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New Aspects of Rayleigh Distribution under Progressive First-Failure Censoring Samples

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Abstract: This article considers the problem of estimating the unknown parameters of the compound Rayleigh distribution with progressive first-failure censoring scheme during step-stress partially accelerated life tests (ALT). Progressive first-failure censoring and accelerated life testing are performed to decrease the duration of testing and to lower test expenses. The maximum likelihood estimators (MLEs) and Bayes estimates (BEs) for the distribution parameters and acceleration factor are obtained. The optimal time for stress change is determined. Furthermore, the approximate, bootstrap and credible confidence intervals (CIs) of the parameters are derived. Methods of Markov chain Monte Carlo (MCMC) are used to obtain the Bayes estimates. Finally, the accuracy of the MLEs and BEs for the model parameters is investigated through simulation studies.

Keywords: Compound Rayleigh distribution; Step stress partially accelerated life test; Progressive first failure censoring; Maximum likelihood estimation; Bayes estimation; MCMC .

1 Introduction

Accelerated life test (ALT) is a popular experimental strategy to obtain information on life distribution of highly reliable products. Test units under normal operating conditions are often extremely reliable, with significant mean times to failure. ALT experiments may be used to obtain reliable information on product components within a short period by subjecting them to higher-than-usual stress (pressure, temperature, voltage, etc.). [1,2] introduced and studied the concept of ALT. Data collected under such accelerated conditions are then extrapolated through appropriate statistical model.

In the ALT, the experiment can either be started at higher stresses than normal and continued under these conditions or it can be stated under normal conditions. Thus, there are two types of ALT. The first is said to be the ordinary accelerated life test (OALT), and the second is the partially accelerated life tests (PALTs). The major assumption in OALT is that the mathematical model relating the lifetime of the unit to the stress must be known or can be assumed. In several cases, this life stress relationships are not known and can't be supposed, i.e. OALT data can't be extrapolated to normal condition. So, PALT is the more proper test to be performed, where, the tested units are undergone by both accelerated and normal conditions. PALT are of two types, constant stress PALTs (see [3,4]) and step stress PALTs (see [5]).

In constant PALTs, every unit is run at constant high stress until either the test terminates or all units fail; for more specifics about constant-stress ALT, see [6,7]. In step PALTs, the stress on each unit is not fixed but is increased step by step at personified times or simultaneously to the appearance of a fixed number of failures. When the test contain two levels of stress, it's indicated to as a simple step-stress ALT. Several authors discuss step-stress PALTs scheme for example, see [8,9, 10,11,12]. Several studies have employed Bayesian estimation based on ALT, for example, see [13,14,15,16, 17,18,19].

Type-I and type-II censoring are the most two common censoring schemes in life testing, but these kind of censoring don't allow units to be extracted from the experiment at any other point than the last termination point. For this reason, the progressive censoring scheme has been very famous for analyzing extremely reliable data.

Progressive censoring schemes introduced by [20], or in the review by [21]. [22] described a life test where the experimenter units set in to different groups, each as an component of test units, and then all of them run until the

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first failure in each cluster. This scheme is called first-failure censoring. [23] studied a sampling experiment for a bearing manufacturer. The bearing test engineer wanted to reduce test time by testing 50 bearings in sets of 10 each, where, the first failure times from each group were observed. If an experimenter wish to take out some sets of test units before the first failures in these sets; this life test experiment is named as a progressive first-failure censoring scheme, as specified by [24].

The compound Rayleigh distribution (α,β), denoted by CRD, supplies a population model which is valuable in life testing and reliability. The probability density function (pdf), and the cumulative distribution function (CDF) are presented, respectively, by

$$f(x)=2\alpha\beta^{\alpha}x(\beta+x^2)^{-(\alpha+1)},\ \alpha,\beta>0,\ x>0, \tag{1}$$

and

$$F(x)=1-\beta^{\alpha}(\beta+x^2)^{-\alpha}. \tag{2}$$

Also the failure rate and the reliability functions, at the specified t, are

$$H(t)=\frac{2\alpha t}{\beta+t^2},\ t>0, \tag{3}$$

$$S(t)=\beta^{\alpha}(\beta+t^2)^{-\alpha},\ t>0, \tag{4}$$

where β and α are the scales and the shape parameters. The CRD is a particular case of the 3-parameter Burr type XII distribution. Many authors studied the 2-parameter CRD, including [25]. The step-stress ALT with progressive first-failure censoring from Weibull distribution are considered in [26]. The MLEs is given for the distribution parameters and the acceleration factor. The point estimation and interval estimation for Lindley distribution parameter and the acceleration factor is obtained with step-stress accelerated life test under progressive first failure sample in [26].

The novelty of this study is the application of the step stress PALT to compound Rayleigh failure time distribution using the progressive first-failure censoring, life test. Maximum likelihood estimators and Bayes estimates for the parameters are then calculated using the method of MCMC.

The paper is organized as follows: Section 2 describes the lifetime model and test assumptions. In Section 3, the MLEs of the model parameters with the simple step-stress ALT are derived. Estimation of optimal time of stress change time is given in Section 4. The Bayes estimates of model parameters using the MCMC method are obtained in Section 5. In Section 6, the approximate, bootstrap and credible confidence intervals are derived. Section 7 discusses the simulation studies. Conclusions are presented in Section 8.

2 Model Description and Basic Assumptions

2.1 A progressive first-failure-censoring scheme

In this subsection, the progressive censoring scheme is jointed with the first-failure censoring scheme as in [24]. Let n independent groups with k items are set in a life test, R_1 groups and the group in which the first failure is observed are randomly removed from the test when the first failure (say $Y^{\mathbf{R}}_{1:m:n:k}$) has occurred, R_2 groups and the group in which the second first failure is observed are randomly removed from the test promptly when the second failure (say $Y^{\mathbf{R}}_{2:m:n:k}$) has happened, and lastly $R_m, m\leq n$ groups and the group in which the $m-th$ first failure is obtained are randomly removed from the test when the $m-th$failure $(Y^{\mathbf{R}}_{m:m:n:k})$ has happened. The $Y^{\mathbf{R}}_{1:m:n:k}<Y^{\mathbf{R}}_{2:m:n:k}<\ldots<Y^{\mathbf{R}}_{m:m:n:k}$are titled progressively first-failure-censored order statistics with the progressive censoring scheme $\mathbf{R}=(R_1,R_2,\ldots,R_m)$. It is clear that $n=m+\sum_{i=1}^{m}R_i$. If the failure times of the $n\times k$ items originally on the test are from a continuous distribution with pdf $f(y)$ and df $F(y)$, the joint probability density function of $Y^{\mathbf{R}}_{1:m:n:k},Y^{\mathbf{R}}_{2:m:n:k},\ldots,Y^{\mathbf{R}}_{m:m:n:k}$ is presented by

$$f_{1,2,\ldots,m}(y^{\mathbf{R}}_{1:m:n:k},y^{R}_{2:m:n:k},\ldots,y^{\mathbf{R}}_{m:m:n:k})=Ck^m\prod_{j=1}^{m}f(y^{R}_{j:m:n:k})\times\left[1-F(y^{R}_{j:m:n:k})\right]^{k(\mathbf{R}_j+1)-1}, \tag{1}$$

$$0<y^{\mathbf{R}}_{1:m:n:k}<y^{\mathbf{R}}_{2:m:n:k}<\ldots<y^{\mathbf{R}}_{m:m:n:k}<\infty,$$

where

$$C=n\prod_{j=1}^{m-1}(n-\sum_{i=1}^{j}R_i-j).$$

Special cases:
The following four censoring schemes are special cases from (5):
(1) The first-failure censored scheme obtained when $R=(0,0,\ldots,0)$.
(2) When $k=1$, we obtained the progressive type-II censored order statistics.
(3) In case of $R=(0,0,\ldots,n-m)$ and $k=1$ we obtained type II censored order statistics.
(4) If $R=(0,0,\ldots,0)$ and $k=1$, we gained the complete sample.
From the distribution function $1-(1-F(x))^k$, $Y^{\mathbf{R}}_{1;m,n,k},Y^{\mathbf{R}}_{2;m,n,k},\ldots,Y^{\mathbf{R}}_{m;m,n,k}$ can be sighted as a progressive type-II censored sample. so that, results for progressive type-II censored can be expanded to progressive first-failure censored order statistics. The progressive first-failure censored order statistics are interesting because they reduce the test time, where, many items are utilized, where, just m of $n\times k$ items are failures.

