Applied Mathematics & Information Sciences
An International Journal

Self-Tuning Weighted Measurement Fusion Kalman Filter for ARMA Signals with Colored Noise

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Received: Received May 02, 2011; Revised July 25, 2011; Accepted September 12, 2011
Published online: 1 January 2012

Abstract: Based on the modern time series analysis method, for single channel autoregressive moving average (ARMA) signals with colored noise, a self-tuning weighted measurement fusion Kalman filter is presented when the model parameters and noise statistics are unknown. By applying the recursive instrumental variable (RIV) algorithm and the Gevers-Wouters (G-W) iterative algorithm with dead band, the local and fused estimates for the unknown model parameters and noise variances can be obtained. Then a self-tuning weighted measurement fusion Kalman filter is obtained by substituting the fused estimates into the corresponding optimal fusion Kalman filter. Further, applying the dynamic error system analysis (DESA) method, it is rigorously proved that the self-tuning weighted measurement fusion Kalman filter has globally asymptotic optimality. A simulation example shows its effectiveness.

Keywords: Weighted measurement fusion, self-tuning Kalman filter, ARMA signal, colored noise.

1. Introduction

Information fusion has become one of the most popular fields for its advantage of overcoming the inaccuracy of single sensor. Two fusion methods were developed after the occurrence of the information fusion theory [1]: the centralized fusion method [2,3] and the distributed fusion method [4-6]. The former needs to combine all the local measurement equations into a high-dimension measurement equation, according to which a centralized fusion estimator is obtained. This estimator has globally optimality but generates large computational burden to solve the high-dimension equation. The latter includes the weighted state fusion method and the weighted measurement fusion method. The weighted state fusion estimator [7,8] is obtained by weighting all local state estimators, it can facilitate fault detection more conveniently and increase the input data rates significantly, but it is just globally suboptimal and requires to compute the cross-covariance among the local estimation errors. The weighted measurement fusion estimator [9,10] is obtained by weighting all local measurement equations, it may increase the computational burden to obtain the fused measurement and the fused measurement noise variance, however, it has asymptotical global optimality as the centralized fusion estimator. In the existing results, most of them are for the single-channel system with white measurement noise [4,6,9], however, the results for the multi-channel system [5,10] or for the system with colored measurement noise [11] are seldom because of their complexity.

In this paper, for the multisensor single channel ARMA signal system with colored noise, when the model parameters and noise variances are unknown, the local and fused estimates of them can be obtained by the RIV algorithm [12] and the Gevers-Wouters algorithm[13] . Further, a self-tuning weighted measurement fusion Kalman filter is obtained by substituting the fused estimates into the optimal weighted measurement fusion Kalman filter. Then applying the DESA method [6,8], it is rigorously proved that the self-tuning weighted measurement fusion Kalman filter converges to the optimal weighted measurement fusion Kalman filter.

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2. Problem Formulation

Consider a multisensor single channel ARMA signal system with the common colored measurement noise

$$A(q^{-1})s(t) = C(q^{-1})e(t) \quad (1)$$

$$y_i(t) = s(t) + \eta(t) + v_i(t), i = 1, \cdots, L \quad (2)$$

$$P(q^{-1})\eta(t) = R(q^{-1})\xi(t) \quad (3)$$

where t is the discrete time, $s(t)$ is the signal, $y_i(t)$ is the measurement of the ith sensor, $\xi(t)$ is white noise, $e(t)$, $v_i(t)$ and $\eta(t)$ are white process noise, white measurement noise and colored measurement noise, respectively. $A(q^{-1})$, $C(q^{-1})$, $P(q^{-1})$ and $R(q^{-1})$ are stable polynomials (i.e. all zeros of each polynomial lie inside the unit circle) of backward shift operator q^{-1} with the following form:

$$X(q^{-1}) = 1 + x_1 q^{-1} + \cdots + x_{n_x} q^{-n_x} \quad (4)$$

and $n_a > n_c, n_p > n_r$.

Assumption 1. $e(t), v_i(t)$ and $\xi(t)$ are uncorrelated white noises with zero mean and variances $\sigma_e^2, \sigma_{vi}^2$ and σ_ξ^2, respectively.

Assumption 2. The model parameters a_j, c_j and the noise variance σ_e^2 are unknown, the parameters p_j, r_j and the noise variances σ_{vi}^2, σ_ξ^2 are known.

