Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/120404

Two-Step Scheme for Implementing N Two-Qubit Quantum Logic Gates Via Cavity QED

T. Said[1,*], *A. Chouikh*[1] *and M. Bennai*[1,2]

[1] Laboratoire de Physique de la Matire Condense, Equipe Physique Quantique et Applications, Facult des Sciences Ben M'sik, B.P. 7955, Universit Hassan II, Casablanca, Maroc
[2] LPHE-Modelisation et Simulation, Facult des Sciences Rabat, Universit de Rabat, Maroc

Received: 21 Sep. 2017, Revised: 28 Nov. 2017, Accepted: 3 Dec. 2017
Published online: 1 Jul. 2018

Abstract: We theoretically present an effective method to realize a two-step NiSWAP, $N\sqrt{i\text{SWAP}}$, NSWAP and $N\sqrt{\text{SWAP}}$ gates based on the qubit-qubit interaction in a cavity QED driven by a strong microwave field. The operation times do not increase with the growth of the qubit number. Due to the virtual excitations of the cavity, the scheme is insensitive to both the cavity decay and the thermal field. Numerical analysis under the influence of the gates operations shows that the our proposal can be implemented with high fidelity. Moreover, the scheme can be realized in the range of current cavity QED techniques.

Keywords: NiSWAP gate, $N\sqrt{i\text{SWAP}}$gate, NSWAP gate, $N\sqrt{\text{SWAP}}$ gate, qubit-qubit interaction, cavity QED

1 Introduction

Quantum computers can be more powerful than classical computers [1]. In addition, the results and limitations of realistic quantum computers gives us overviews of the fundamentals of quantum mechanics. The implementation of quantum logic gates is the basis of inventing quantum computers [2]. Thus, how to implement various quantum gates, such as SWAP gate [3], iSWAP gate [4], $\sqrt{i\text{SWAP}}$ gate [5] and $\sqrt{\text{SWAP}}$ gate [6] in different quantum systems, such as cavity QED system [7], linear optics [8], trapped ions [9], quantum dots system [10] and NMR [11], attracts much attention.

Recently, Liu et al. [12] presented a scheme for implementing two-atom $\sqrt{\text{SWAP}}$ gate in a cavity QED. In ref. [13] the authors proposed a method to implement a **three**-step n qubits SWAP gate simultaneously between one qubit with n qubits coupled to a cavity. In this paper, we propose a method for realizing an NiSWAP, $N\sqrt{i\text{SWAP}}$, NSWAP and $N\sqrt{\text{SWAP}}$ gates of one qubit (such as atoms, quantum dots, and superconducting qubits) simultaneously controlling N target qubits in cavity QED with nearest dipole-dipole interaction (qubit-qubit interaction) by adding a strong microwave field, we calculate the evolution operator in **two**-step, we use the overall evolution operator for obtaining these logic gates, we also calculate the implementation time and discuss the result. Numerical analysis shows that the scheme can be implemented with high fidelity. On the other hand, the presence of both qubit-qubit interaction and Jaynes-Cummings model make our system powerful. We also notice that to enhance the performance of our schemes, implementation of our logic gates with the total operation time should be much shorter than the energy relaxation time and the cavity decay time is the main way. The rest of the paper is organized as follows: In Section 2, we calculate the evolution operator of the whole system. In Section 3, a two-step scheme for implementation of quantum logic gates is presented. In Section 4, we analyze the performance of our scheme numerically under parameters currently reachable and then discuss its feasibility based on current experiments in cavity QED. Finally, a conclusion appears in Section 5.

