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# Contribution of Omega (782)- meson to the incoherent $\pi^0$ Electroproduction Off the Deuteron

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**Abstract:** The effect of the omega- meson ( $\omega$ -meson) on the unpolarized semi-exclusive structure functions for the incoherent neutral pion electroproduction form the deuteron at different values for the squared four-momentum transfer ( $K^2$ ) and the virtual photon lab energy ( $k_0^{(ab)}$ ). is studied. The study is carried out in the impulse approximation (IA) i.e. the final state interactions are neglected. The elementary amplitude for pion electroproduction is taken from the MAID-2007 model. The contribution of  $\omega$ -meson to the structure functions is noticeable which decrease, it increasing ( $k_0^{(ab)}$ ). While increasing ( $K^2$ ) lead to a small increase in the effect of  $\omega$ -meson contribution to the structure functions.

**Keywords:** pion electroproduction; the  $\omega$  (782) resonance, Invariant Amplitudes, CGLN amplitudes, Structure functions, the impulse approximation (IA).

## **1** Introduction

The present paper is considered as an extension of previous work on single pion electroproduction on the deuteron [1][2][3]. In the first paper [1] the incoherent single pion electroproduction on the deuteron with polarization effects is studied then in the second one [2] the contribution of single pion electroproduction to the generalized Gerasimov-Drell-Hearn sum rule for the deuteron is studied where in the third [3] the effect of the  $\Delta(1232)$ the incoherent resonance on pion electroproduction off the deuteron is studied. As an extension to studying the contributions of the different resonances to the incoherent pion electroproduction off the deuteron, in this paper the contribution of  $\omega$ -meson decay on the process will be systematically studied.

Deuteron was chosen since it allows one to study this reaction on a bound nucleon in the simplest nuclear environment, so that one can take into account medium effects in a reliable manner, at least in the nonrelativistic domain. It provides important information on this reaction on the neutron. In view of this latter aspect, the deuteron is often considered as an effective neutron target assuming that binding effects can be neglected to a large extent. In the incoherent process[4][5], the nucleus ruptures and thus fails to maintain its initial identity. The meson is produced in association with a nucleon (or an excited state of the nucleon) and some new recoil "daughter" hadronic system. Thus, the interaction starts with a a virtual photon and some nucleus and ends up with a meson, a free nucleon (or an excited state of it) and a new hadronic system, i.e.  $\gamma^* X_A \longrightarrow \pi N X_{A-1}$ . The process is labeled as "incoherent" because it occurs in kinematic and physical circumstances similar to those of the process that produces a meson from a free nucleon.

The pion electroproduction on the deuteron near threshold has been studied in the impulse approximation using an approach based on the unitary transformation method both experimentally [6][7][8] as well as theoretically [9][10][11].

Nucleon resonances are excited states of nucleon particles, often corresponding to one of the quarks having a flipped spin state, or with different orbital angular momentum when the particle decays. The symbol format is given as  $N(M)L_{2I2J}$ , where M is the particle's approximate mass, L is the orbital angular momentum of the Nucleon-meson pair produced when it decays, and I and J are the particle's isospin and total angular momentum respectively. Since nucleons are defined as

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having 1/2 isospin, the first number will always be 1, and the second number will always be odd. When discussing nucleon resonances, sometimes the N is omitted and the order is reversed, giving  $L_{2I2J}(M)$ . For example, a proton can be symbolized as " $N(939)S_{11}$ " or " $S_{11}(939)$ ", Delta resonances can be symbolized as" $S_{33}(1232)$ " and Omega can be symbolized as " $\omega(782)$ ".