Assumption 3. For each sensor i $(i = 1, \cdots, L)$, the measurement data $y_i(t)$ are bounded, i.e. a realization of the stochastic process $y_i(t)$ is bounded for t.

The problem is to find a self-tuning weighted measurement fusion Kalman filter $\hat{s}^s(t|t)$ and prove its asymptotic optimality.

3. Optimal Weighted Measurement Fusion Kalman Filter

3.1. The conversion of signal model into state space model

Making $e(t) = \underline{e}(t-1)$ and from (1), we have

$$A(q^{-1})s(t) = \underline{C}(q^{-1})\underline{e}(t) \quad (5)$$

where $\underline{C}(q^{-1}) = q^{-1} + c_1 q^{-2} \cdots + c_{n_c} q^{-n_c - 1}$. Then we have the state space model [6] as

$$\alpha(t+1) = \bar{A}\alpha(t) + \bar{C}\underline{e}(t) \quad (6)$$

$$s(t) = H_1 \alpha(t) \quad (7)$$

where

$$\bar{A} = \begin{bmatrix} -a_1 & & \\ -a_2 & I_{n_a-1} & \\ \vdots & & \\ -a_{n_a} & 0 \cdots & 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} 1 \\ c_1 \\ \vdots \\ c_{n_a-1} \end{bmatrix}, \quad (8)$$

$H_1 = \begin{bmatrix} 1 \ 0 \cdots 0 \end{bmatrix}$

with $c_j = 0 (j > n_c)$.

Similarly, making $\xi(t) = \underline{\xi}(t-1)$, another state space model with following form can be obtained from (3):

$$\beta(t+1) = \bar{P}\beta(t) + \bar{R}\underline{\xi}(t) \quad (9)$$

$$\eta(t) = H_2 \beta(t) \quad (10)$$

where

$$\bar{P} = \begin{bmatrix} -p_1 & & \\ -p_2 & I_{n_p-1} & \\ \vdots & & \\ -p_{n_p} & 0 \cdots & 0 \end{bmatrix}, \bar{R} = \begin{bmatrix} 1 \\ r_1 \\ \vdots \\ r_{n_p-1} \end{bmatrix}, \quad (11)$$

$H_2 = \begin{bmatrix} 1 \ 0 \cdots 0 \end{bmatrix}$

with $r_j = 0 (j > n_r)$. According to the definition of $\underline{e}(t)$ and $\underline{\xi}(t)$, we have $\sigma_{\underline{e}}^2 = \sigma_e^2$ and $\sigma_{\underline{\xi}}^2 = \sigma_\xi^2$.

Then an augmented system can be obtained from (2) and (6) – (11) as

$$x(t+1) = \Phi x(t) + \Gamma \bar{w}(t) \quad (12)$$

$$y_i(t) = Hx(t) + v_i(t) \quad (13)$$

$$s(t) = \bar{H}x(t) \quad (14)$$

where

$$x(t) = \begin{bmatrix} \alpha(t) \\ \beta(t) \end{bmatrix}, \bar{w}(t) = \begin{bmatrix} \underline{e}(t) \\ \underline{\xi}(t) \end{bmatrix}, \Phi = \begin{bmatrix} \bar{A} & 0 \\ 0 & \bar{P} \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} \bar{C} & 0 \\ 0 & \bar{R} \end{bmatrix},$$

$$H = \begin{bmatrix} H_1 \ H_2 \end{bmatrix}, \bar{H} = \begin{bmatrix} H_1 \ 0 \end{bmatrix} \quad (15)$$

It is obvious that $\bar{w}(t)$ and $v_i(t)$ are uncorrelated white noises with zero mean, and

$$\mathrm{E}\left\{ \begin{bmatrix} \bar{w}(t) \\ v_i(t) \end{bmatrix} \begin{bmatrix} \bar{w}^T(k) & v_j^T(k) \end{bmatrix} \right\} = \begin{bmatrix} Q_{\bar{w}} & 0 \\ 0 & \sigma_{vi}^2 \delta_{ij} \end{bmatrix} \delta_{tk} \quad (16)$$

$$Q_{\bar{w}} = \begin{bmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_\xi^2 \end{bmatrix} \quad (17)$$

where E denotes the mathematical expectation, T denotes the transpose, $\delta_{tt} = 1, \delta_{tk} = 0 (t \neq k)$.

