will follow that B is homogeneous. Since Φ is a basis for B, there exist elements $r_1, r_2, \cdots, r_n$ in R and there exist elements $b_1, b_2, \cdots, b_n$ in Φ such that $p = \sum_{i=1}^{n} r_i b_i$. Let $\sum_{j=1}^{n_i} {r_j}^i$ be the decomposition of r_i into its homogeneous components. Clearly,

$$p = \sum_{i=1}^{n} r_i b_i = \sum_{i=1}^{n} (\sum_{j=1}^{n_i} {r_j}^i) b_i$$
$$= \sum_{i=1}^{n} (\sum_{j=1}^{n_i} {r_j}^i b_i),$$

and ${r_j}^i b_i$ is a homogeneous element in R. Thus, the above sum is the sum of homogeneous elements. If all terms of the same degree are combined into one term, it is clear that $\sum_{i=1}^{n} (\sum_{j=1}^{n_i} {r_j}^i b_i)$ is the decomposition of p into its homogeneous components. Consequently, each p_{q_i} must be a linear combination in Φ. Thus, each $p_{q_i} \in B$. □

Theorem 4.8 indicates a method for generating homogeneous ideals in a graded semiring with an identity. In view of Theorem 4.2, Example 4.3 and the above result, it is clear that the homogeneous ideals form a large class of ideals in graded semirings.

The following theorem will show that the class of homogeneous ideals in a graded semiring with zero is closed under the standard ideal-theoretic operations.

Theorem 4.9. *Let A and B be ideals in a graded semiring R. If A and B are homogeneous, then $A+B$ and $A \cap B$ are homogeneous ideals in R.*

Proof. It is clear that $A + B$ and $A \cap B$ are ideals in R. Let p be a non-zero element in $A + B$. Thus, $p = a + b$ where $a \in A$ and $b \in B$. Let $\sum {p_{q_i}}^a$ and $\sum {p_{q_j}}^b$ be the decompositions of a and b, respectively, into their homogeneous components. Since A and B are homogeneous, it is clear that each ${p_{q_i}}^a \in A$ and each ${p_{q_j}}^b \in B$. Clearly,

$$p = a + b$$
$$= \sum {p_{q_i}}^a + \sum {p_{q_j}}^b = \sum p_{q_m},$$

where the terms of like degree are combined to form a single term in the last sum. Thus, $\sum p_{q_m}$ is the decomposition of p into its homogeneous components, and it is clear that each $p_{q_m} \in A + B$. Thus, $A + B$ is a homogeneous ideal

in R. The assertion relative to $A \cap B$ results trivially from the definition of homogeneous ideal. □

In [1], the authors defined and investigated several notions from ideal theory in commutative semirings. An ideal in the semiring R is said to be *prime* provided (1) $P \neq R$; and (2) if A and B are ideals in R such that $AB \subset P$, then $A \subset P$ or $B \subset P$, where $AB = \{ab \,|\, a \in A \text{ and } b \in B\}$. The semiring R is said to be an *A-semiring* provided (1) R has commutative multiplication and (2) every proper ideal in R is contained in a prime ideal of R. Whenever B is a proper ideal in an R, the *radical* of B is denoted by $\sqrt{B}$ and is defined to be the intersection of all prime ideals in R that contain B.

An ideal B in the semiring R is said to be a *k-ideal* in R if $b \in B$ and $r \in R$, then $b + r \in B$ (or $r + b \in B$) implies $r \in B$. Such ideals were of special interest to S. Bourne [3], M. Henrikson [7], D. R. La Torree [8] and P. J. Allen [2].

Theorem 4.10. *Let B be a proper, homogeneous ideal in a graded, R. If $\sqrt{B}$ is a k-ideal in R, then $\sqrt{B}$ is a homogeneous ideal in R.*

Proof. Let p be a non-zero element in $\sqrt{B}$ and let $\sum_{i=1}^{n} r_{q_i}$ be the decomposition of p into its homogeneous components, where $q_1 < q_2 < \cdots < q_n$. Since $p \in \sqrt{B}$, there exists a positive integer m such that $p^m \in B$. Thus,

$$r_{q_1}{}^m + X = (\sum_{i=1}^{n} r_{q_i})^m = p^m \in B,$$

where X consists of terms of degree greater that mq_1. Thus, $r_{q_1}{}^m$ is the homogeneous component of p^m of degree mq_1. Since B is homogeneous, it is clear that $r_{q_1}{}^m \in B$. Thus, $r_{q_1} \in \sqrt{B}$. Since $p, r_{q_1} \in \sqrt{B}$ and $\sqrt{B}$ is a k-ideal in R, $p = r_{q_1} + \sum_{i=2}^{n} r_{q_i} \in \sqrt{B}$ implies $\sum_{i=2}^{n} r_{q_i} \in \sqrt{B}$. By the same argument, $r_{q_2} \in \sqrt{B}$ and $\sum_{i=3}^{n} r_{q_i} \in \sqrt{B}$. Continuing in this fashion, it is clear that the homogeneous components of p belong to $\sqrt{B}$. □

To conclude this paper, necessary and sufficient conditions will be given in order that a homogeneous k-ideal in a graded semiring be completely prime.