The decay  $\omega \to \pi^0 + \gamma$  studied intensively experimentally by the CBELSA/TAPS collaboration in photoproduction reactions on nuclei [12][13][14][15]. Moreover,  $\omega$  mesons in cold nuclear matter have been investigated via the dilepton decay channel by E325 at KEK [16], CLAS at JLAB [17][18] and HADES at GSI [19]. While E325 claimed a  $\omega$  mass shift without any broadening, the CLAS data indicate a broadening of the  $\rho$  and  $\omega$ . The experimental study of  $\omega \to \pi^0 + \pi^0 + \gamma$  and  $\rho \to \pi^0 + \pi^0 + \gamma$  decays by SND Collaboration obtained the value B ( $\omega \to \pi^0 + \pi^0 + \gamma$ ) = (6.60.6)10<sup>-5</sup> for the branching ratio of the ( $\omega \to \pi^0 + \pi^0 + \gamma$ ) decay [20].

Their result is in good agreement with GAMS Collaboration measurement of B ( $\omega \rightarrow \pi^0 + \pi^0 + \gamma$ ) = (7.22.5)10<sup>-5</sup> [21], but it has a higher accuracy.

On the theoretical side,  $\omega \to \pi^0 + \pi^0 + \gamma$  decay was first studied by Singer [22] who postulated that this transition proceeds through the  $\omega \to \pi^0 + \rho \to \pi^0 + \pi^0 + \gamma$ mechanism involving  $\omega$  meson intermediate state. The contribution of intermediate vector mesons (VMD) to the vector meson decays into two pseudoscalars and a single photon  $V \rightarrow PP\gamma'$  was also considered by Bramon et al.[23] using standard Lagrangians obeying the SU (3)-symmetry, and in particular for the branching ratio of the decay  $\omega \to \pi^0 + \pi^0 + \gamma$  they obtained the result B (  $\omega \to \pi^0 + \pi^0 + \gamma$ ) =2.810<sup>-5</sup>. The  $V \to PP\gamma'$  decays have also been considered within the framework of chiral effective Lagrangians using chiral perturbation theory. Bramon et al. [24] studied various such decays using this approach and they noted that if chiral perturbation theory Lagrangians is used there is no tree-level contribution to the amplitudes for the decay processes  $V \to PP\gamma'$ , and moreover the one-loop contributions are finite and to this order no counter terms are required. Since  $\omega$  meson has the possibility to decay as follow: [21][22][25][26].

$$\omega \longrightarrow \pi^0 + \pi^0 + \gamma \tag{1}$$

It is interesting to study the effect of  $\omega$  meson decay on the structure functions of  $\pi^0$  electroproduction from the deuteron.

In this paper the effect of  $\omega$  on the unpolarized semi-exclusive structure functions of the incoherent  $\pi^0$ -meson electroproduction off the deuteron is studied at 0.01, 0.05 and 0.1  $GeV^2$  squared four momentum transfer,  $(K^2)$ , and different values of the incident virtual photon lab energy  $(k_0^{lab})$  The present paper is organized as follows; the formalism of  $\pi^0$  electroproduction off the deuteron in the IA is briefly given in section 2. The results are summarized and some discussions are presented in

section 3. At the end the conclusion and an outlook are presented.

## **2 FORMALISMS**

The basic formalism for electromagnetic single pion electroproduction on the deuteron has been presented in detail in previous work [1]. Therefore, we review here only the most important formulas for reaction kinematics, T-matrix and cross section.

#### 2.1 Kinematics

The following theorem characterizes the nature of the equilibrium point  $k_*$ . The kinematics of the neutral pion electroproduction in the one-photon exchange approximation is very similar to photoproduction in replacing the real photon by a virtual one with transverse [1].

$$\gamma^*(k) + d(p_d) \longrightarrow \quad n(p_1) + p(p_2) + \pi^0(q), \qquad (2)$$

Where  $K = (k_0, \mathbf{k})$ ,  $p_d = (E_d, \mathbf{d}), q = (\omega, \mathbf{q})$ , and  $p_1 = (E_1, \mathbf{p}_1), p_2 = (E_2; \mathbf{p}_2)$  denote the four-momenta of the the incoming virtual photon ,initial deuteron, the outgoing pion and the two outgoing nucleons, respectively.