2.2 Basic assumptions and test procedure

Throughout the paper the following assumptions are used in the scope of step stress PALT:

1. n identical and independent groups with k items within each set are put on a life test; each unit has CRD.
2. The test is concluded at the $m-th$ failure, where m is prespecified $(m \leq n)$.
3. The units are first put in normal condition, if it does not fail or removed from the test by presignified time τ, it's run under accelerated condition.
4. At the $i-th$ failure, a random number of the surviving groups $R_i, i = 1,2,...,m-1$, and the group in which the failure $Y^R_{i;m,n,k}$ has occurred are randomly removed from the test. Finally, at the m_{th} failure the remaining surviving group's $R_m = n-m-\sum_{i=1}^{m-1} R_i$ are all extracted from the test, and the test is concluded.
5. Let $N_1 = \sum_{i=1}^{n_1}$ be the number of failures before time τ_κ at normal condition, and $N_2 = \sum_{i=n_1+1}^{m}$ be the number of failures before(after) time τ_κ at stress condition and $N = N_1 + N_2$, with this procedure the spotted progressive first-failure censored data are

$$y^R_{1:m:n:k} < ... < y^R_{n_1:m:n:k} < \tau < y^R_{n_1+1:m:n:k} < ... < y^R_{m:m:n:k},$$

Where $\sum_{i=1}^{m} R_i = n-m$ and $R = (R_1, R_2, ..., R_m)$.
Then the influence of this process is to multiply the remaining lifetime of the item by the inverse of the acceleration factor. Where, the changing to the higher stress level will shorten the life of test item. The total lifetime Y passes through two stages: normal and accelerated conditions.
Thus the total lifetime Y of the groups in SSPALT is presented as follows:

$$Y = \begin{cases} T, & T < \tau \\ \tau + \lambda^{-1}(T-\tau), & T > \tau, \end{cases} \tag{6}$$

where T is the first failure lifetime of a group under the conditions used, λ is the acceleration factor, and τ is the time-change stress. $\lambda > 1$ is the ratio of mean life under the conditions used to that under accelerated conditions. Suppose that the lifetime of the test item follows CRD. Thus, the pdf of the total lifetime Y is presented by

$$f(y) = \begin{cases} 0, & y < 0 \\ f_1(y), & 0 < y \leq \tau \\ f_2(y), & y > \tau, \end{cases} \tag{7}$$

where $f_1(y)$, is presented by (1) and $f_2(y)$, given by

$$f_2(y) = 2\alpha\beta^{\alpha}\lambda(\tau+\lambda(y-\tau))(\beta+[\tau+\lambda(y-\tau)]^2)^{-(\alpha+1)}. \tag{8}$$

is obtained by the transformation variable technique using equations (7) and (8). The cumulative distribution function cdf, reliability function $S_2(t)$, and hazard rate function $h_2(t)$ are presented by

$$F_2(x) = 1-\beta^{\alpha}(\beta+[\tau+\lambda(y-\tau)]^2)^{-\alpha}, \tag{9}$$

$$S_2(t) = \beta^{\alpha}(\beta+[\tau+\lambda(y-\tau)]^2)^{-\alpha}, \tag{10}$$

and

$$h_2(t) = \frac{2\alpha\lambda(\tau+\lambda(y-\tau))}{(\beta+[\tau+\lambda(y-\tau)]^2)}. \tag{11}$$

In progressive first failure censoring, the test terminates when the first failure censoring numbers reach to $m < n$. From the total lifetime Y, the observed values are $y_{1;m,n,k} < y_{2;m,n,k} < ... < y_{n_1;m,n,k} < \tau < y_{n_1+1;m,n,k} < ... < y_{m;m,n,k}$ where n_1 is the number of groups failed under normal conditions and $m-n_1$ under accelerated conditions. Let us determine the indicator functions

$$\delta_i = \begin{cases} 1 & y_{i;m,n,k} \leq \tau \\ 0 & \text{otherwise} \end{cases}, \; i = 1,\ 2,\ ...,\ m \tag{12}$$

For simplicity let us assume the first failure lifetimes $y_1 < y_2 < ... < y_m$ other than $y_{1;m,n,k} < y_{2;m,n,k} < ... < y_{m;m,n,k}$ of m groups are identically and independent distribution, consequently the likelihood function is presented by

$$L(\alpha,\beta,\lambda|\underline{y}) = Ck^m \prod_{i=1}^{m} \left[f_1(y_i)[S_1(y_i)]^{k(R_i+1)-1} \right]^{\delta_i} \times \left[f_2(y_i)[S_2(y_i)]^{k(R_i+1)-1} \right]^{1-\delta_i} \tag{2}$$

$$0 < y_1 < y_2 < ... < y_{n_1} < \tau < y_{n_1+1} < ... < y_m < \infty,$$

where C given in (5).
Special cases

1) If $\tau \to 0$, then the experiment run only under accelerate conditions.
2) If $\tau \to \infty$, then the experiment runs only under use conditions.

3 Maximum Likelihood Estimation

The aims of MLE is to specify the parameters which maximize the likelihood function of the sample data. The method of ML is used to be more robust and produces estimators with perfect statistical properties. Numerical techniques are used to compute them, which is based on progressive first-failure censoring data under step stress PALT.

Let $y_{i,m,n,k} = y_i$ be the observed values of the lifetime T given from the progressive first failure censoring. The likelihood function $L(\alpha,\beta,\lambda|\underline{y})$ in (13) with two distributions (7) and (8), with censoring scheme $\mathbf{R} = (R_1,$ R_2, ..., $R_m)$ is given by

$$L(\alpha,\beta,\lambda|\underline{y}) \propto \alpha^m\beta^{kn\alpha}\lambda^{m-n_1} \exp\left\{ -\sum_{i=1}^{n_1}(\alpha k(R_i+1)+1)\times\log\left[\beta+y_i^2\right] + \sum_{i=n_1+1}^{m}\log\left[\tau+\lambda(y_i-\tau)\right] - \sum_{i=n_1+1}^{m}(\alpha k(R_i+1)+1)\log\left[\beta+\left(\tau+\lambda(y_i-\tau)\right)^2\right] \right\}, \tag{3}$$

then the log-likelihood function of $L(\alpha,\beta,\lambda|\underline{y})$ is specified by

$$\ell(\alpha,\beta,\lambda|\underline{y}) = m\log\alpha + kn\alpha\log\beta + (m-n_1)\log\lambda - \sum_{i=1}^{n_1} (\alpha k(R_i+1)+1)\log\left[\beta+y_i^2\right] + \sum_{i=n_1+1}^{m}\log\left[\tau+\lambda(y_i-\tau)\right] - \sum_{i=n_1+1}^{m}(\alpha k(R_i+1)+1)\log\left[\beta+(\tau+\lambda(y_i-\tau))^2\right] \quad (4)$$