3.2. The weighted measurement fusion model

From (13), the centralized fusion measurement equation is obtained by combining all the measurement equations as

$$y^{(0)}(t) = \mathrm{H}^{(0)} x(t) + v^{(0)}(t) \quad (18)$$

where

$$y^{(0)}(t) = [y_1(t), \cdots, y_L(t)]^T \quad (19)$$

$$H^{(0)} = [H^{\mathrm{T}}, \cdots, H^{\mathrm{T}}]^T \quad (20)$$

$$v^{(0)}(t) = [v_1(t), \cdots, v_L(t)]^T \quad (21)$$

where the fused measurement noise $v^{(0)}(t)$ is a white noise with zero mean and variance matrix

$$R^{(0)} = \begin{bmatrix} \sigma_{v1}^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_{vL}^2 \end{bmatrix} \tag{22}$$

The centralized fusion measurement equation (18) can be viewed as a measurement model for $Hx(t)$, so it can be rewritten as

$$y^{(0)}(t) = eHx(t) + v^{(0)}(t) \tag{23}$$

with the definition $e^T = [1, \cdots, 1]$.

Then the Gauss-Markov estimate of $Hx(t)$ can be obtained by applying the weighted least squares (WLS) method to (23):

$$y(t) = (e^T R^{(0)-1} e)^{-1} e^T R^{(0)-1} y^{(0)}(t) \tag{24}$$

From (19), (24) and Assumption 3, it yields that a realization of the stochastic process $y(t)$ is bounded for t. Thus, the weighted measurement fusion equation is given by

$$y(t) = Hx(t) + \bar{v}(t) \tag{25}$$

From (18), (24) and (25), it yields that

$$\bar{v}(t) = (e^T R^{(0)-1} e)^{-1} e^T R^{(0)-1} v^{(0)}(t) \tag{26}$$

with the noise variance

$$\sigma_{\bar{v}}^2 = (e^T R^{(0)-1} e)^{-1} \tag{27}$$

3.3. The optimal weighted measurement fusion Kalman filter

From (12), (15) and (25), we have

$$\bar{A}(q^{-1})y(t) = \bar{B}(q^{-1})\bar{w}(t) + \bar{A}(q^{-1})\bar{v}(t) \tag{28}$$

where

$$\bar{A}(q^{-1}) = A(q^{-1})P(q^{-1}) \tag{29}$$

$$\begin{aligned} \bar{B}(q^{-1}) &= [\bar{C}(q^{-1}), \bar{R}(q^{-1})]q^{-1}, \bar{C}(q^{-1}) \\ &= C(q^{-1})P(q^{-1}), \bar{R}(q^{-1}) = A(q^{-1})R(q^{-1}) \end{aligned} \tag{30}$$

with $\bar{a}_0 = \bar{c}_0 = \bar{r}_0 = 1$,$n_{\bar{a}} = n_a + n_p, n_{\bar{c}} = n_c + n_p, n_{\bar{r}} = n_a + n_r$.

Thus the ARMA innovation model is obtained as

$$\bar{A}(q^{-1})y(t) = D(q^{-1})\varepsilon(t) \tag{31}$$

where $D(q^{-1}) = 1 + d_1 q^{-1} + \cdots + d_{n_d} q^{-n_d} (n_d = n_{\bar{a}})$ is a stable polynomial, and innovation process $\varepsilon(t) \in R$ is white noise with zero mean and variance σ_ε^2.

$$D(q^{-1})\varepsilon(t) = \bar{B}(q^{-1})\bar{w}(t) + \bar{A}(q^{-1})\bar{v}(t) \tag{32}$$

$D(q^{-1})$ and σ_ε^2 can be obtained by Gevers-Wouters iterative algorithm [13].