The energies are given by:

$$E_d = \sqrt{M_d^2 + \mathbf{d}^2}, \quad E_1 = \sqrt{M^2 + \mathbf{p}_1^2},$$
 (3)

$$E_2 = \sqrt{M^2 + \mathbf{p}_2^2}$$
 and  $\omega = \sqrt{m_\pi^2 + \mathbf{q}^2}$ . (4)

As coordinate system we choose a right-handed



Fig. 1: Kinematics of single pion electroproduction on the deuteron.

orientation with z-axis along the photon momentum **q** and y-axis perpendicular to the scattering plane along  $\mathbf{k}_e \times \mathbf{k}_{e'}$ . We distinguish in general three planes: (i) the scattering plane spanned by the incoming and scattered

electron momenta,(ii) the pion plane, spanned by the photon and pion momenta, which intersects the scattering plane along the z-axis with an angle  $\phi_{\pi}$  and (iii) the nucleon plane spanned by the momenta of the two outgoing nucleons intersecting the pion plane along the total momentum of the two nucleons. This is illustrated in Figure.(1)

For the description of the two outgoing nucleons we use the relative  $\left(p_{r}\right)$  and the total  $\left(p_{t}\right)$  momenta as defined by

$$\mathbf{p}_t = \mathbf{p}_1 + \mathbf{p}_2 = -\mathbf{q}$$
$$\mathbf{p}_r = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) \tag{5}$$

Then one can express the nucleon three-momenta as

$$\mathbf{p}_1 = \frac{1}{2}\mathbf{p}_t + \mathbf{p}_r \tag{6}$$

$$\mathbf{p}_2 = \mathbf{1} 2 \mathbf{p}_t - \mathbf{p}_r \tag{7}$$

For their Energies of these particles are then fixed by

$$E_1^2 = \mathbf{p}_1^2 + M = \mathbf{p}_r^2 + \frac{\mathbf{q}^2}{4} - \mathbf{p}_r \cdot \mathbf{q} + M^2$$
(8)

And

$$E_2^2 = \mathbf{p}_2^2 + M = \mathbf{p}_r^2 + \frac{\mathbf{q}^2}{4} + \mathbf{p}_r \cdot \mathbf{q} + M^2$$
(9)

from subtract of Eq.(9) and Eq.(8) we get

$$E_1^2 - E_2^2 = E_{12}(E_1 - E_2) = 2\mathbf{p}_r \cdot \mathbf{p}_t \tag{10}$$

Then from Eq.(10) one has

$$E_1 - E_2 = \frac{2\mathbf{p}_r \cdot \mathbf{p}_t}{E_{12}} \tag{11}$$

and thus finally for the single-nucleon energies

$$E_1 = \frac{E_{12}}{2} + \frac{2\mathbf{p}_r \cdot \mathbf{p}_t}{E_{12}}$$
(12)

And

$$E_2 = \frac{E_{12}}{2} - \frac{2\mathbf{p}_r \cdot \mathbf{p}_t}{E_{12}}$$
(13)

In order to determine the absolute value of  $\mathbf{p}_r$  we evaluate using Eq.(5)

$$\mathbf{p}_{t}^{2} = E_{1}^{2} + E_{2}^{2} - 2M^{2} + 2\mathbf{p}_{1} \cdot \mathbf{p}_{2}$$
$$= \frac{E_{12}^{2}}{2} + 2\mathbf{p}_{r}^{2} (\frac{\mathbf{p}_{t}^{2}}{E_{12}^{2}} \cos\theta_{r} - 1) - 2M^{2} + \frac{\mathbf{p}_{t}^{2}}{2} \qquad (14)$$

from which one finds as final result using  $\mathbf{p}_t = -\mathbf{q}$ 

$$\mathbf{p}_{r}^{2} = \frac{1}{4} \frac{E_{12}^{2} (E_{12}^{2} - \mathbf{q}^{2} - 4M^{2})}{E_{12}^{2} - \mathbf{q}^{2} cos \theta_{r}}$$
(15)