After calculating the first partial derivatives of (15) on α, β and λ and equating each to zero, we obtain the likelihood equations as

$$\frac{\partial\ell(\alpha,\beta,\lambda|\underline{y})}{\partial\alpha} = \frac{\partial\ell(\alpha,\beta,\lambda|\underline{y})}{\partial\beta} = \frac{\partial\ell(\alpha,\beta,\lambda|\underline{y})}{\partial\lambda} = 0 \quad (16)$$

hence

$$\alpha(\beta,\lambda) = \frac{m/k}{D_1+D_2-n\log\beta} \quad (17)$$

where

$$D_1 = \sum_{i=1}^{n_1}(R_i+1)\log\left[\beta+y_i^2\right], \quad (18)$$

$$D_2 = \sum_{i=n_1+1}^{m}(R_i+1)\log\left[\beta+(\tau+\lambda(y_i-\tau))^2\right]. \quad (19)$$

Also

$$\frac{kn}{\beta} - \sum_{i=1}^{n_1}\frac{k(R_i+1)+\frac{1}{\alpha}}{\beta+y_i^2} - \sum_{i=n_1+1}^{m}\frac{k(R_i+1)+\frac{1}{\alpha}}{\beta+(\tau+\lambda(y_i-\tau))^2} = 0, \quad (20)$$

and

$$\frac{m-n_1}{\lambda} + \sum_{i=n_1+1}^{m}\frac{(y_i-\tau)}{\tau+\lambda(y_i-\tau)} - 2\sum_{i=n_1+1}^{m}\frac{(\alpha k(R_i+1)+1)(y_i-\tau)(\tau+\lambda(y_i-\tau))}{\beta+(\tau+\lambda(y_i-\tau))^2} = 0 \quad (5)$$

Consequently, the likelihoods equations are written in the two nonlinear equation (20) and (21). In order to solve it numerically on β and λ we use quasi-Newton Raphson, to obtain the MLE, $\hat{\beta}$ and $\hat{\lambda}$ and the MLE of α say $\hat{\alpha}$ by substituting of $\hat{\beta}$ and $\hat{\lambda}$.

4 Estimation of Optimal Time-Change Stress

The optimal time-change stress τ^* is determined in this section by minimizing the asymptotic variance of MLEs of the acceleration factor and the model parameters. The asymptotic variance of $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\lambda}$ is obtained using the diagonal entries of the inverse of the Fisher information matrix. With the assumption that $\alpha = 0.1$,$\beta = 1.5$ and $\lambda = 2.5$, then for $k = 3, n = 50, m = 30$ and $C.SI$, which are the true values of the population parameters and the acceleration factor. The minimum option is used in Mathematica 9 to specify the time τ^*, τ^* minimizes the asymptotic variance of MLEs of the acceleration factor and the model parameters. So $\tau^* = 1.1261$. see [27].

5 Bayes Estimation of the Model Parameters

In this section, both symmetric loss (square error loss (SEL)) function and asymmetric loss (linear exponential (LINEX) and general entropy (GE)) loss function are investigated to get (BEs) of the parameters (α, β, and λ) with progressive first-failure censoring. In different effective cases, information about the parameters is available independently. So the independent prior is specified for β and α, and the noninformative prior (NIP) for the acceleration factor λ. We used the gamma priors for the shape and the scale parameters because it's wealthy to cover the previous trust of the experimenter. The independent gamma prior for β and α is given, respectively, as follows:

$$\pi_1^*(\alpha) \propto \alpha^{a-1}exp(-b\alpha), (\alpha>0), \quad (22)$$

and

$$\pi_2^*(\beta) \propto \beta^{c-1}exp(-d\beta), (\beta>0), \quad (23)$$

Also, the NIP for the acceleration factor λ is given by

$$\pi_3^*(\lambda) \propto \frac{1}{\lambda}, (\lambda>0), \quad (24)$$

consequently, from equations (22), (23) and (24) the joint prior can be accurate as

$$\pi^*(\alpha,\beta,\lambda) \propto \alpha^{a-1}\beta^{c-1}\lambda^{-1}exp(-b\alpha-d\beta), (\lambda,\alpha,\beta>0), \quad (25)$$

The joint posterior density of β,α and λ, indicated by $\pi(\alpha,\beta,\lambda|\underline{y})$ are apparent as:

$$\pi(\alpha,\beta,\lambda|\underline{y}) = \frac{L(\alpha,\beta,\lambda|\underline{y})\pi^*(\alpha,\beta,\lambda)}{\int_0^\infty\int_0^\infty\int_0^\infty L(\alpha,\beta,\lambda|\underline{y})\pi^*(\alpha,\beta,\lambda)d\alpha d\beta d\lambda}, \quad (26)$$

Thus, the Bayes estimate of α, β and λ, say $\varphi(\alpha,\beta,\lambda)$, with the squared error loss function (SEL), is gained by

$$\hat{\varphi}(\alpha,\beta,\lambda) = E_{\alpha,\beta,\lambda|\underline{y}}(\varphi(\alpha,\beta,\lambda)) = \frac{\int_0^\infty\int_0^\infty\int_0^\infty \varphi(\alpha,\beta,\lambda)L(\alpha,\beta,\lambda|\underline{y})\pi^*(\alpha,\beta,\lambda)d\alpha d\beta d\lambda}{\int_0^\infty\int_0^\infty\int_0^\infty L(\alpha,\beta,\lambda|\underline{y})\pi^*(\alpha,\beta,\lambda)d\alpha d\beta d\lambda}.$$

The ratio of three integrals given by (27) can't be gained in a closed form. In this case, we use the MCMC method to create samples from the posterior distributions to compute the Bayes estimator of $\varphi(\alpha,\beta,\lambda)$ with SEL function.
By choosing $\Phi(u(\phi),\delta) = e^{a(\delta-u(\phi))} - a(\delta-u(\phi)) - 1$, we get the $LINEX$ loss function written as

$$\delta_{a,\Psi,\delta_o}(\underline{y}) = -\frac{1}{a}ln[E(e^{-au(\phi)}|\underline{y})], \quad (28)$$

Where $a \neq 0$ is the shape parameter of $LINEX$ loss function. The Bayes estimate of $u(\phi)$ gathers the form

$$\hat{\phi}_{BL} = -\frac{1}{a}\ln[\int_0^\infty\int_0^\infty\int_0^\infty e^{-a\phi}\pi^*(\phi|\underline{y})d\alpha d\beta d\lambda]. \quad (29)$$

The GE loss function is formed as

$$\delta_{a,\Psi,\delta_o}(\underline{y}) = [E(u(\phi)^{-a}|\underline{y})]^{-\frac{1}{a}}, \quad (30)$$

Where $a \neq 0$ is the shape parameter of GE loss function. For which the Bayes estimate of $u(\phi)$ takes the form

$$\hat{\phi}_{GE} = [\int_0^\infty\int_0^\infty\int_0^\infty u(\phi)^{-a}\pi^*(\phi|\underline{y})d\alpha d\beta d\lambda]^{-\frac{1}{a}}. \quad (31)$$

Unfortunately, we can't calculate the integrals in equations (28), (29) and (31) exactly. So, the MCMC technique is applied to approximate the integrals.