Lemma 1 [14]. For the state space model (12) and (25), the steady-state optimal weighted measurement fusion Kalman filter is given by

$$\hat{x}(t|t) = \Psi_f \hat{x}(t-1|t-1) + K_f y(t) \tag{33}$$

$$\Psi_f = [I_{n_{\bar{a}}} - K_f H]\Phi \tag{34}$$

where Ψ_f is a stable matrix. The filter gain K_f is given as

$$K_f = \begin{bmatrix} H \\ H\Phi \\ \vdots \\ H\Phi^{n_d - 1} \end{bmatrix}^{-1} \begin{bmatrix} 1 - \sigma_{\bar{v}}^2/\sigma_\varepsilon^2 \\ m_1 \\ \vdots \\ m_{n_d - 1} \end{bmatrix} \tag{35}$$

where the m_j can recursively be computed as

$$m_j = -\bar{a}_1 m_{j-1} - \cdots - \bar{a}_{n_d} m_{j-n_d} + d_j, j = 1, \cdots, n_d \tag{36}$$

with $m_j = 0 (j < 0), m_0 = 1$.

Theorem 1. For the multisensor single channel ARMA signal with colored noise, the steady-state optimal weighted measurement fusion Kalman signal filter is given by

$$\hat{s}(t|t) = \bar{H}\hat{x}(t|t) \tag{37}$$

Proof. Making the projection to (14) directly yields (37). The proof is completed.

4. Estimates of the Unknown Model Parameters and Noise Variance

4.1. The estimate of $A(q^{-1})$ based on the RIV algorithm

Introduce a new measurement

$$z_i(t) = P(q^{-1})y_i(t) \tag{38}$$

From (1), (2) and (3), we have

$$A(q^{-1})z_i(t) = \bar{C}(q^{-1})e(t) + \bar{R}(q^{-1})\xi(t) + \bar{A}(q^{-1})v_i(t) \tag{39}$$

where $\bar{A}(q^{-1})$, $\bar{C}(q^{-1})$ and $\bar{R}(q^{-1})$ are defined in (29) and (30).

Hence for the ith sensor, we have the corresponding least squares (LS) structure as

$$z_i(t) = \varphi_i^T(t)\theta + h_i(t) \tag{40}$$

$$\varphi_i(t) = [-z_i(t-1), \cdots, -z_i(t-n_a)]^T \tag{41}$$

$$\theta = [a_1, \cdots, a_{n_a}]^T \tag{42}$$

$\mathrm{h}_i(t) = e(t) + \bar{c}_1 e(t-1) + \cdots + \bar{c}_{n_{\bar{c}}} e(t - n_{\bar{c}}) + \xi(t) + \bar{r}_1 \xi(t-1) + \cdots + \bar{r}_{n_{\bar{r}}} \xi(t - n_{\bar{r}}) + v_i(t) + \bar{a}_1 v_i(t-1) + \cdots + \bar{a}_{n_{\bar{a}}} v_i(t - n_{\bar{a}})$ Applying the recursive instrument variable (RIV) algorithm [12], the local estimates $\hat{\theta}_i(t)$ of θ are obtained and they converge to the true value with probability one, i.e.

$$\hat{\theta}_i(t) \to \theta, as \quad t \to \infty, w.p.1, i = 1, \cdots, L \tag{43}$$

Then the fused estimate $\hat{\theta}_f(t)$ of θ is defined as

$$\hat{\theta}_f(t) = \frac{1}{L}\sum_{i=1}^{L}\hat{\theta}_i(t) \tag{44}$$

So it is obvious that $\hat{\theta}_f(t)$ is also strongly consistent, i.e.

$$\hat{\theta}_f(t) \to \theta, as \quad t \to \infty, w.p.1 \tag{45}$$

Hence, the strongly consistent fused estimates $\hat{a}_{jf}(t)(j = 1,\cdots,n_a)$ are obtained, i.e. $\hat{A}(q^{-1})$ is obtained. Then substituting $\hat{a}_{jf}(t)$ into (29) and (30), $\hat{\bar{a}}_j(t)(j = 1,\cdots,n_{\bar{a}})$ and $\hat{\bar{r}}_j(t)(j = 1,\cdots,n_{\bar{r}})$ are obtained, i.e. $\hat{\bar{A}}(q^{-1})$ and $\hat{\bar{R}}(q^{-1})$ are obtained.