For the semi-exclusive reaction, where besides the scattered electron only the produced pion is measured,

one can determine the quasi-free lab pion energy  $E_{\pi}^{qf,lab}$  and finds [1]

$$E_{\pi}^{qf,lab}(\theta_{\pi}^{lab}) = \frac{1}{2((E_{\pi}^{qf,lab})^2 - k_0^{lab}cos^2\theta_{\pi}^{lab})} \times C_{qf}^{lab} E_{\pi N}^{qf,lab} \pm k_0^{lab}cos\theta_{\pi}^{lab} \times \sqrt{(C_{qf}^{lab})^2 - 4m_{\pi}^2((E_{\pi}^{qf,lab})^2 - k_0^{lab}cos^2\theta_{\pi}^{lab})}$$
(16)

Where we have introduced

$$\begin{split} C^{lab}_{qf} &= (E^{lab}_{qf})^2 + m_\pi^2 - M^2 - k_0^{lab} \\ &= (M^{qf}_\pi)^2 - m_\pi^2 - M^{lab} \end{split}$$

With  $M_{\pi}^{qf}$  as invariant mass of the active quasi-free ?Nsystem. In 16 the "plus" sign should be taken for  $0 \le \theta_{\pi} \le \pi$  otherwise the "minus" -sign. The corresponding quasifree missing mass  $M_x^{qf}$  is given by.

$$M_x^{qf} = \sqrt{2M(M_d + k_0^{lab} - E_{\pi N}^{qf, lab})}$$

### 2.2 The T-matrix

As in photoproduction, all observables are determined by the T -matrix elements of the electromagnetic pion production current  $J_{\gamma\pi}$  between the initial deuteron and the final  $\pi NN$  states.

$$T_{sm_s,\mu m_d} = -^{(-)} \langle \mathbf{p}_1 \mathbf{p}_2 sm_s, \mathbf{p}_\pi | J_{\gamma \pi,\mu}(0) | \mathbf{p}_d 1m_d \rangle \quad (17)$$

where *s* and  $m_s$  denote the total spin and its projection on the relative momentum **p** of the outgoing two nucleons, and md correspondingly the deuteron spin projection on the z-axis as quantization axis. In the expression on the rhs of [4] non-covariant normalization for the initial deuteron and the final  $\pi NN$ -states is adopted. As already mentioned, all kinematic quantities related to the T -matrix refer to the  $\gamma^* - d$  c.m. system.

Introducing a partial wave decomposition of the final states, one finds

$$T_{sm_s,\mu m_d}(W, Q^2, p_\pi, \Omega_\pi, \Omega_p) = e^{i(\mu + m_d - m_s)\phi_\pi} \times t_{sm_s,\mu m_d}(W, Q^2, p_\pi, \theta_\pi, \theta_p, \phi_{p\pi})$$
(18)

where the small t-matrix depends besides W,  $Q^2$  and  $p_{\pi}$  only on  $\theta_{\pi}, \theta_p$ , and the relative azimuthal angle  $\phi_{p\pi} = \phi_p - \phi_{\pi}$ .

It had shown in [4] that, if parity is conserved, the following symmetry relation holds for  $\mu = \pm 1$ 

$$t_{s-m_s-\mu-m_d}(W,Q^2,p_{\pi},\theta_{\pi},\theta_p,\phi_{p\pi}) = (-)^{s+m_s+\mu+m_d}$$

$$\times t_{sm_s,\mu m_d}(W,Q^2,p_{\pi},\theta_{\pi},\theta_p,-\phi_{p\pi})$$
(19)

In the present work we include as e.m.current the elementary one-body pion production current of MAID-2007 [27] which has been developed for nuclear applications for photon energies up to 2 GeV. It contains Born terms, nucleon resonances  $P_{33}(1232)$ ,  $P_{11}(1440)$ ,  $D_{13}(1520)$ ,  $S_{11}(1535)$ ,  $S_{31}(1620)$ ,  $S_{11}(1650)$ ,  $D_{15}(1675)$ ,  $F_{15}(1680)$ ,  $D_{33}(1700)$ ,  $P_{13}(1720)$ ,  $F_{35}(1905)$ ,  $P_{31}(1910)$ ,



**Fig. 2:** Diagrammatic representation of the elementary pion electroproduction on the nucleon. (a) Kroll-Rudermant term, (b) Born direct term, (c) Born crossed term, (d) Pion Pole term, (e) Nucleon resonance exchange (direct), (f) Nucleon resonance exchange (crossed), (g) Vector meson t-channel term.