5.1 Bayesian estimation using MCMC method

The MCMC technique is a good method for parameter estimation. Several schemes of MCMC are obtainable, one significant sub-class of MCMC methods is Gibbs sampling and more general Metropolis within-Gibbs samplers. MCMC has the feature over the MLE method that by structuring the probability intervals based on the experimental posterior distribution, we get a appropriate interval estimate of the parameters.
The joint posterior density function of β,α, and λ are:

$$\pi(\alpha,\beta,\lambda_y)\alpha^{m+a-1}\beta^{kn\alpha+c-1}\lambda^{m-n_1-1}$$
$$e^{-b\alpha-d\beta-\Sigma_{i=1}^{n_1}(\alpha k(R_i+1)+1)\log[\beta+y_i^2]}$$

$$+\sum_{i=n_1+1}^{m}\log[\tau+\lambda(y_i-\tau)] - \sum_{i=n_1+1}^{m}(\alpha k(R_i+1)+1)\times\log\left[\beta+(\tau+\lambda(y_i-\tau))^2\right]\Bigg\}.$$

The conditional posterior pdf's of the parameters α,β, and λ using the conjugate prior can be computed by

$$\pi_1(\alpha|\beta,\lambda,\underline{y}) \propto Gamma(m+a, b-knlog\beta - k\sum_{i=1}^{n_1}(R_i+1)$$
$$\times\log[\beta+y_i^2] - k\sum_{i=n_1+1}^{m}(R_i+1)\log[\beta+(\tau+\lambda(y_i-\tau))^2]$$

where

$$\pi_2(\beta|\alpha,\lambda,y)\beta^{\alpha kn+c-1}e^{-d\beta-\sum_{i=1}^{n_1}(\alpha k(R_i+1)+1)\times\log[\beta+y_i^2]-\sum_{i=n_1+1}^{m}(\alpha k(R_i+1)+1)\times\log[\beta+(\tau+\lambda(y_i-\tau))^2]}.$$

and

$$\pi_3(\lambda|\alpha,\beta,\underline{y}) \propto \lambda^{m-n_1-1}\exp\left\{\sum_{i=n_1+1}^{m}\log[\tau+\lambda(y_i-\tau)] - \sum_{i=n_1+1}^{m}(\alpha k(R_i+1)+1)\log[\beta+(\tau+\lambda(y_i-\tau))^2]\right. \quad (6)$$

Figures(1-3)shows the number of simulation of CRD parameters generated by the MCMC method and the corresponding histogram. The plots of (34) show that they are identical to a normal distribution, see Figure(2). Therefore to generate these distributions, we utilize the Metropolis-Hastings method [28] together with normal proposal distribution.
The algorithm of Metropolis-Hastings method is as follows
1. Start with initial guess of $\beta^{(0)} = \beta, \alpha^{(0)} = \alpha$ and $\lambda^{(0)} = \lambda$.
2. Set $I = 1$.
3. Generate $\alpha^{(I)}$ from Gamma distribution $\pi_1^*(\alpha|\beta^{I-1},\lambda^{I-1},\underline{y})$.
4. Using Metropolis-Hastings, generate $\beta^{(I)}$ from $\pi_2^*(\beta|\alpha^{I-1},\lambda^{I-1},\underline{y})$ with the $N(\beta^{(I-1)},v_{22})$ proposal distribution. Where v_{22} is from the variances-covariances matrix.
5. Using Metropolis-Hastings, generate $\lambda^{(I)}$ from $\pi_3^*(\lambda|\alpha^{I-1},\beta^{I-1},\underline{y})$ with the $N(\lambda^{(I-1)},v_{33})$ proposal distribution. Where v_{33} is from the variances-covariances matrix.
6. Compute $\alpha^{(I)}$, $\beta^{(I)}$, and $\lambda^{(I)}$.
7. Put $I = I+1$.
8. Repeat steps 3-6 N times.
9. The approximate Bayes MCMC point estimate of $\phi_I(\phi_1 = \alpha, \phi_2 = \beta, \phi_3 = \lambda)$ under SEL and $LINEX$ loss functions, respectively, are expressed by

$$E(\phi_I|data) \propto \frac{1}{N-M}\sum_{i=M+1}^{N}\phi_I^{(i)}, \quad (36)$$

$$E(\exp[-c\phi_I]|data) \propto \frac{1}{N-M}\sum_{i=M+1}^{N}\exp[-c\phi_I^{(i)}], \quad (37)$$

Where M is the burn-in period (that is, some iterations before the stationary distribution is carried out) and the posterior variance of ϕ_I becomes

$$\hat{\nabla}(\phi_I|data) \propto \frac{1}{N-M}\sum_{i=M+1}^{N}(\phi_I^{(i)} - \hat{E}(\phi_I|data))^2, \quad (38)$$

6 Interval Estimation

The approximate, credible and bootstrap confidence intervals (CIs) of the parameters α, β, and λ are discussed in this section.

6.1 Approximate confidence intervals CIs

In this subsection, the approximate CIs of the parameters are obtained using the asymptotic distributions of the elements of the vector $\varphi(\alpha,\beta,\lambda)$. The asymptotic distribution of the MLEs of φ is obtained by

$$((\hat{\alpha}-\alpha),(\hat{\beta}-\beta),(\hat{\lambda}-\lambda)) \mapsto N(0,I^{-1}(\alpha,\beta,\lambda)), \quad (39)$$

where $I^{-1}(\alpha,\beta,\lambda)$ is the variance-covariance matrix. The 3×3 symmetric matrix of negative second partial derivatives of the log-likelihood function on α, β, and λ is the Fisher information matrix $I(\alpha,\beta,\lambda)$, for the MLEs ($\hat{\alpha}$, $\hat{\beta}$ and $\hat{\lambda}$), see [29]. The performance of $I_{ij}(\alpha,\beta,\lambda)), i,j = 1,2,3,$, which is written by $I_{ij}(\hat{\alpha},\hat{\beta},\hat{\lambda}))$, where

$$I_{ij}(\varphi) = -\frac{\partial^2\ell(\varphi|\underline{y})}{\partial\varphi_i\partial\varphi_j} \tag{40}$$

$$I_0^{-1} = \begin{bmatrix} -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\alpha 2} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\alpha\partial\beta} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\alpha\partial\lambda} \\ -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\beta\partial\alpha} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\beta 2} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\beta\partial\lambda} \\ -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\lambda\partial\alpha} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\lambda\partial\beta} & -\frac{\partial^2\ell(\alpha,\beta,\lambda|\underline{t})}{\partial\lambda 2} \end{bmatrix}^{-1}_{(\hat{\alpha},\hat{\beta},\hat{\lambda})}. \tag{41}$$

Thus, the 100(1-γ)% approximate CIs for α,β, and λ are obtained as

$$\hat{\alpha} \mp z_{\frac{\gamma}{2}}\sqrt{v_{11}},\ \hat{\beta} \mp z_{\frac{\gamma}{2}}\sqrt{v_{22}} \text{ and } \hat{\lambda} \mp z_{\frac{\gamma}{2}}\sqrt{v_{33}} \tag{42}$$

Where v_{11}, v_{22}, and v_{33} are the elements on the main diagonal of the variance-covariance matrix $I^{-1}(\hat{\alpha},\hat{\beta},\hat{\lambda})$ and $z_{\frac{\gamma}{2}}$ is the percentile of the standard normal distribution with right-tail probability $\frac{\gamma}{2}$.