4.2. The estimates of $C(q^{-1})$ and σ_e^2 based on the Gevers-Wouters algorithm with dead band

From (38), (39) and (4.1) we have

$$h_i(t) = \bar{A}(q^{-1})y_i(t) \tag{46}$$

It is clearly that $\bar{A}(q^{-1})$ is a stable polynomials of q^{-1}, $y_i(t)$ is a stationary stochastic process, so it yields that $r_i(t)$ is also a stationary stochastic process with correlation function as

$$R_{hi}(k) = E[h_i(t)h_i(t-k)], \quad k = 0,1,\cdots,n_{\bar{a}} \tag{47}$$

with a cut-off as lag $n_{\bar{a}}$. At time t, the estimate of the measurement process $h_i(t)$ is defined as

$$\hat{h}_i(t) = \hat{\bar{A}}(q^{-1})y_i(t) \tag{48}$$

then the on-line sample estimate of $R_{hi}(k)$ is defined as

$$\hat{R}^t_{hi}(k) = \frac{1}{t}\sum_{\alpha=1}^{t}\hat{h}_i(\alpha)\hat{h}_i(\alpha-k) \tag{49}$$

which has the recursive formula

$$\hat{R}^t_{hi}(k) = \hat{R}^{t-1}_{hi}(k) + \frac{1}{t}[\hat{h}_i(t)\hat{h}_i(t-k) - \hat{R}^{t-1}_{hi}(k)], \quad t = 2,3\cdots \tag{50}$$

with the initial value $\hat{R}^1_{hi}(k) = \hat{h}_i(1)\hat{h}_i(1-k)$.

Defining $m(t) = \bar{C}(q^{-1})e(t)$, then its correlation function $R_m(k) = E[m(t)m(t-k)]$ has the formula as

$$R_m(k) = \sigma_e^2\sum_{j=k}^{n_{\bar{c}}}\bar{c}_j\bar{c}_{j-k}, k = 0,1,\cdots,n_{\bar{c}} \tag{51}$$

Computing the correlation function of the two sides of (4.1), we have that

$$R_m(k) = R_{hi}(k) - \sigma_\xi^2\sum_{j=k}^{n_{\bar{r}}}\bar{r}_j\bar{r}_{j-k} - \sigma_{vi}^2\sum_{j=k}^{n_{\bar{a}}}\bar{a}_j\bar{a}_{j-k}, k = 0,1,\cdots,n_{\bar{c}} \tag{52}$$

Substituting $\hat{\bar{a}}_j(t), \hat{\bar{r}}_j(t)$ and the sample estimates $\hat{R}^t_{hi}(k)$ into (52), the local estimates $\hat{R}_{mi}(k)$ of $R_m(k)$ are obtained. Applying Gevers-Wouters algorithm with dead band, the local on-line estimates $\hat{\bar{c}}_{ji}(t), \hat{\sigma}^2_{ei}(t)$ at time t are obtained. Then the local estimates $\hat{c}_{ji}(t)$ of c_j can be obtained by solving the linear equation (30). Further, the information fusion estimates $\hat{\bar{c}}_{jf}(t), \hat{c}_{jf}(t), \hat{\sigma}^2_{ef}(t)$ based on all sensors are defined as

$$\begin{aligned}\hat{\bar{c}}_{jf}(t) &= \frac{1}{L}\sum_{i=1}^{L}\hat{\bar{c}}_{ji}(t), \hat{c}_{jf}(t) \\ &= \frac{1}{L}\sum_{i=1}^{L}\hat{c}_{ji}(t), \hat{\sigma}^2_{ef}(t) \\ &= \frac{1}{L}\sum_{i=1}^{L}\hat{\sigma}^2_{ei}(t), j = 1,\cdots,n_{\bar{c}}\end{aligned} \tag{53}$$

with $\hat{c}_{jf}(t) = 0(j > n_c)$.

Remark: The Gevers-Wouters algorithm with dead band means running Gevers-Wouters algorithm every T steps, where T is a domain selected ahead. Compared with running Gevers-Wouters algorithm every time, this method can reduce the computational burden.

5. Self-tuning Weighted Measurement Fusion Kalman Filter

For the multisensor single channel ARMA signal (1)-(3) with Assumptions 1-3, the self-tuning weighted measurement fusion Kalman signal filter $\hat{s}^s(t|t)$ can be realized by the following four parts:

Step 1. From (44), the fused estimates $\hat{a}_{jf}(t)$ at time t can be obtained by taking the average of all local estimates, which are obtained by applying the RIV algorithm. Then from (29) and (30), the estimates $\hat{\bar{a}}_j(t)$ and $\hat{\bar{r}}_j(t)$ are obtained.