 $F_{37}(1950)$  and vector meson exchange, see Fig. (2).

As in [1] we split the T -matrix into the impulse approximation (IA)  $T^{IA}$ , where final state interaction effects are neglected, and the rescattering contribution  $T^{NN}$  and  $T^{\pi N}$  of the two-body *NN*- and  $\pi N$ -subsystems, respectively.

$$T_{sm_s\mu m_d} = T^{IA}_{sm_s\mu m_d} + T^{NN}_{sm_s\mu m_d} + T^{\pi N}_{sm_s\mu m_d}$$
(20)

For the IA contribution, where the final state is described by a plane wave, antisymmetrized with respect to the two outgoing nucleons, one has[?][2][3].

$$T_{sm_{s}\mu m_{d}}^{IA} = \langle \mathbf{p}sm_{s}, \mathbf{p}_{\pi} | [J_{\gamma\pi,\mu}(1) + J_{\gamma\pi,\mu}(2)] | 1m_{d} \rangle$$
  
=  $\sqrt{2} \sum_{m'_{s}} (\langle sm_{s} | \langle \mathbf{p}_{\pi} | J_{\gamma\pi,\mu}(W_{\gamma N_{1}}, Q^{2}) | \mathbf{p}_{d} - \mathbf{p}_{2} \rangle (21)$   
 $\times \phi_{m'_{s}m_{d}}(\frac{1}{2}\mathbf{p}_{d} - \mathbf{p}_{2}) | 1m'_{s} \rangle - (1 \longleftrightarrow 2))$ 

where  $J_{\gamma\pi,\mu}J_{\gamma\pi,\mu}$  denotes the elementary pion photoproduction operator of the MAID-2007 model,  $W_{\gamma N_1}$ the invariant energy of the  $\gamma N_1$  system,  $\mathbf{p}_{1/2} = (\mathbf{q} + \mathbf{p}_d - \mathbf{p}_{\pi})/2 \pm \mathbf{p}$ .

This expression reflects the well-known spectator model in which the pion production takes place on a single nucleon inside the deuteron while the other nucleon acts as a pure spectator (see Figure (3))

Furthermore,  $\phi_{m_sm_d}$  is related to the internal deuteron wave function in momentum space by.

$$\langle \mathbf{p}, 1m_s | 1m_s \rangle^{(d)} = \phi_{m_s m_d}(\mathbf{p}) = \sum_{L=0,2} \sum_{m_L} i^L (Lm_L 1m_s | 1m_d) \mu_L(p) Y_{Lm_L} \mathbf{p}^{(22)}$$

The two rescattering contributions have a similar structure.

$$T_{sm_s\mu m_d}^{NN} = \langle \mathbf{p} \, sm_s, \mathbf{p}_{\pi} | T_{NN} G_{NN} [ J_{\gamma\pi,\mu} (W_{\gamma N_1}, Q^2) + J_{\gamma\pi,\mu} (W_{\gamma N_2}, Q^2) ] | 1m_d \rangle, \qquad (23)$$



Fig. 3: Spectator model diagram for pion electroproduction off the deuteron.

$$T_{sm_s\mu m_d}^{\pi N} = \langle \mathbf{p} \, sm_s, \mathbf{p}_{\pi} | T_{\pi N} G_{\pi N} [ J_{\gamma \pi, \mu} (W_{\gamma N_1}, Q^2) + J_{\gamma \pi, \mu} (W_{\gamma N_2}, Q^2) ] | 1m_d \rangle, \qquad (24)$$

where  $T^{NN}$  and  $T^{\pi N}$  denote respectively the *NN*- and  $\pi N$  scattering matrices and  $G^{NN}$  and  $G^{\pi N}$  the corresponding free two-body propagators.

