Journal of Statistics Applications & Probability Letters
An International Journal

http://dx.doi.org/10.18576/jsapl/050303

On Progressive-Stress Accelerated Life Testing for Power Generalized Weibull Distribution under Progressive Type-II Censoring

M. M. Mohie El-Din[1], *A. M. Abd El-Raheem*[2,*] *and S. O. Abd El-Azeem*[3]

[1] Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt
[2] Department of Mathematics, Faculty of Education, Ain Shams University, Cairo, Egypt
[3] Department of Basic Sciences, Faculty of Engineering, MTI University, Cairo, Egypt

Received: 20 Apr. 2018, Revised: 2 Jun. 2018, Accepted: 15 Jun. 2018
Published online: 1 Sep. 2018

Abstract: In this paper, progressive-stress accelerated life testing (ALT) is studied when the lifetime of test units follows power generalized Weibull distribution (PGW). The maximum likelihood estimates (MLEs) and Bayes estimates (BEs) of the model parameters are obtained under type-II progressive censoring. Moreover, the approximate and credible confidence intervals (CIs) of the estimators are derived. Furthermore, a real dataset is analyzed to show the suggested methods. Also, simulation studies are conducted to demonstrate the precision of the MLEs and BEs for the parameters of PGW distribution. Finally, some interesting conclusions are obtained.

Keywords: progressive-stress model; progressive type-II censoring; power generalized Weibull distribution; maximum likelihood estimation; Bayes estimation; interval estimations; simulation study.

1 Introduction

Experiments of reliability and life testing are done to investigate data of failure time which occurs under the normal operating conditions. Due to the hardness of collecting such data which needs too long time, we have tended to use ALT in order to obtain adequate failure data in a compact time. In ALT, experiments are done at greater than normal levels of stress to expedite failure occurring. Then, the collected life data is investigated and used to estimate the life characteristics under normal operating conditions. The stress in ALT can be applied in different ways, the most commonly used methods are constant-stress, step-stress and progressive-stress. Nelson [1] discussed the advantages and disadvantages of each of such methods.

The constant-stress ALT is practiced by operating every unit at a constant high stress till either failure occurs or the test is stopped. Constant-stress models were discussed by various authors; see Kim and Bai [2], Watkins and John [3]. Abdel-Hamid [4] reviewed the constant-partially accelerated life tests for Burr type-XII distribution with type-II progressive censoring. Guan et al. [5] discussed the optimal constant-stress accelerated life tests with uncensored sampling for the generalized exponential distribution. Jaheen et al. [6] tested the constant partially ALT under progressive type-II censoring for generalized exponential distribution. Mohie El-Din et al. [7] studied the constant-stress accelerated life tests for extension of the exponential distribution under progressive censoring. Mohie El-Din et al. [8] discussed the optimal plans of constant-stress accelerated life tests for the Lindley distribution. Mohie El-Din et al. [9] introduced the geometric process as a constant-stress accelerated model. Abd El-Raheem [10] derived the optimal designs of constant-stress ALTs for the extension of the exponential distribution. Abd El-Raheem [11] expanded his results in Abd El-Raheem [10] to the censored data.

The second method is the step-stress. In step-stress ALT, the applied stress on every unit is not fixed but is increasing step by step at prespecified times or simultaneous the occurrence of a fixed number of failures. The step-stress models

* Corresponding author e-mail: a_m2am@yahoo.com

were discussed in the literature; see Miller and Nelson [12] and Gouno et al. [13]. Balakrishnan et al. [14] studied the simple step-stress ALT under type-II censoring by using a cumulative exposure model for exponential distribution. Mohie El-Din et al. [15] obtained BE for step-stress ALT of PGW distribution under progressive censoring by using a tampered random variable model. Mohie El-Din et al. [16] discussed the simple step-stress ALT under progressive first-failure censoring by using a tampered random variable model for Weibull distribution. Mohie El-Din et al. [17] discussed the parametric inference on step-stress accelerated life testing for the extension of exponential distribution under progressive type-II censoring.

The third method is the progressive-stress. In progressive-stress ALT, the applied stress on test units is continuously increasing in time. If an ALT has a continuous linearly increasing stress, this test is called a ramp-stress test. Yin and Sheng [18] obtained the MLEs of parameters of the exponential progressive-stress model. Abdel-Hamid and AL-Hussaini [19] carried out the progressive-stress ALT under progressive censoring for Weibull distribution. Abdel-Hamid and Abushul [20] obtained the BE of exponentiated exponential distribution under type-II progressive hybrid censoring by using the inverse power law and the cumulative exposure model. Mohie El-Din et al. [21] studied progressive-stress ALT for the extension of the exponential distribution.