2 Model and evolution operator

We consider $(N+1)$ qubits each having two levels, whose states are designated by a ground state $|g_j\rangle$ and an excited state $|e_j\rangle$, interacting with a single mode cavity simultaneously and driven by a conventional field added to the system. The N qubits are very close together, so the

* Corresponding author e-mail: taoufik.said81@gmail.com

qubit-qubit interaction should be included in the cavity QED. The Hamiltonian of the whole system in the rotating wave approximation (assuming $\hbar = 1$) [14,15] is given by

$$H = \omega_0 \sum_{j=1}^{N+1} S_{z,j} + \omega_a a^+ a + \Omega \sum_{j=1}^{N+1} (S_j^+ e^{-i\omega t} + S_j^- e^{i\omega t}) + g \sum_{j=1}^{N+1} (a^+ S_j^- + a S_j^+) + \Gamma \sum_{\substack{j,k=1 \\ j\neq k}}^{N+1} S_j^+ S_k^-, \quad (1)$$

$S_{z,j}$, S_j^-, and S_j^+ are the collective operators for the $(1,2,...,N+1)$ qubits, where $S_{z,j} = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, $S_j^+ = (|e_j\rangle\langle g_j|)$, $S_j^- = (|g_j\rangle\langle e_j|)$ with $|e_j\rangle(|g_j\rangle)$ is the excited state (ground state) of the qubit. ω_0, ω_a and ω are the transition frequencies between the two levels $|g\rangle$ and $|e\rangle$ of each qubit, the cavity mode and the conventional field, respectively. a^+, a are the creation and annihilation operators to the cavity mode, g is intensity qubit-cavity coupling, Ω the Rabi frequency of the classical field, and Γ is the force dipole-dipole coupling. Here, we consider the case of $\omega_0 = \omega$. Define the new basis $|+_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle)$, $|-_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle - |e_j\rangle)$. In the strong driving region $\Omega \gg \delta, g, \Gamma$ [16], we can eliminate the terms oscillating fast. Then, we have H_I in the interaction picture [17]

$$H_I = 2\Omega \sum_{j=1}^{N+1} \sigma_{z,j} + g(a^+ e^{i\delta t} + a e^{-i\delta t}) \sum_{j=1}^{N+1} \sigma_{z,j} + \frac{\Gamma}{4} \sum_{\substack{j,k=1 \\ j\neq k}}^{N+1} \left[4\sigma_{z,j}\sigma_{z,k} + \sigma_j^+ \sigma_k^- + \sigma_j^- \sigma_k^+\right], \quad (2)$$

where $\delta = \omega_0 - \omega_a$ is the detuning between the frequency of the cavity ω_0 and the frequency of the qubit ω_a and $\sigma_{z,j} = \frac{1}{2}(|+_j\rangle\langle +_j| - |-_j\rangle\langle -_j|)$, $\sigma_j^+ = |+_j\rangle\langle -_j|$, $\sigma_j^- = |-_j\rangle\langle +_j|$. Then, the evolution operator of the system can be given by [18]

$$U(\tau) = \exp\left[-2i\Omega\tau \sum_{j=1}^{N+1} \sigma_{z,j}\right] \exp\left[-i4\lambda\tau \left(\sum_{j=1}^{N+1} \sigma_{z,j}\right)^2\right] \times \exp\left[-i\frac{\Gamma}{4}\tau \sum_{\substack{j,k=1 \\ j\neq k}}^{N+1} \left[4\sigma_{z,j}\sigma_{z,k} + \sigma_j^+ \sigma_k^- + \sigma_j^- \sigma_k^+\right]\right], \quad (3)$$

where $\lambda = \frac{g}{8}$ and $t = \tau = 2\pi/|\delta|$. In the following, we demonstrate how to realize a two-step NiSWAP, $N\sqrt{i\text{SWAP}}$, NSWAP and $N\sqrt{\text{SWAP}}$ gates of one qubit simultaneously controlling N qubits in a cavity QED.