### 2.3 Cross Section and Structure Functions

The well-known spectator model Figure3 In which the pion production takes place on a single nucleon inside the deuteron while the other nucleon acts as a pure spectator is used to produce the matrix element of electroproduction off the deuteron.

The final expression for the semi-exclusive differential cross section is defined in Ref.[?], and the reader is referred to this work for full details of the next expressions.

$$\frac{d^{3}\sigma}{dE'_{e}d\Omega'_{e}d\Omega^{c,m}_{\pi}} = \frac{\alpha}{(2\pi)^{2}(K^{2})^{2}} \frac{p'_{e}}{p_{e}} (\bar{\rho}_{T}R_{T} + \frac{1}{2}\bar{\rho}_{L}R_{L} - \frac{1}{\sqrt{2}}\bar{\rho}_{LT}R_{LT}cos\phi^{c,m}_{\pi} - \bar{\rho}_{TT}R_{TT}cos2\phi^{c,m}_{\pi})$$
(25)

where the structure functions  $R_{\alpha}(\alpha = L, T, LT, TT)$  are given in detail by

$$R_L = W_{00}, \quad R_T = W_{11},$$
  
 $R_{LT} = -\sqrt{2}ReW_{10}, \quad R_{TT} = W_{1-1}$ 

These structure functions depend on the invariant mass W, the squared four-momentum transfer  $K^2$ , and on the pion angle  $\theta_{\pi}^{c.m}$ .

# **3 RESULTS AND DISCUSSION**

In this section we present and discuss our numerical results for the unpolarized semi-exclusive structure functions of the neutral pion electroproduction from the deuteron in the IA. As already mentioned, the realistic MAID-2007 model [27] has been used for the evaluation of the elementary pion electroproduction operator on the free nucleon. The electromagnetic production amplitude is parameterized in term of CGLN amplitudes given as numerical tables in the pion-nucleon c.m. frame. This

amplitude had to be generalized to an arbitrary frame of reference in order to be incorporated into the reaction on the deuteron. This was achieved by constructing from the MAID-2007 model Lorentz invariant amplitudes. This generalized elementary production operator was then used to evaluate pion electroproduction off the deuteron. The numerical evaluation is based on Gauss integration for the calculation of the matrix element of the MAID operator using for the deuteron wave function an analytical parameterization of the S- and D-waves of the Bonn potential in momentum space [28].

In Figures (4-8), the angular distribution for the four unpolarized semi-exclusive structure functions  $R_L,R_T$ ,  $R_{TT}$  and  $R_{LT}$  at different values for the squared four momentum transfer ( $K^2$ ) and the virtual photon lab energy ( $k_0^{lab}$ ) shown. The full lines indicate the situation when the  $\omega$  meson contribution is included in our calculations and the dashed ones are when this contribution is eliminated. It is clear that the four structure functions contribute to the reaction; $R_L$ ,  $R_T$  and  $R_{LT}$  are in the positive direction while  $R_{TT}$  is in the negative direction.

Firstly, the effect of  $\omega$  meson on the four unpolarized semi-exclusive structure functions of  $d(e, e'\pi^0)pn$  reaction at different values of the virtual photon lab energy  $(k_0^{lab})=250$ , 300 and 400 MeV and constant squared four momentum transferee  $(K^2 = 0.01 GeV^2)$  is studied to show its dependence on  $(k_0^{lab})$ .

Figure (4) shows angular distribution for the four



**Fig. 4:** Angular dependence of the four semi-exclusive structure functions of  $d(e, e'\pi^0)np$  at  $k_0^{lab} = 250$  MeV and squared four-momentum transfer  $K^2 = 0.01(GeV)^2$ , full lines where the  $\omega(782)$  is included and dashed lines where  $\omega(782)$  is eliminated.

unpolarized semi-exclusive structure functions of the neutral pion electroproduction from the deuteron without FSI at squared four momentum transferee  $(K^2 = 0.01 GeV^2)$  and virtual photon lab energy  $(k_0^{lab}) = 250$  MeV, the contribution of  $\omega$  results in a notcible increase in  $R_L$ ,  $R_T$  and  $R_{TT}$ , it reaches its maximum effect at  $\theta_{\pi} = 60^{\circ}$ 