Step 2. From (48) and (50), the estimates $\hat{h}_i(t)$ and the sampled correlation function estimates $\hat{R}^t_{hi}(k)$ are obtained. Applying these estimates and the Gevers-Wouters algorithm to (52) and (51) respectively, the local estimates $\hat{\bar{c}}_{ji}(t)$ and $\hat{\sigma}^2_{ei}(t)$ at time t are obtained. Then the local estimates $\hat{c}_{ji}(t)$ can be obtained by solving linear equation (30). Further, the fused estimates $\hat{\bar{c}}_{jf}(t)$, $\hat{c}_{jf}(t)$ and $\hat{\sigma}^2_{ef}(t)$ are obtained by taking the average of all corresponding local estimates.

Step 3. Substituting all the fused estimates obtained in step 1 and 2 into (8), (11), (15) and (16), $\hat{\bar{\Phi}}(t), \hat{\bar{\Gamma}}(t), \hat{Q}_{\bar{w}}(t)$ are obtained. Then from (30), $\hat{\bar{B}}(q^{-1})$ are obtained. Further, from (32), based on the Gevers-Wouters iterative algorithm with dead band, the estimates $\hat{d}_j(t)$ and $\hat{\sigma}^2_{\varepsilon}(t)$ of the ARMA innovation model parameters and noise variance are obtained.

Step 4. Applying (33)-(37), the self-tuning weighted measurement fusion Kalman signal filter is given by

$$\hat{s}^s(t|t) = \bar{H}\hat{x}^s(t|t) \tag{54}$$

$$\hat{x}^s(t|t) = \hat{\Psi}_f(t)\hat{x}^s(t-1|t-1) + \hat{K}_f(t)y(t) \quad (55)$$

$$\hat{\Psi}_f(t) = [I_{n_{\bar{a}}} - \hat{K}_f(t)H]\hat{\Phi}(t) \quad (56)$$

$$\hat{K}_f(t) = \begin{bmatrix} H \\ H\hat{\Phi}(t) \\ \vdots \\ H\hat{\Phi}(t)^{n_d-1} \end{bmatrix}^{-1} \begin{bmatrix} 1-\sigma_{\bar{v}}^2(t)/\hat{\sigma}_\varepsilon^2(t) \\ \hat{m}_1(t) \\ \vdots \\ \hat{m}_{n_d-1}(t) \end{bmatrix} \quad (57)$$

$$\hat{m}_j(t) = -\hat{\bar{a}}_1(t)\hat{m}_{j-1}(t) - \cdots - \hat{\bar{a}}_{n_d}(t)\hat{m}_{j-n_d}(t) + \hat{d}_j(t) \quad (58)$$

with $\hat{m}_j(t) = 0 (j < 0), \hat{m}_0(t) = 1$.

6. Convergence Analysis

Theorem 2. For the multisensor system (1)-(3) with Assumptions 1-3, we have $\hat{a}_{jf}(t) \to a_j, \hat{\bar{a}}_j(t) \to \bar{a}_j, \hat{\bar{r}}_j(t) \to \bar{r}_j, \hat{\bar{c}}_{jf}(t) \to \bar{c}_j, \hat{c}_{jf}(t) \to c_j, \hat{\sigma}_{ef}^2(t) \to \sigma_e^2,$ $as \quad t \to \infty, i.a.r.$

$$\hat{d}_j(t) \to d_j, \hat{\sigma}_\varepsilon^2(t) \to \sigma_\varepsilon^2, as \quad t \to \infty, i.a.r. \quad (59)$$

$$\hat{\Phi}(t) \to \Phi, \hat{\Gamma}(t) \to \Gamma, \hat{K}_f(t) \to K_f, \hat{\Psi}_f(t) \to \Psi_f, as \quad t \to \infty, i.a.r. \quad (60)$$

where "i.a.r." denotes "in a realization".