3 Implementation of quantum logic gates

3.1 NiSWAP and $N\sqrt{i\text{SWAP}}$ gates

It is demonstrated that more complex quantum gates can be created by iSWAP and $\sqrt{i\text{SWAP}}$ gates [19]. Furthermore, two iSWAP gates together with several one-qubit rotation gates can construct the CNOT gate, which is considered as a universal two-qubit gate [20]. The $\sqrt{i\text{SWAP}}$ gate forms a universal gate set, a CNOT gate for example can be obtained from single-qubit rotations and two $\sqrt{i\text{SWAP}}$ gates [20]. Recently, it has been shown that the iSWAP and $\sqrt{i\text{SWAP}}$ gates can be very useful for applications in quantum information process QIP and quantum computing [21]. The iSWAP and $\sqrt{i\text{SWAP}}$ gates are appropriate elementary two-qubit gates, these quantum logic gates can be represented by the operators:

$$i\text{SWAP} \equiv |00\rangle\langle 00| + i|01\rangle\langle 10| + i|10\rangle\langle 01| + |11\rangle\langle 11|, \quad (4)$$

$$\sqrt{i\text{SWAP}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (5)$$

The goal of this subsection is to demonstrate how the NiSWAP and $N\sqrt{i\text{SWAP}}$ gates can be realized. The proposed quantum logic gates provides a vivid blueprint for future quantum computation, that is deterministic and scalable. Let us now consider an $(N+1)$ qubits placed in single-mode cavity, where the first qubit as the controlling qubit and the other N qubits as the target qubits. The NiSWAP and $N\sqrt{i\text{SWAP}}$ gates gates can be realized by using the evolution operator in equation (3). We now consider two special cases: $\delta > 0$ as well as $\delta < 0$. The results from the unitary evolution, obtained for these two special cases, are employed below for the gates implementation. The detailed procedure is given as follows.

Step 1: Let us begin with the case $\delta = \omega_0 - \omega_a \sim 2g > 0$ (see Fig.1(A)): adjust the transition frequencies of $(N+1)$ qubits with the pulse Rabi frequency Ω in a period of evolution time $\tau = \pi/g$. Then, the evolution operator can be described by

$$U_1(\tau) = \exp\left[-2i\Omega\tau \sum_{j=1}^{N+1} \sigma_{z,j}\right] \exp\left[-i4\lambda\tau \left(\sum_{j=1}^{N+1} \sigma_{z,j}\right)^2\right] \times \exp\left[-i\frac{\Gamma}{4}\tau \sum_{\substack{j,k=1 \\ j\neq k}}^{N+1} \left[4\sigma_{z,j}\sigma_{z,k} + \sigma_j^+ \sigma_k^- + \sigma_j^- \sigma_k^+\right]\right]. \quad (6)$$

Step 2: Let us now consider the case $\delta < 0$ where $-\delta = \omega_0 - \omega_a' \sim 2g > 0$ (see Fig.1(B)): by setting $\Omega' = 0$

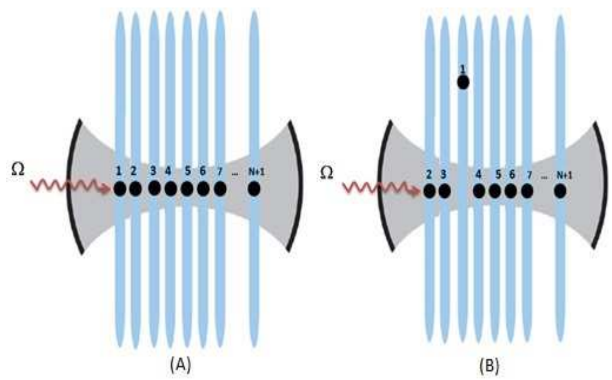

Fig. 1: Proposed the two-setup for an N-two-qubit quantum logic gates with $N+1$ qubits and a cavity, where the $N+1$ qubits are very close together. Figure (A) corresponds to the first step where Ω is the pulse Rabi frequency applied to $(1,2,3,...,N+1)$ qubits. Figure (B) corresponds to the second step where the pulse Rabi frequency Ω is now applied to $(2,3,...,N+1)$ qubits, where the pulse Rabi frequency applied to qubit 1 is $\Omega' = 0$.