6.2 Credible confidence intervals CIs

A 100(1-γ)% Bayesian credible for a random quantity φ is the interval that has the posterior probability $(1-\gamma)$ that $\varphi(\alpha,\beta,\lambda)$ lies in the range such that

$$p(L \le \varphi \le U) = \int_L^U \pi^*(\varphi|\underline{y})d\varphi = 1-\gamma. \tag{43}$$

There are several types of the credible interval, including a central interval of posterior probability which is the range of values between the $(\frac{\gamma}{2})$ and $(1-\frac{\gamma}{2})$ percentiles.
The following procedures are performed to get credible CIs of α, β, and λ.
1. Set the initial guess of $\beta^{(0)} = \beta, \alpha^{(0)} = \alpha$ and $\lambda^{(0)} = \lambda$.
2. put $I = 1$.
3. Generate $\alpha^{(I)}$ from Gamma distribution $\pi_1^*(\alpha|\beta^{I-1},\lambda^{I-1},\underline{y})$.
4. Using Metropolis-Hastings, output $\beta^{(I)}$ from $\pi_2^*(\beta|\alpha^{I-1},\lambda^{I-1},\underline{y})$ with the $N(\beta^{(I-1)},v_{22})$ proposal distribution. Where v_{22} is possessed from the variances-covariances matrix.
5. Using Metropolis-Hastings, generate $\lambda^{(I)}$ from $\pi_3^*(\lambda|\alpha^{I-1},\beta^{I-1},\underline{y})$ with the $N(\lambda^{(I-1)},v_{33})$ proposal distribution, where v_{33} is taken from a variances-covariances matrix.
6. Compute $\alpha^{(I)}$, $\beta^{(I)}$ and $\lambda^{(I)}$.
7. Repeat step (1-6), U times and sort each estimate in ascending order as $[\hat{\varphi}_{iSE}^{[1]},\hat{\varphi}_{iSE}^{[2]},...,\hat{\varphi}_{iSE}^{[U]}]$, $i = 1,2,3$, where $\hat{\varphi}_{1SE} \equiv \hat{\alpha}_{SE}$,$\hat{\varphi}_{2SE} \equiv \hat{\beta}_{SE}$ and $\hat{\varphi}_{3SE} \equiv \hat{\lambda}_{SE}$, Then, the 100(1-$\gamma$)% credible CIs for φ_i is presented by

$$(\hat{\varphi}_{iSE}^{[\gamma\frac{U}{2}]},\hat{\varphi}_{iSE}^{[(1-\frac{\gamma}{2})U]}), i = 1,2,3. \tag{44}$$

10. To compute the credible intervals of ϕ_I, we commonly choose the quartiles of the sample as the endpoints of the interval, or as $\phi_I^{M+1},\phi_I^{M+2},\phi_I^{M+3},...,\phi_I^N$ as $\phi_{I(1)},\phi_{I(2)},\phi_{I(3)},...,\phi_{I(N-M)}$. Then the $100(1-\gamma)$ % symmetric credible interval is

$$(\phi_{I[\frac{\gamma}{2}(N-M)]},\phi_{I[(1-\frac{\gamma}{2})(N-M)]}), \tag{45}$$

6.3 Bootstrap confidence intervals CIs

We propose to use CIs based on the parametric bootstrap methods. It's known that CIs with the asymptotic results don't implement very well for small samples. We use the parametric percentile bootstrap(Boot-p)CIs method based on the concept of [30]. The following procedures are followed to obtain the progressive first-failure censoring bootstrap sample from CRD.
1. Compute the MLEs of the parameters α, β and λ from equations (17)-(21), using the original data set, $\underline{y} \equiv (y_{1:m:n:k},...,y_{n_1:m:n:k},y_{n_1+1:m:n:k},...,y_{m:m:n:k})$.
2. Use $\hat{\alpha}_{ML}$, $\hat{\beta}_{ML}$ and $\hat{\lambda}_{ML}$ to generate a bootstrap sample y^* with same $R_i, (i = 1,2,...,m)$ using the algorithm of [34].
3. As in step one, using y^* compute the bootstrap sample estimates of $\hat{\alpha}_{ML}$, $\hat{\beta}_{ML}$, and $\hat{\lambda}_{ML}$ say $\hat{\alpha}^*$, $\hat{\beta}^*$, and $\hat{\lambda}^*$.
4. Repeat steps (2)-(3), G times.
5. Sort each estimate in ascending order to gain the bootstrap samples $[\hat{\alpha}^{*[1]},\hat{\alpha}^{*[2]},...,\hat{\alpha}^{*[G]}]$, $[\hat{\beta}^{*[1]},\hat{\beta}^{*[2]},...,\hat{\beta}^{*[G]}]$ and $[\hat{\lambda}^{*[1]},\hat{\lambda}^{*[2]},...,\hat{\lambda}^{*[G]}]$.
Then, the 100(1-γ)% percentile bootstrap CIs for φ_i is presented by

$$(\hat{\varphi}_{iL}^*,\hat{\varphi}_{iG}^*) = (\hat{\varphi}_i^{*[\gamma\frac{G}{2}]},\varphi_i^{*[(1-\frac{\gamma}{2})G]}), i = 1,2,3, \tag{46}$$

where $\hat{\varphi}_1^* \equiv \hat{\alpha}^*$, $\hat{\varphi}_2^* \equiv \hat{\beta}^*$, and $\hat{\varphi}_3^* \equiv \hat{\lambda}^*$.

7 Simulation Studies

The aim of the simulation is to see the effect of the MLEs and BEs with SEL, LINEX and GE loss functions of the suggested methods. Monte Carlo simulations are carried out employing 1000 progressively first-failure censored samples from a CRD (α,β). We use the algorithm described in [23] to simulate the samples. Different effective samples of sizes m, different samples of sizes n, and different k has been used. The study is done to calculate the MLEs, BEs, MSEs, and RABs, based on

$N = 11000$ and $M = 1000$ Monte Carlo simulations, where the computations are performed using (MATHEMATICA ver.9). The implementation of the resulting estimators of the acceleration factor (λ) and the distribution parameters (α, β) has been discussed in terms of their absolute relative bias (RABias); this is the ultimate difference between the mean estimates, and its true value divided by the true value of the parameter (i.e. $RABias(\hat{\theta}) = \frac{\hat{\phi}-\phi}{\phi}$) and mean square error MSE; which is the sum squares of the difference between the true value and its estimated parameter divided by the number of the sample (i.e. $MSE(\hat{\phi}) = E[(\hat{\phi} - \phi)^2]$). Table.2 includes MSEs and RABs of the MLEs and BEs of α, β, λ.

Furthermore, the approximate bootstrap and credible CIs of the acceleration factor, the scale parameter, and the shape parameters obtained. Table 1 introduces the lengths and the coverage probabilities of 95% and 90% approximate, credible and percentile bootstrap CIs of the model parameters.
The simulation procedure is performed according to the following algorithm:
1- Specify the values of n, m, k, and τ.
2- Specify the values of α, β, and λ as case.1: $(\alpha = 0.1, \beta = 1.5, \lambda = 2.5)$ and case.2: $(\beta = 0.1, \alpha = 0.2, \lambda = 1.5)$.
3- For specific values of the prior parameters a, b, c, and d generate $\pi_1(\alpha)$ and $\pi_2(\beta)$.
4- Generate a sample of size $(n \times k)$ from the random variable Y presented by equation (5) and arrange it. The CRD can be generated easily, for instance, if U indicates a uniform random variable from [0,1], and if $y \implies U62f,$, then $Y = [[\beta^{-\alpha}[1-U]]^{-\frac{1}{\alpha}} - \beta]^{\frac{-1}{2}}$ has CRD with pdf specified by (13).
5- Generate progressively first-failure censored data for specified n, m Using the model given by equation (14), we consider the set of data: $Y^R_{1;m,n,k} < \ldots < Y^R_{n1;m,n,k} < \tau < Y^R_{n1+1;m,n,k} < \ldots < Y^R_{m;m,n,k}$, where $R = (R_1, R_2, \ldots, R_m)$ and $\sum_{i=1}^{m} R_i = n - m$.
6- Use the progressive first-failure censored data to calculate the MLEs of the model parameters; the Newton-Raphson method is used for resolving equations (17)-(21) to get the MLEs of the unknown parameters.
7- Compute the BEs of the unknown parameters with SE and LINEX loss functions, where N = 11000 and M = 1000.
8- Compute the approximate CI with $\gamma = 0.95$ and $\gamma = 0.90$ for the unknown parameters.
9- Replicate steps (4)-(9), 1000 times.
10- Find the average values of the (MSEs) and (RABs) attached with the MLEs and BEs of α, β, and λ.
11- Perform steps 1-10 with several values of n, m, and τ.
Tables 1-3 summarize the simulation results. Table 1 displays the approximated CI at 95% and 90% for α, β, and λ.
We apply the algorithm proposed by [35] to generate progressive first failure censored samples from CRD.