Theorem 4.11. *Let P be a homogeneous k-ideal in the graded semiring R. In order that P be completely prime in R, it is necessary and sufficient that $fg \in P$ implies $f \in P$ or $g \in P$ for homogeneous elements in R.*

Proof. Let F and G be elements in $R - P$ and let $\sum f_i$ and $\sum g_j$ be the decompositions of F and G, respectively, into their homogeneous components. Clearly, $F \notin P$ and $G \notin P$ imply there exist homogeneous components of least degree f_{i_0} and g_{j_0} of F and G, respectively, which are not in P. Consequently, $f_{i_0} g_{j_0} \notin P$. Assume $(\sum_{i \geq i_0} f_i)(\sum_{j \geq j_0} g_j) \in P$. Thus, $f_{i_0} g_{j_0} + X \in P$ where X consists of terms of degree greater that $i_0 + j_0$. Since $f_{i_0} g_{j_0}$ is the homogeneous component of degree $i_0 + j_0$ of an element in the homogeneous ideal P, it is clear that $f_{i_0} g_{j_0} \in P$, a contradiction. Therefore, $(\sum_{i \geq i_0} f_i)(\sum_{j \geq j_0} g_j) \notin P$. Assume $FG \in P$. Thus, the sum

$$(\sum_{i<i_0} f_i)(\sum_{j<j_0} g_j) + (\sum_{i<i_0} f_i)(\sum_{j \geq j_0} g_j) +$$
$$(\sum_{i \geq i_0} f_i)(\sum_{j<j_0} g_j) + (\sum_{i \geq i_0} f_i)(\sum_{j \geq j_0} g_j)$$

is equal to the product

$$(\sum_{i<i_0} f_i + \sum_{i \geq i_0} f_i)(\sum_{j<j_0} g_j + \sum_{j>j_0} g_j)$$

which is in turn equal to FG an element in P. Since the first three terms of the above sum are elements in P, and since P is a k-ideal in R, it follows that

$$(\sum_{i \geq i_0} f_i)(\sum_{j>j_0} g_j) \in P,$$

a contradiction. Therefore, $F, G \notin P$ imply $FG \notin P$, and it follows that P is a completely prime ideal in R. □

Let R be a graded semiring. Suppose that $B = P_1 \cap \cdots \cap P_n$, where the P_i are prime homogeneous k-ideals. Suppose also that $fg \in B$, $f_i \notin P_i$ implies $g \in P_i$ for $i = 1, \cdots, n$ when f and g are homogeneous elements. Then it follows that $FG \in B$ implies, for each $i = 1, \cdots, n$, if $F \notin P_i$ then $G \in P_i$. Furthermore, $B = \sqrt{B}$ and $B = \sqrt{C} \cap \sqrt{D} = C \cap D$, where $C = \cap\{P_i | F \in P_i\}$, $D = \{P_j | G \in P_j\}$. Here if $\cap\{P_i | F \in P_i\}$ contains no P_i, we let $C = R$, while if $\cap\{P_i | G \in P_j\}$ contains no P_j, we let $D = R$.

5. Conclusion

In this paper we have considered the situation where R is an A-semiring and where the semiring is graded so that one may consider proper homogeneous ideals B and their radicals $\sqrt{B}$ which were shown to be homogeneous as well if they were k-ideals and furthermore necessary and sufficient conditions were found for a homogeneous k-ideal P to be completely prime. Looking forward questions arise concerning the softening of conditions under which the equivalents (i.e., generalizations) of Theorems 4.10 and 4.11 may be deduced. Thus, one may look at the role played by A-semirings in the arguments above and of course the question about the condition on ideals being k-ideals as being replaceable through conditions on the semiring R itself. The extents to which all this may be done is a set of questions and topics of possible interest in future investigations.

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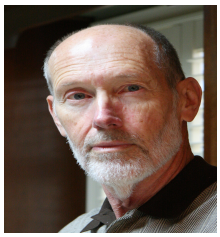

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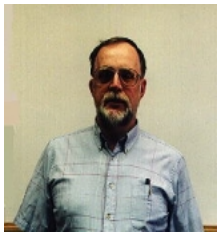

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