for qubit 1, the coupling of qubit 1 with the cavity mode becomes negligibly small, one can adjust the level spacings of qubit 1 such that it is decoupled from the cavity mode, so the cavity mode is coupled to target qubits $(2,3,...,N+1)$ with the pulse Rabi frequency Ω in a period of time $\tau = \pi/g$, the evolution operator can be described by

$$U_2(\tau) = \exp\left[-2i\Omega\tau\sum_{j=2}^{N+1}\sigma_{z,j}\right]\exp\left[-i4\lambda\tau\left(\sum_{j=2}^{N+1}\sigma_{z,j}\right)^2\right] \times \exp\left[-i\frac{\Gamma}{4}\tau\sum_{\substack{j,k=2\\ j\neq k}}^{N+1}\left[4\sigma_{z,j}\sigma_{z,k}+\sigma_j^+\sigma_k^- +\sigma_j^-\sigma_k^+\right]\right]. \tag{7}$$

After the above two steps (after a period of time 2τ), the evolution operator of the system is

$$U(2\tau) = e^{-2i\Omega\tau\sigma_{z,1}}\prod_{j=2}^{N+1}e^{-8i\lambda\tau\sigma_{z,1}\sigma_{z,j}} \times \prod_{j=2}^{N+1}e^{-i\frac{\Gamma}{4}\tau(4\sigma_{z,1}\sigma_{z,j}+\sigma_1^+\sigma_j^-+\sigma_1^-\sigma_j^+)}. \tag{8}$$

When the condition $\Omega = 2N\lambda$ is satisfied, one can find that the joint time-evolution operator, after the above two steps of operation, is given by

$$U = \prod_{j=2}^{N+1}e^{-i\xi\tau}, \tag{9}$$

with
ξ=$4\lambda(\sigma_{z,1}+2\sigma_{z,1}\sigma_{z,j})+\frac{\Gamma}{4}(4\sigma_{z,1}\sigma_{z,j}+\sigma_1^+\sigma_j^-+\sigma_1^-\sigma_j^+)$, $j = 2,3,...,N+1$. According to the operator $e^{-i\xi\tau}$ on the basis $(|+_1\rangle|+_j\rangle$, $|+_1\rangle|-_j\rangle$, $|-_1\rangle|+_j\rangle$, $|-_1\rangle|-_j\rangle)$, we can obtain following evolutions

$$\begin{aligned}
|+_1\rangle|+_j\rangle &\longrightarrow e^{-i(\frac{\Gamma}{4}+2\lambda)\tau}|+_1\rangle|+_j\rangle\\
|+_1\rangle|-_j\rangle &\longrightarrow e^{i\eta\pi}[\cos(\Gamma\tau/4)|+_1\rangle|-_j\rangle\\
&\qquad +i\sin(\Gamma\tau/4)|-_1\rangle|+_j\rangle]\\
|-_1\rangle|+_j\rangle &\longrightarrow e^{i\eta\pi}[\cos(\Gamma\tau/4)|-_1\rangle|+_j\rangle\\
&\qquad +i\sin(\Gamma\tau/4)|+_1\rangle|-_j\rangle]\\
|-_1\rangle|-_j\rangle &\longrightarrow |-_1\rangle|-_j\rangle,
\end{aligned} \tag{10}$$

where a phase factor $\eta\pi$ in the previous evolutions can be produced by several different proposals [22]. By choosing $\Gamma\tau = 2\pi$, $\eta = 2k$ (with k being an integer) and $2\lambda\tau = -\frac{\pi}{2}$, an N-two-qubit iSWAP operations are simultaneously performed on N qubit pairs $(1,2)$, $(1,3)$, ..., $(1,N+1)$, written as

$$\begin{aligned}
|+_1\rangle|+_j\rangle &\longrightarrow |+_1\rangle|+_j\rangle\\
|+_1\rangle|-_j\rangle &\longrightarrow i|-_1\rangle|+_j\rangle\\
|-_1\rangle|+_j\rangle &\longrightarrow i|+_1\rangle|-_j\rangle\\
|-_1\rangle|-_j\rangle &\longrightarrow |-_1\rangle|-_j\rangle.
\end{aligned} \tag{11}$$