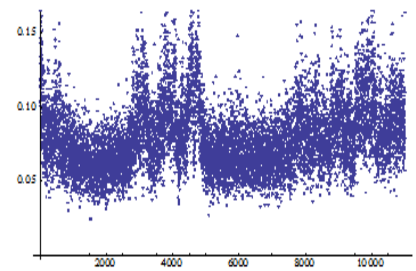

Fig. 1: Simulation number of α obtained by MCMC method.

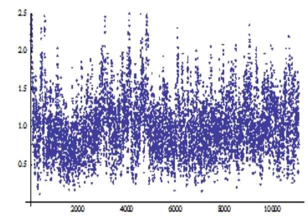

Fig. 2: Simulation number of β_1 obtained by MCMC method.

Whereas the number of items setting in a life test is $(n \times k)$ items, where n indicates the group number and k indicates the items number in every group. We consider the following progressive CSs(I, II, III), for the simulation studies, to compare the performances of the estimation discussed in this study.
Scheme I: $R_m = n - m$, $R_1 = 0$ for $i \neq m$.
Scheme II: $R_1 = n - m$, $R_1 = 0$ for $i \neq 1$.
Scheme III: $R_{\frac{m+1}{2}} = n - m$, $R_1 = 0$ for $i \neq \frac{m+1}{2}$; if m odd, and $R_{\frac{m+2}{2}} = n - m$, $R_1 = 0$ for $i \neq \frac{m+2}{2}$; if m even.
The three censoring schemes are coincide with the cases of all remaining items which are extracted from the test at the first failure point, last failure point, and midpoint, respectively. Furthermore, it is notable that Type-II first failure censored scheme is the scheme-I.

8 Conclusion

In this study, we have considered a progressive first-failure censored samples, this study represents maximum likelihood and Bayes methods for the analysis of the SSPALT, using the compound Rayleigh failure model. We employed the MCMC technique to obtain the Bayes estimates and it has been shown the Bayes estimate concerning informative prior performs very well in this study. The simulation is performed to compare the

Table 1: 90% and 95% approximate, credible and bootstrap CIs for α,β and λ

m	n	SC	Par	Approximate CI	L	Credible CI	L	bootstrap CI	L
15	30	I	α	(-0.0305, 0.2182)	0.2487	(0.0282, 0.1355)	0.1073	(0.0351, 0.1804)	0.1453
				(-0.0544, 0.2421)	0.2965	(0.0237, 0.1394)	0.1157	(0.0172, 0.1946)	0.1774
			β	(-2.2495, 3.1147)	5.3642	(0.3165, 1.9184)	1.6019	(0.2680, 2.3851)	2.1171
				(-3.0507, 3.9159)	6.9666	(0.2395, 2.1044)	1.8649	(0.1853, 2.6173)	2.4320
			λ	(-0.5819, 2.5748)	3.1567	(0.4230, 4.1491)	3.7189	(0.2581, 4.8039)	4.5458
				(-0.8843, 2.8772)	3.7615	(0.3502, 4.5984)	4.2482	(0.1740, 3.9594)	3.7854
		II	α	(-0.0870, 0.6859)	0.7729	(0.0567, 0.1708)	0.1141	(0.0236, 0.3813)	0.3577
				(-0.1610, 0.7599)	0.9209	(0.0511, 0.1878)	0.1367	(0.0207, 0.4572)	0.4365
			β	(-1.0578, 5.6142)	6.6720	(0.3485, 1.4014)	1.0529	(0.1547, 2.3168)	2.1621
				(-1.7084, 5.2648)	6.9732	(0.2905, 1.5451)	1.2546	(0.2083, 2.7205)	2.5122
			λ	(-0.1444, 1.2993)	1.4437	(0.5111, 3.9317)	3.4206	(0.3058, 3.5197)	3.2139
				(-0.2827, 1.4376)	1.7203	(0.3999, 4.0930)	3.6931	(0.2507, 4.1159)	3.8652
		III	α	(0.0257, 0.2175)	0.1918	(0.0365, 0.2048)	0.1583	(0.0316, 0.1386)	0.1070
				(0.0073, 0.2359)	0.2286	(0.0332, 0.2550)	.2118	(0.0271, 0.1649)	0.1448
			β	(-1.0120, 4.4845)	5.4965	(0.3429, 1.8725)	1.5296	(0.2057, 2.7534)	2.5477
				(-2.1132, 4.5857)	6.6989	(0.2835, 1.9153)	1.6318	(0.1503, 2.6103)	2.4600
			λ	(-0.5689, 4.3249)	4.8938	(0.4865, 3.7296)	3.2431	(0.2179, 4.8603)	4.6424
				(-0.8460, 4.6022)	5.4482	(0.3733,4.1816)	3.8083	(0.1066,5.6308)	5.5242
25	30	I	α	(0.0264, 0.1832)	0.1568	(0.0591, 0.1568)	0.0977	(0.0308, 0.1645)	0.1337
				(0.0114, 0.1982)	0.1868	(0.0536,0.1700)	0.1164	(0.0307,0.2060)	0.1753
			β	(-0.4819, 4.2027)	4.6846	(0.7292, 2.2895)	1.5603	(0.6292, 2.6915)	2.0623
				(-0.9306, 4.6514)	5.5820	(0.6286, 2.4790)	1.8504	(0.4893, 2.9820)	2.4927
			λ	(-0.4248, 4.3755)	4.8003	(0.3483, 3.2107)	2.8624	(0.2591, 3.4002)	3.1411
				(-0.6929, 4.6437)	5.3366	(0.2838, 3.6524)	3.3686	(0.1906, 3.8527)	3.6621
		II	α	(0.0339, 0.1475)	0.1136	(0.0510, 0.1321)	0.0811	(0.0447, 0.1383)	0.0936
				(0.0231, 0.1583)	0.1352	(0.0464, 0.1446)	0.0982	(0.0564, 0.1531)	0.0967
			β	(-0.6650, 3.8668)	4.5318	(0.5153, 1.9301)	1.4148	(0.4038, 2.1435)	1.7397
				(-1.0991, 4.3009)	5.4000	(0.4415, 2.0936)	1.6521	(0.3391, 2.4107)	2.0716
			λ	(-0.5346, 4.2067)	4.7413	(0.6675, 4.0904)	3.4229	(0.2571, 3.5300)	3.2729
				(-0.9888, 4.6608)	5.6496	(0.5607, 4.6681)	4.1074	(0.0792, 3.9017)	3.8225
		III	α	(0.0423, 0.1766)	0.1343	(0.0685, 0.1706)	0.1021	(0.0481, 0.1735)	0.1254
				(0.0294, 0.1895)	0.1601	(0.0627, 0.1858)	0.1231	(0.0520, 0.1817)	0.1297
			β	(-0.3507, 3.3368)	2.9861	(0.8144, 2.4047)	1.5903	(0.6011, 2.5907)	1.9896
				(-0.7039, 3.6900)	4.3939	(0.6907, 2.6017)	1.9110	(0.4003, 2.8164)	2.4161
			λ	(-0.6894, 3.9735)	4.6629	(0.4817, 3.0441)	2.5624	(0.2751, 3.4602)	3.1851
				(-1.1360, 4.4202)	5.5562	(0.3762, 3.4646)	3.0884	(0.2283, 4.0715)	3.8432
30	30		α	(0.0539, 0.1754)	0.1215	(0.0711, 0.1688)	0.0977	(0.0618, 0.1704)	0.1086
				(0.0423, 0.1871)	0.1448	(0.0656, 0.1822)	0.1166	(0.0752, 0.1905)	0.1153
			β	(-0.1716, 3.6709)	3.8425	(0.8878, 2.4168)	1.2590	(0.8006, 2.2734)	1.4728
				(-0.5397, 4.0389)	4.5786	(0.8009, 2.5729)	1.7720	(0.7403, 2.6279)	1.8876
			λ	(-0.0747, 4.24329)	4.3180	(0.7686, 4.0232)	3.2546	(0.6960, 4.3008)	3.6048
				(-0.4884, 4.6569)	5.1553	(0.6517, 4.0628)	3.4111	(0.6025, 4.6671)	4.0646
50	30	I	α	(-0.0122, 0.2384)	0.1284	(0.0598, 0.1614)	0.1016	(0.0671, 0.1845)	0.1174
				(-0.0381, 0.2624)	0.2085	(0.0552, 0.1759)	0.1207	(0.0462, 0.1903)	0.1441
			β	(-0.1031, 5.2501)	5.3532	(1.3419, 3.0827)	1.7408	(1.1407, 3.2550)	2.1143
				(-0.0948, 4.2418)	4.3366	(1.1895, 3.4462)	2.2567	(1.1453, 3.6002)	2.4549
			λ	(-0.2857, 2.6819)	2.9676	(0.5837, 3.3341)	2.7504	(0.4816, 3.5131)	3.0315
				(-0.5699, 2.9662)	3.5361	(0.5024, 3.5851)	3.0827	(0.4520, 3.6530)	3.2010
		II	α	(0.0474, 0.1476)	0.1002	(0.0760, 0.1817)	0.1057	(0.0471, 0.1846)	0.1375
				(0.0359, 0.1491)	0.1132	(0.0705, 0.1962)	0.1257	(0.0415, 0.2136)	0.1721
			β	(-0.2925, 3.0314)	3.3239	(1.0122, 2.6975)	1.6853	(1.1302, 2.9495)	1.8193
				(-0.6109, 3.3499)	3.9608	(0.9231, 2.8643)	1.9412	(0.8671, 3.0651)	2.1980
			λ	(-0.1159, 3.5450)	3.6609	(0.9566, 3.9771)	3.0205	(1.2548, 4.0357)	2.7809
				(-0.6581, 3.0872)	3.7453	(0.8056, 3.7391)	2.9335	(0.9865, 4.001)	3.0145