Proof. From (42) and (45), we have $\hat{a}_{jf}(t) \to a_j$, as $t \to \infty$, i.a.r. Then from (29) and (30), $\hat{\bar{a}}_j(t) \to \bar{a}_j, \hat{\bar{r}}_j(t) \to \bar{r}_j$. From (46), (48) and Assumption 3 we have $\hat{h}_i(t) \to h_i(t)$, then $\hat{R}_{hi}^t(k) \to R_{hi}(k)$. From (51), (52) and the existence theorem of implicit function, it follows that in a sufficiently small neighborhood of the elements of $R_{hi}(k)$, $\bar{c}_j$ and σ_e^2 are the continuous functions of $R_{hi}(k), \bar{a}_j, \bar{r}_j$ and $\sigma_\xi^2, \sigma_{vi}^2$. Substituting $\hat{R}_{hi}^t(k), \hat{\bar{a}}_j(t), \hat{\bar{r}}_j(t)$ into (51) and (52), applying the Gevers-Wouters algorithm to (51) will yield $\hat{\bar{c}}_{ji}(t), \hat{\sigma}_{ei}^2(t)$, then by solving the linear equation (30), $\hat{c}_{ji}(t)$ are obtained. From (53), the fused estimates $\hat{\bar{c}}_{jf}(t), \hat{c}_{jf}(t), \hat{\sigma}_{ef}^2(t)$ are obtained. Therefore, according to the continuity of function, it follows that $\hat{\bar{c}}_{jf}(t) \to \bar{c}_j$,$\hat{c}_{jf}(t) \to c_j$, $\hat{\sigma}_{ef}^2(t) \to \sigma_e^2$. Further from (30) and (32), $\hat{d}_j(t) \to d_j, \hat{\sigma}_\varepsilon^2(t) \to \sigma_\varepsilon^2$. From (15) and Lemma 1, it can be easily obtained that $\hat{\Phi}(t) \to \Phi, \hat{\Gamma}(t) \to \Gamma, \hat{K}_f(t) \to K_f, \hat{\Psi}_f(t) \to \Psi_f$. The proof is completed.

Theorem 3. For the multisensor single channel ARMA signal (1) - (3) with Assumptions 1-3, the self-tuning weighted measurement fusion Kalman signal filter $\hat{s}^s(t|t)$ converges to the steady-state optimal weighted measurement fusion Kalman signal filter $\hat{s}(t|t)$ in a realization

$$[\hat{s}^s(t|t) - \hat{s}(t|t)] \to 0, as \quad t \to \infty, i.a.r. \quad (61)$$

Proof. Setting $\hat{K}_f(t) = K_f + \Delta\hat{K}_f(t), \hat{\Psi}_f(t) = \Psi_f + \Delta\hat{\Psi}_f(t)$, from (60) we have that $\Delta\hat{K}_f(t) \to 0, \Delta\hat{\Psi}_f(t) \to 0$. Denoting $\delta(t) = \hat{x}^s(t|t) - \hat{x}(t|t)$, subtracting (33) from (55) yields a dynamic error system

$$\delta(t) = \Psi_f \delta(t-1) + u(t) \quad (62)$$

$$u(t) = \Delta\hat{\Psi}_f(t)\hat{x}^s(t-1|t-1) + \Delta\hat{K}_f(t)y(t) \quad (63)$$

Applying (60) and the boundness of $y(t)$ yields that $\hat{K}_f(t)y(t)$ is bounded. Noting that $\hat{\Psi}_f(t) \to \Psi_f$, and Ψ_f is a stable matrix, from (55) it yields that $\hat{x}^s(t|t)$ is bounded in a realization [6,8]. Hence from (63), it yields that $u(t) \to 0$ based on the boundness of $y(t)$ and $\hat{x}^s(t|t)$. Then from (62) we that $\delta(t) \to 0$ [6,8]. Further, from (37) and (54), it can be easily obtained that (61) holds. The proof is completed.

7. Simulation Example

Consider a 3-sensor single channel ARMA signal system with colored measurement noise

$$(1 + a_1 q^{-1} + a_2 q^{-2})s(t) = (1 + c_1 q^{-1})e(t) \quad (64)$$

$$y_i(t) = s(t) + \eta(t) + v_i(t), i = 1, 2, 3 \quad (65)$$

$$(1 + p_1 q^{-1} + p_2 q^{-2})\eta(t) = (1 + r_1 q^{-1})\xi(t) \quad (66)$$

where $y_i(t)$ is the measurement of the ith sensor, $e(t), v_i(t), \xi(t)$ are independent Gaussian white noises with zero mean and variances $\sigma_e^2, \sigma_{vi}^2, \sigma_\xi^2$, respectively. In simulation we take $a_1 = -1.7$, $a_2 = 0.72$, $c_1 = 0.4$, $p_1 = 1$, $p_2 = 0.21$, $r_1 = -0.2$, $\sigma_w^2 = 2$,$\sigma_{v1}^2 = 0.2$, $\sigma_{v2}^2 = 0.4$, $\sigma_{v3}^2 = 0.6$, $\sigma_\xi^2 = 0.3$.