By choosing $\Gamma\tau = \pi$, $\eta = 2k'$ (with k' being an integer) and $2\lambda\tau = -\frac{\pi}{4}$, we obtain N-two-qubit $\sqrt{i\text{SWAP}}$ operations (from equation (10)) as

$$\begin{aligned}
|+_1\rangle|+_j\rangle &\longrightarrow |+_1\rangle|+_j\rangle\\
|+_1\rangle|-_j\rangle &\longrightarrow \frac{1}{\sqrt{2}}[|+_1\rangle|-_j\rangle+i|-_1\rangle|+_j\rangle]\\
|-_1\rangle|+_j\rangle &\longrightarrow \frac{1}{\sqrt{2}}[|-_1\rangle|+_j\rangle+i|+_1\rangle|-_j\rangle]\\
|-_1\rangle|-_j\rangle &\longrightarrow |-_1\rangle|-_j\rangle.
\end{aligned} \tag{12}$$

Then, the purely quantum NiSWAP and $N\sqrt{i\text{SWAP}}$ gates can be implemented simultaneously between the first qubit and the N qubits.

3.2 *NSWAP and $N\sqrt{SWAP}$ gates*

The SWAP gate is equivalent to three CNOT gates. It is an appropriate elementary two-qubit gate. On the other hand, the $\sqrt{\text{SWAP}}$ gate constitutes a universal set of quantum gates together with single-qubit rotations around an arbitrary axis [6]. After the successive application of two $\sqrt{\text{SWAP}}$ gate the states of the qubits are interchanged. Thus, the SWAP and $\sqrt{\text{SWAP}}$ gates are useful in quantum computation and quantum information processing [20], such as establishing the universality of two-qubit gates [23], programmable gate arrays [24], and constructing quantum circuits [25], these type of quantum logic gates can be represented by the operators:

$$\mathrm{SWAP} \equiv |00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01| + |11\rangle\langle 11|, \tag{13}$$

$$\sqrt{\mathrm{SWAP}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1+i}{2} & \frac{1-i}{2} & 0 \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{14}$$

The main object of this subsubsection is to realize NSWAP and $N\sqrt{\mathrm{SWAP}}$ gates using the evolutions of the subsection 2.1 (see equation (10)). The proposed quantum logic gates provides a vivid blueprint for future quantum computation and quantum information processing, that is deterministic and scalable.

By selecting $\Gamma\tau = 2\pi$, $\eta = \left(2k - \frac{1}{2}\right)$ (with k being an integer) and $2\lambda\tau = -\frac{\pi}{2}$ (from equation (10)), an N two qubit SWAP operations are simultaneously performed on N qubit pairs $(1,2)$, $(1,3)$, ..., $(1,N+1)$ (from equation (10)), written as

$$\begin{aligned} &|+_1\rangle|+_j\rangle \longrightarrow |+_1\rangle|+_j\rangle \\ &|+_1\rangle|-_j\rangle \longrightarrow |-_1\rangle|+_j\rangle \\ &|-_1\rangle|+_j\rangle \longrightarrow |+_1\rangle|-_j\rangle \\ &|-_1\rangle|-_j\rangle \longrightarrow |-_1\rangle|-_j\rangle. \end{aligned} \tag{15}$$

By selecting $\Gamma\tau = -\pi$, $\eta = \left(2k' + \frac{1}{4}\right)$ (with k' being an integer) and $2\lambda\tau = \frac{\pi}{4}$, we obtain N-two-qubit $\sqrt{\mathrm{SWAP}}$ operations (from equation (10)) as

$$\begin{aligned} &|+_1\rangle|+_j\rangle \longrightarrow |+_1\rangle|+_j\rangle \\ &|+_1\rangle|-_j\rangle \longrightarrow \frac{1+i}{2}|+_1\rangle|-_j\rangle + \frac{1-i}{2}|-_1\rangle|+_j\rangle \\ &|-_1\rangle|+_j\rangle \longrightarrow \frac{1+i}{2}|-_1\rangle|+_j\rangle + \frac{1-i}{2}|+_1\rangle|-_j\rangle \\ &|-_1\rangle|-_j\rangle \longrightarrow |-_1\rangle|-_j\rangle. \end{aligned} \tag{16}$$

Hence, it is clear that the NSWAP and $N\sqrt{\mathrm{SWAP}}$ gates can be realized after the two-step process.