Table 1 Continued

	III	α	(0.0477, 0.2693)	0.2216	(0.0760, 0.1817)	0.1057	(0.0510, 0.1973)	0.1463
			(0.0265, 0.2905)	0.2640	(0.0705, 0.1962)	0.1257	(0.0481, 0.2119)	0.1638
		β	(-0.2509, 3.9037)	4.1546	(1.0122, 2.6975)	1.6853	(0.9716, 2.715)	1.7434
			(-1.1279, 4.7806)	5.9085	(0.9231, 2.8643)	1.9412	(0.9231, 2.7690)	1.8459
		λ	(-0.1137, 2.0706)	2.1843	(1.0970, 3.2647)	2.1677	(1.0566, 3.6250)	2.5684
			(-0.3229, 2.2798)	2.6027	(0.9865, 3.5301)	2.5436	(1.2604, 3.8601)	2.5997
50	50	α	(0.0562, 0.1316)	0.0754	(0.0884, 0.1643)	0.0759	(0.0805, 0.1729)	0.0924
			(0.0490, 0.1389)	0.0899	(0.0825, 0.1733)	0.0908	(0.0937, 0.1831)	0.0894
		β	(0.0977, 2.6299)	2.5322	(2.0651, 3.8525)	1.7874	(1.9641, 3.7401)	1.776
			(-0.1449, 2.8725)	3.0197	(1.9296, 4.0463)	2.1740	(1.8907, 4.1626)	2.2719
		λ	(0.5121, 3.8443)	3.3322	(1.0257, 3.1036)	2.0779	(1.0049, 3.3106)	2.3057
			(0.0972, 4.2594)	4.1622	(0.9254, 4.0703)	3.1449	(0.9703, 4.0922)	3.1219

Table.2: MSEs and RABs inside the parentheses of the MLE and BEs with $\alpha = 0.1, \beta = 1.5$ and $\lambda = 2.5$

m	n	SC	Par	ML	SEL	LINEX			GE		
						c=-2	c=0.01	c=2	c=-2	c=0.01	c=2
15	30	I	α	0.0713	0.0121	0.0072	0.0169	0.0257	0.0112	0.0121	0.0130
				0.7132	0.1211	0.0721	0.1686	0.2569	0.1120	0.1211	0.1297
			β	2.8389	0.2531	0.1780	0.3330	0.0257	0.4183	0.2540	0.0235
				1.8926	0.1687	0.1186	0.2220	0.3387	0.2789	0.1694	0.0157
			λ	0.5741	1.1848	1.7799	0.5381	0.7056	6.257	1.1613	0.7145
				0.2297	0.4739	0.7119	0.2152	0.2822	2.5028	0.4645	0.2858
		II	α	0.2264	0.0366	0.0172	0.0298	0.0436	0.0347	0.0366	0.0386
				2.2638	0.3660	0.1723	0.2981	0.4364	0.3468	0.3659	0.3863
			β	4.1597	0.3834	0.5902	0.4506	0.3184	0.5109	0.3841	0.1959
				2.7732	0.2556	0.3934	0.3004	0.2123	0.3406	0.2561	0.1306
			λ	1.1837	1.0047	1.4909	0.4907	0.5893	4.9800	0.9866	0.6103
				0.4735	0.4019	0.5963	0.1963	0.2357	1.992	0.3947	0.2441
		III	α	0.2744	0.0227	0.0186	0.0341	0.0266	0.0220	0.0233	0.0227
				2.7435	0.2265	0.1858	0.3414	0.2664	0.2200	0.2329	0.2266
			β	12.9007	0.4981	0.5801	0.7640	0.4221	0.4989	0.6330	0.2999
				8.6004	0.3321	0.3867	0.5093	0.2814	0.3326	0.4220	0.1999
			λ	1.5687	0.2642	0.0742	0.7818	0.566	0.2554	0.7837	2.0587
				0.6275	0.1057	0.0297	0.3127	0.2264	0.1022	0.3135	0.8235
25	30	I	α	0.0253	0.0239	0.0207	0.0270	0.0324	0.0234	0.0239	0.0244
				0.2529	0.2394	0.2075	0.2697	0.324	0.2343	0.2394	0.2442
			β	0.0009	0.3252	0.5605	0.4038	0.2481	0.4787	0.3261	0.0865
				0.0006	0.2168	0.3737	0.2692	0.1654	0.3191	0.2174	0.0577
			λ	1.2606	1.7004	2.3660	0.9941	0.1869	7.213	1.6704	0.446
				0.5042	0.6802	0.9464	0.3977	0.0748	2.8852	0.6682	0.1784
		II	α	0.0403	0.0466	0.0450	0.0482	0.0511	0.0464	0.0466	0.0467
				0.4031	0.4657	0.4496	0.4815	0.5113	0.4640	0.4658	0.4675
			β	0.8029	1.7626	1.977	1.8296	1.7001	1.8465	1.7631	1.6522
				0.3212	0.7051	0.7908	0.7319	0.6801	0.7386	0.7052	0.6609
			λ	1.8754	2.9798	3.7172	2.1508	0.6898	8.3671	2.9443	0.4891
				1.2503	1.9865	2.4781	1.4339	0.4599	5.5781	1.9628	0.3260
		III	α	0.0138	0.0133	0.0102	0.0164	0.0223	0.0128	0.0133	0.01390
				0.1383	0.1332	0.1018	0.1643	0.2235	0.1276	0.1332	0.1387
			β	0.1238	0.1969	0.132	0.2648	0.4048	0.0028	0.1978	0.3476
				0.0825	0.1313	0.088	0.1766	0.2698	0.0019	0.13190	0.2318
			λ	0.4329	0.3223	0.8919	0.2648	1.0148	4.0009	0.3048	0.9759
				0.1732	0.1289	0.3568	0.0938	0.4059	1.6004	0.12190	0.3904