For  $R_T$ ,  $R_{TT}$  and at the forward angles for  $R_L$  where its effect is quite small for  $R_{LT}$ . Increasing virtual photon lab energy to  $(k_0^{lab}) = 300$  MeV and keeping the four-momentum transfer  $(K^2 = 0.01 GeV^2)$  Figure (5), the absolute size of the four structure functions increase while the contribution of  $\omega$  is somewhat smaller than what was found in Figure (4). For  $R_L$ , the effect is still so small. In Figure (6) the virtual photon lab energy is



Fig. 5: Notation as in Figure (4) at  $k_0^{lab} = 300$  MeV.

increased to  $(k_0^{lab}) = 400$  MeV at the same value of the four-momentum transfer  $(K^2 = 0.01 GeV^2)$ , the absolute size of  $R_T$ ,  $R_{TT}$  and  $R_{LT}$  also increased and the contributions of  $\omega$  became smaller than what was found in Figures (4) and (5).

This means, increasing the virtual photon lab energy results in decreasing the effect of  $\omega$  meson in neutral pion electroproduction of the deuteron. One readily notices a large dependence off all structure functions on  $(k_0^{lab})$ , which increase with increasing  $(k_0^{lab})$ , especially in the peak region. This behavior has its origin in the  $\Delta$ -resonance contribution to the transverse current because with increasing  $(k_0^{lab})$  one approaches the  $\Delta$  resonance peak. As next, the effect of  $\omega$  meson on the four unpolarized semi-exclusive structure functions of reaction at different values for the squared four momentum



**Fig. 6:** Notation as in Figure (4) at  $k_0^{lab} = 400$  MeV.



**Fig. 7:** Angular dependence of the four semi-exclusive structure functions of  $d(e, e'\pi^0)np$  at  $k_0^{lab}$  =400 MeV and squared four-momentum transfer  $K^2 = 0.05(GeV)^2$ , full lines where the  $\omega(782)$  is included and dashed lines where  $\omega(782)$  is eliminated.

transfer ( $K^2$  and a constant virtual photon lab energy ( $k_0^{lab}$ ) is discussed, where ( $K^2$  takes the values 0.01, 0.05 and 0.1 GeV<sup>2</sup> and the virtual photon lab energy is kept constant (( $k_0^{lab}$ )= 400 MeV), Figures (6,7) and (8) One notices, increasing ( $K^2$  results in increasing the effect of  $\omega$  meson on the four unpolarized semi-exclusive structure



**Fig. 8:** Notation as in Figure (7) at  $K^2 = 0.1 (GeV)^2$ .

functions, especially for  $R_L$ ,  $R_T$  and  $R_{TT}$ . Furthermore, the structure functions show quite a strong  $K^2$ dependence, increasing  $K^2$  from 0.01 to  $0.1 GeV^2$  results in reduction of the magnitude. Re- sensible for this reduction is the strong fall-off of the absolute size of the elementary production amplitudes with increasing  $K^2$ .

## **4 CONCLUSION AND OUTLOOK**

A systematic study for the contribution of  $\omega$  meson on the unpolarized semi-exclusive structure functions of the neutral pion electroproduction off the deuteron is done.

This study is made in the IA without adding NN final state interactions. Since the structure functions depend on the squared four momentum transfer  $K^2$ , the invariant energy or equivalently the virtual photon laboratory energy and the outgoing pion angle in the final hadronic c.m. system, three values for  $K^2$  and  $(k_0^{lab})$  have been selected for the presentation of the results. The results show a big effect of  $\omega$  meson on  $R_L$ ,  $R_T$  and  $R_{TT}$  while a quit small effect is seen for  $R_{LT}$ . This effect decreases with the increase of  $(k_0^{lab})$  and increase by increasing  $K^2$ . In future work it will be interesting to study the effect of  $\omega$  meson on the charged pion electroproduction from the deuteron.

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