4 Fidelity and Discussion

In order to check the validity of our proposal, we study the following fidelity of the system for finding the target state. We assume that the cavity QED is initially in a Fock state $|n\rangle$, the fidelity of implementing the conditional phase operation is given by [26,27]

$$F = |\langle\Psi(t)|U(t)|\Psi(0)\rangle|^2, \tag{17}$$

where $|\Psi(t)\rangle$ is the state of the whole system after the above two-step gate operations, in the ideal case without considering the dissipation of the system during the entire gate operation that the initial state $|\Psi(0)\rangle$ followed by an ideal phase operation, and $U(t)$ represents the overall evolution operator of the whole system after the gate operations are performed in a real situation.

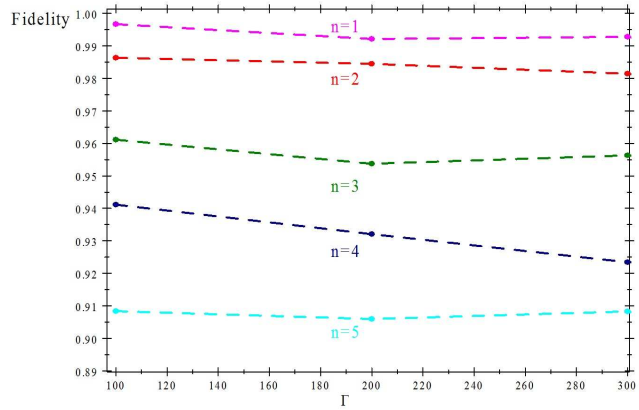

Fig. 2: Numerical results represent the fidelity of the gate operations versus the force qubit-qubit coupling $\Gamma(Hz)$ with the increase of the photon number n.

We numerically simulate the relationship between the fidelity of the system and the force qubit-qubit coupling Γ. Numerical simulations shows the influence of the photon number operations on the fidelity. Even for $n = 5$, the fidelity $F > 90\%$ (see Fig. 2).

Now, we discuss some issues which are relevant for future experimental implementation of our proposal. For our scheme to work, the total operation time $t_{op} = 2\tau + \tau_{aj} + 3\tau_{mo}$ should be much smaller than the cavity decay time κ^{-1}, so that the cavity dissipation is negligible. Here, t_{op} is independent of the number of target qubits N, τ_{aj} is the typical time required for adjusting the cavity mode frequency during step 2 and τ_{mo} is the typical time required for moving atoms into or out of the cavity. In addition, the t_{op} needs to be much smaller than the energy relaxation time of the level $|e_j\rangle$, such that the decoherence induced by the spontaneous decay of the level $|e_j\rangle$ is negligible. In principle, these conditions can be satisfied by choosing a cavity with a high quality factor Q and Rydberg atoms with a sufficiently long energy relaxation time where the principal quantum numbers are 50 and 51 (respectively, corresponding to the levels $|g_j\rangle$ and $|e_j\rangle$). The transition frequency is $\omega_0/2\pi \sim 51.1GHZ$, the radiative time of the level $|e_j\rangle$ is $T_r = 3.0\times 10^{-2}$s [12] and the coupling strength is $g = 2\pi\times 50K$Hz [28], which is experimentally available. We consider the conservative case of a larger cavity frequency $\omega_c/2\pi \sim 51.2GHZ$, the photon lifetime in the cavity or cavity decay time is $T_c = Q/\omega_c \sim 13.1\times 10^{-2}$s for a cavity with a high-quality factor $Q \sim 4.2\times 10^{10}$ which have been reported in experiments [29]. The direct calculation shows that the time required to implement an NiSWAP, $N\sqrt{i\mathrm{SWAP}}$, NSWAP and $N\sqrt{\mathrm{SWAP}}$ gates is $t_{op} = 24\mu$s for $\tau_{aj} = \tau_{mo} \sim 1\mu$s, then the N-two-qubit quantum logic gates operations time t_{op} is much shorter than the T_r and T_c, which satisfies our experimental requirement. Also, we note that the decoherence time of field state inside a cavity depends on the initial field state. However, the gate operation is independent of the initial state of the resonator because of the operator $U(2\tau)$ does not incluse the photon operator a and a^+ of the cavity. Our numerical calculation shows that a high fidelity $\sim 98\%$ can be achieved when the photon number $n \leqslant 5$ (Fig. 2).