Table 2 continued

30	30		α	0.1470	0.1476	0.0339	0.1086	0.1867	0.1386	0.1476	0.1569
				0.0147	0.0148	0.0034	0.0109	0.0187	0.0139	0.0148	0.0157
			β	0.1664	0.0513	0.0954	0.0055	0.0858	0.2077	0.0506	0.0717
				0.2496	0.0769	0.1431	0.0082	0.1288	0.3116	0.0759	0.1075
			λ	0.1663	0.1210	0.3919	0.1202	0.4133	2.0008	0.1125	0.4002
				0.4157	0.3024	0.9798	0.3005	1.0332	5.0021	0.2813	1.0006
30	50	I	α	0.0133	0.0302	0.0166	0.0255	0.0349	0.0290	0.0302	0.0315
				0.1326	0.3020	0.1659	0.2552	0.3488	0.2898	0.3019	0.3146
			β	0.0035	1.3242	1.4015	1.2421	1.0663	1.819	1.3219	0.9256
				0.0024	0.8828	0.9343	0.8280	0.7109	0.6170	0.8813	1.2126
			λ	0.6625	1.1663	1.4855	1.2831	1.0452	1.4056	1.168	0.7038
				0.2650	0.4665	0.5942	0.5132	0.4181	0.5622	0.4672	0.2815
		II	α	0.0042	0.0003	0.0032	0.0038	0.0097	0.001	0.0003	0.0004
				0.0417	0.0029	0.0320	0.0383	0.0967	0.0102	0.0028	0.0043
			β	0.1968	0.0792	0.1525	0.1633	0.0013	0.3431	0.1272	0.0780
				0.1312	0.0528	0.1017	0.1089	0.0009	0.2288	0.0848	0.0520
			λ	0.4175	2.2448	3.1451	1.3384	0.0415	9.8165	2.1985	0.3276
				0.1670	0.8979	1.258	0.5354	0.0166	3.9266	0.8794	0.1310
		III	α	0.1882	0.0305	0.0292	0.0319	0.0345	0.0304	0.0305	0.0307
				1.8822	0.3054	0.2920	0.3189	0.3453	0.3035	0.3054	0.3073
			β	3.0990	0.1749	0.2446	0.1034	0.0410	0.4568	0.1737	0.0264
				2.0660	0.1166	0.1630	0.0689	0.0273	0.3045	0.1158	0.0176
			λ	2.7225	1.3199	1.3203	1.3194	1.3185	1.3233	1.3199	1.3164
				1.0890	0.5280	0.5281	0.5278	0.5274	0.5293	0.5279	0.5266
50	50		α	0.0061	0.0228	0.0164	0.0206	0.0250	0.0223	0.0228	0.0233
				0.0610	0.228	0.1640	0.2063	0.2496	0.2226	0.2279	0.2334
			β	0.1362	1.419	1.3657	1.3657	1.2604	1.7579	1.4174	1.1490
				0.0908	0.9460	0.9105	0.9105	0.8403	1.1719	0.9450	0.7660
			λ	0.1783	0.2538	0.4569	0.7783	0.4569	1.8694	0.8141	0.2592
				0.0713	0.1015	0.1828	0.3113	0.1828	0.7478	0.3256	0.1037

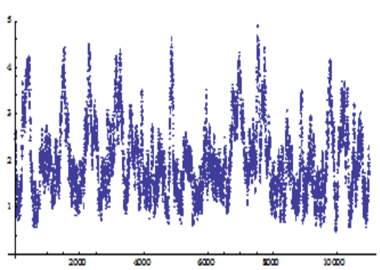

Fig. 3: Simulation number of λ_2 obtained by MCMC method. Simulation number of β_1 obtained by MCMC method.

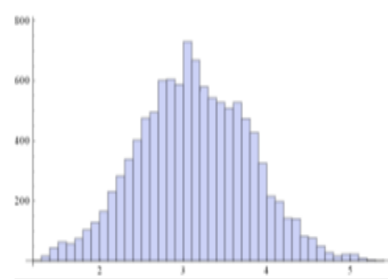

Fig. 5: Histogram of β_1 obtained by MCMC method.

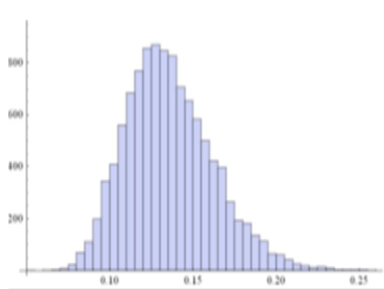

Fig. 4: Histogram of α obtained by MCMC method.

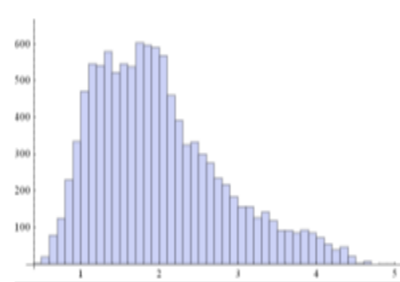

Fig. 6: Histogram of λ_2 obtained by MCMC method.

suggested methods for several accelerated factors, several parameter values, several sample sizes (n,m) and three several CSs (I, II, III). Approximate, credible and bootstrap CIs have been determined for α, β, and λ. From Tables (1), (2) and (3) we have realized that:
1. In most of the cases the length of approximate, bootstrap and credible CIs are decrease when the sample size is increases, excepting a little cases; this is probably due to the inconstancy in the data.
2. Overall, the MCMC credible CIs of α, β, and λ yield good results than approximate and bootstrap CIs for the length of CIs. While, the bootstrap CIs of α, β, and λ yield more convenient results than approximate CIs and for the length of CIs, for various sample sizes, various observed failures, and various schemes.
3. It may be noted that for fixed observed failures and sample sizes, the first scheme-I, yield lower lengths for the three methods of the CIs in contrast to the other two schemes.
4. Also, the MCMC credible intervals give lower lengths for the three schemes, in case of small sample sizes.
5. It can also be seen that the Bayes estimates of α, β, and λ give better results for the MSEs and RABs than for MLEs in most of the cases considered.
6. In general, when $(c^aU2c6'2)$ the Bayes estimates of α under LINEX loss function and GE loss function provides the smallest MSEs and RABs as discussed with the estimates under SEL, LINEX loss function (c= 0.01, 2), GE loss function (c= 0.01, 2)and MLEs.
7. In general, the Bayes estimates of β and λ under LINEX loss function (c=2), and GE loss function (c=2) have the smallest MSEs and RABs as discussed with the estimates under SEL, LINEX loss function (c= 0.01, -2), GE loss function (c= 0.01, -2), and MLEs.
8. For specific values of the sample and failure time sizes, the scheme-I implements better than Scheme-II and Scheme-III, in the concept of having smaller MSEs in most of the cases considered.
9. In general, as sample size m/n increases, the MSEs and RABs of MLEs and Bayes estimates of α, β, and λ decrease, except for few.

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