The problem is to find the self-tuning weighted measurement fusion Kalman signal filter $\hat{s}^s(t|t)$, when the model parameters a_1, a_2, c_1 and noise variance σ_e^2 are unknown.

The simulation results are shown in Figure 1-6.

Applying the RIV algorithm, the estimates of a_j are obtained in Figure 1 and Figure 2. Then applying the Gevers-Wouters algorithm with the dead band $T_d = 400$, the estimates of c_1 and σ_e^2 are obtained in Figure 3. The dotted curves denote the local estimates, the solid curves denote the fused estimates, the straight lines denote the true values in Figure 1 - Figure 3. The curves of the signal $s(t)$ and measurement $y_i(t)$ are given in Figure 4. Applying the optimal and the self-tuning weighted measurement fusion Kalman filtering algorithm, we can obtain the optimal and self-tuning filters $\hat{s}(t|t)$ and $\hat{s}^s(t|t)$ in Figure 5. Compared Figure 4 with Figure 5, it is obvious that the accuracies of the optimal and self-tuning fusers are higher than that of local measurements. The error curve between the optimal and self-tuning fused Kalman signal filters is shown in Figure 6, from which we see that the self-tuning fused filter converges to the optimal fused filter, i.e. the error converges to zero.

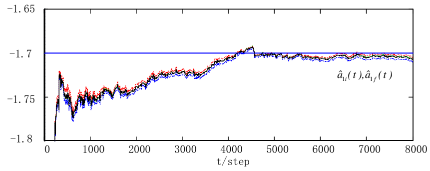

Figure 1 The local and fused estimates of a_1

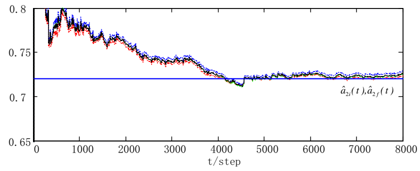

Figure 2 The local and fused estimates of a_2

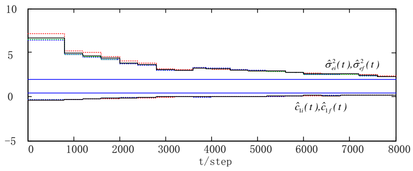

Figure 3 The local and fused estimates of c_1 and σ_e^2

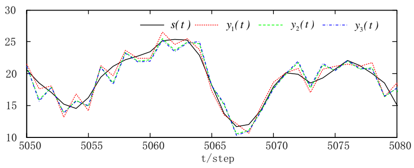

Figure 4 The signal $s(t)$ and the measurement $y_i(t)$

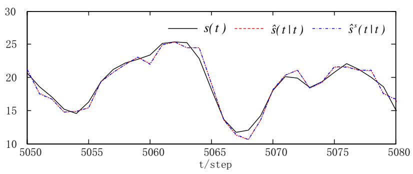

Figure 5 The signal $s(t)$, the optimal fused filter $\hat{s}(t|t)$ and the self-tuning fused filter $\hat{s}^s(t|t)$

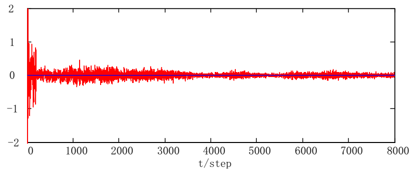

Figure 6 The error curves between the self-tuning and optimal weighted measurement fusion Kalman signal filters

8. Conclusion

For the multisensor single ARMA signal system with colored measurement noise, when the model parameters and the noise variance are unknown, the local and fused estimates of the model parameters and noise variances can be obtained by the RIV algorithm and the Gevers-Wouters algorithm with dead band. Then a self-tuning weighted measurement fusion Kalman filter is presented by applying the fused estimates as the true values. Further, applying the DESA method, it is rigorously proved that the self-tuning weighted measurement fusion Kalman filter converges to the corresponding optimal fusion Kalman filter in a realization, i.e. it has asymptotical optimality.

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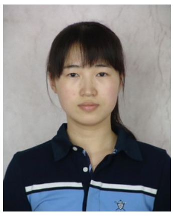

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