5 Conclusion

In Summary, We have proposed an effective method to realize a two-step NiSWAP, $N\sqrt{i\text{SWAP}}$, NSWAP and $N\sqrt{\text{SWAP}}$ gates with $(N+1)$ qubits in a cavity QED by introducing the qubit-qubit interaction. As shown above, quantum logical gates are implemented by driving the resonator with microwave fields. The scheme is insensitive to the thermal field. The system requires no disagreement between the qubits and the cavity. In addition, the operation time is only dependent on the detuning and the time can be controlled by adjusting the frequency between the $|g_j\rangle$ and $|e_j\rangle$. However, we have presented a method to obtaine these logic gates, and we have calculated an evolution operator in two-step. Finally, we have applied the overall evolution operator to the working basis of the qubit 1 and the qubits j $(j = 2, ..., N+1)$ to find our quantum logic gates. The essential advantage of the scheme is that the our logic gates can be realized in a time much smaller than radiative time and lifetime of the cavity photons. In addition, numerical simulation of the gate operations shows that the scheme could be achieved with high fidelity under current state-of-the-art technology.

References

[1] T. D. Ladd and al., Quantum computers, Nature 464, 45-53 (2010).
[2] X. Lin, R-C. Yang, and X. Chen, Implementation of a quantum $\sqrt{\text{swap}}$ gate for two distant atoms trapped in separate cavities, Int. J. Quantum Inform. 13, 1550003 (2015).
[3] L. Liang and C. Li, Realization of quantum SWAP gate between flying and stationary qubits, Phys. Rev. A 72, 024303 (2005).
[4] A. Chouikh, T. Said, K. Essammouni, M. Bennai, Implementation of universal two- and three-qubit quantum gates in a cavity QED, Opt Quant Electron, 48:463 (2016).
[5] J. Ferrando-Soria et al., A modular design of molecular qubits to implement universal quantum gates, Nat. Commun. 7, 11377, (2016).
[6] K. Eckert et al., Quantum computing in optical microtraps based on the motional states of neutral atoms, Phys. Rev. A 66, 042317 (2002).
[7] L. Isenhower , E. Urban, X. Zhang et al., Demonstration of a neutral atom controlled-NOT quantum gate, Phys Rev Lett, 104: 010503 (2010).
[8] N. Kiesel, C. Schmid, U. Weber, et al., Linear optics controlled-phase gate made simple, Phys Rev Lett, 95: 210505 (2005).
[9] C. Monroe, D. M. Meekhof, B. E. King, W.M. Itano and D. J. Wineland, Demonstration of a fundamental quantum logic gate, Phys Rev Lett, 75: 4714– 4717 (1995).
[10] I. Fushman, D. Englund, A. Faraon et al., Controlled phase shifts with a single quantum dot, Science, 2008, 320: 769–772 (2008).
[11] J. A. Jones et al., Implementation of a quantum search algorithm on a quantum computer, Nature (London) 393 344 (1998).
[12] Q. Liu and L. Ye, Implementation of a Two-Atom $(\text{swap})^{1/2}$ Gate in Cavity QED, Chin. Phys. Lett. 24, 599 (2007).
[13] K. H. Song, Y. J. Zhao, Z. G. Shi, S. H. Xiang, X. W. Chen, Simultaneous implementation of n SWAP gates using superconducting charge qubits coupled to a cavity, Optics Communications 10 1016 (2010).
[14] E. Solano et al., Strong-driving-assisted multipartite entanglement in cavity QED, Phys. Rev. Lett. 90, 027903 (2003).
[15] S.B. Zheng, Generation of entangled states for many multilevel atoms in a thermal cavity and ions in thermal motion, Phys. Rev. A 68 035801 (2003).
[16] Z. J. Deng, M. Feng, and K. L. Gao, Simple scheme for the two-qubit Grover search in cavity QED, Phys. Rev. A 72, 034306 (2005).
[17] L. Ye and G.C. Guo, Scheme for implementing quantum dense coding in cavity QED, Phys. Rev. A 71, 034304 (2005).
[18] T. Said , A. Chouikh, K. Essammouni and M. Bennai, Realizing an N-two-Qubit Quantum Logic Gates in a Cavity QED with Nearest Qubit-Qubit Interaction, Quantum Information and Computation, Vol. 16 No. 5&6, 0465–0482 (2016).
[19] T. Tanamoto, Y. X. Liu, X. Hu, and F. Nori, Efficient Quantum Circuits for One-Way Quantum Computing, Phys. Rev. Lett., 102(10):100501 (2009).
[20] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, Cambridge Series on Information and the Natural Sciences, Cambridge University Press, Cambridge, England (2004).
[21] S. Rips and M. J. Hartmann, Quantum information processing with nanomechanical qubits, Phys. Rev. Lett. 110, 120503 (2013).
[22] M. Everitt, B. Garraway, Multiphoton resonances for all-optical quantum logic with multiple cavities, Phys. Rev. A 90, 012335 (2014).
[23] D. Deutsch, A. Barenco, A. Ekert, Universality in quantum computation, Proc. R. Soc. London A 449 669 (1995).
[24] M. A. Nielsen, I. L. Chuang, Programmable quantum gate arrays, Phys. Rev. Lett. 79 321 (1997).
[25] F. Vatan and C. Williams, Optimal quantum circuits for general two-qubit gates, Phys. Rev. A 69 032315 (2004).

[26] M. Grochol and C. Piermarocchi, Multispin errors in the optical control of a spin quantum lattice, Physical Review B B 78, 165324 (2008).
[27] S. M. Barnett et al., Fidelity and the communication of quantum information, J. Phys. A: Math. Gen. 34 6755 (2001).
[28] S. Osnaghi, P. Bertet, A. Auffeves, P. Maioli, M. Brune, J. M. Raimond, and S. Haroche, Coherent control of an atomic collision in a cavity, Phys. Rev. Lett. 87, 037902 (2001).
[29] S. Kuhr et al., Ultrahigh finesse Fabry-Perot superconducting resonator, Appl. Phys. Lett. 90, 164101 (2007).

Taoufik Said received his master degree in Systems and materials at Ibno Zohr University. He also received his PhD in the Laboratory of Physics of Condensed Matter at Ben M'sik Sciences Faculty, Casablanca, Morocco. His research interests include modeling of quantum information processing systems, implementation of quantum algorithm in a system QED, realization of quantum gates in a system QED, and quantum cryptography.

Abdelhaq Chouikh received his Graduate degree (D.E.S) in physics of materials at Casablanca An Chock Faculty of Sciences in 1996. He also received his PhD in the Laboratory of Physics of Condensed Matter at Faculty of Ben M'sik Sciences, Casablanca, Morocco. His research interests include modeling of quantum information processing systems, implementation of quantum algorithm in a system QED, realization of quantum gates in a system QED.

Mohamed Bennai is a Professor of quantum physics at Hassan II University Casablanca Morocco. He is also a visiting Professor of Theoretical Physics at Mohamed V University Rabat Morocco. He received the CEA (1990), Doctorat de 3me Cycle (1992) and the PhD (2002) from Hassan II University in Theoretical Physics. He is actually a Research Director of physics and quantum technology group at the Laboratory of Condensed Matter Physics of Ben M'sik Sciences Faculty, Casablanca, Morocco.