In life testing, tests are often terminated before all units fail. As a result, the censored data is used to reduce test time and cost. The most two common censoring schemes (CSs) in life testing and reliability experiments are type-I and type-II censoring. Progressive type-II CS has became more common in analyzing highly reliable data. This type of CS can be defined as follows: Assume n identical items are set on a life test, the integer $m<n$ is a pre-specified number of failures and $R_1,R_2,\ldots,R_m$ are m pre-fixed integers satisfying $R_1+R_2+\ldots+R_m+m=n$. At the time of the first failure $t_{1:m:n}, R_1$ of the remaining units is randomly removed. Also, at the time of the second failure $t_{2:m:n}, R_2$ of the remaining units is randomly removed and so on. At the time of the $m-th$ failure $t_{m:m:n}$, the test is terminated and all remaining $R_m=n-m-(R_1+\ldots+R_{m-1})$ units are removed. For further information about progressive type-II censoring, see Balakrishnan and Aggarwala [22].

The purpose of this study is to apply the progressive-stress ALT to units whose lifetime follows PGW distribution under type-II progressive censoring. MLEs, BEs and some inferences for the parameters of the supposed model are studied.

The paper is organized as follows: In Section 2, a description of the lifetime model and the test assumptions. In Section 3, the MLEs of the model parameters are obtained. In Section 4, the BEs of model parameters are obtained. In Section 5, interval estimations for the model parameters are established. In Section 6, a real dataset is analyzed to illustrate the suggested methods in Sections 3, 4 and 5. In Section 7, the simulation outcomes are represented. In Section 8, the conclusion is introduced.

2 Model description and test assumptions

2.1 Power generalized Weibull distribution

In this subsection, PGW distribution is an extension of Weibull distribution. It was founded by Bagdonavicius and Nikulin [23] as a baseline distribution for the accelerated failure time model. It contains distributions with unimodal and bathtub hazard shape. Moreover, PGW distribution provides a broader class of monotone hazard rate. Besides, it is a right skewed heavy tailed distribution which is not very common in lifetime model. The PGW distribution can be a possible alternative to the exponentiated Weibull distribution for modeling lifetime data, see Nikulin and Haghighi [24]. Many authors considered the PGW distribution as lifetime model, see for example, Nikulin and Haghighi [25], Nikulin and Haghighi [26], Bagdonavicius and Nikulin [27], Voinov et al. [28], Mohie El-Din et al. [15] and Kumar and Dey [29].

The PGW distribution is defined by the probability density function (PDF):

$$f(t)=\gamma\nu\sigma^{\nu}t^{\nu-1}(1+(\sigma t)^{\nu})^{\gamma-1}\exp\{1-(1+(\sigma t)^{\nu})^{\gamma}\}, \qquad t,\gamma,\nu,\sigma>0, \tag{1}$$

the corresponding cumulative distribution function (CDF) is

$$F(t)=1-\exp\{1-(1+(\sigma t)^{\nu})^{\gamma}\}, \qquad t,\gamma,\nu,\sigma>0, \tag{2}$$

and the corresponding hazard rate function (hrf) is given by

$$h(t)=\gamma\nu\sigma^{\nu}t^{\nu-1}(1+(\sigma t)^{\nu})^{\gamma-1}. \tag{3}$$

There exist three special cases of the PGW distribution which are:

1-Weibull distribution when $\gamma = 1$.
2-Extension of the exponential distribution when $\nu = 1$, see Nadarajah and Haghighi [30].
3-Exponential distribution when $\gamma = 1$ and $\nu = 1$.

2.2 Assumptions and test procedures

In this subsection, the progressive-stress ALT under a progressive type-II is constructed by the following assumptions:

1-The lifetime of the test units follows the PGW with shape parameter γ and scale parameter σ.
2-The progressive-stress $S(t)$ is a function of time and directly proportional to time with a constant rate β, i.e, $S(t) = \beta t$, $\beta > 0$.
3-The inverse power law is satisfied, i.e,

$$\sigma(t) = \frac{1}{a(s(t))^b}, \qquad a,b > 0, \tag{4}$$

4-The linear cumulative exposure model is applied to show the effect of changing the stress from one level to another level, see Nelson [1].
5-Every group contains n_i unit which is under the progressive-stress $S_i(t) = \beta_i t$, $0 < \beta_1 < ... < \beta_k$ for $i = 1,...,k$.
6-All units have the same failure mechanism at any stress rate β_i, $i = 1,...,k$.

The type-II progressive censoring is applied where n is the total number of units under the test. $S_0 < S_1(t) < ... < S_k(t)$ are the stress levels in the test and S_0 is the use-stress. n_i identical units are tested under each progressive-stress level $S_i(t)$ where $i = 1,2,...,k$. When the first failure occurs $t_{i1:m_i:n_i}$, R_{i1} units are randomly removed from the remaining $n_i - 1$ units. When the second failure occurs $t_{i2:m_i:n_i}$, R_{i2} units are randomly removed from the remaining $n_i - 2 - R_{i1}$ units. When the m_i-th failure occurs $t_{im_i:m_i:n_i}$, the test is finished and whole remaining $R_{im_i} = n_i - m_i - \sum_{j=1}^{m_i-1} R_{ij}$units are removed.

It is shown that the progressively censored data under the progressive-stress $S_i(t)$ are $t_{i1:m_i:n_i} < t_{i2:m_i:n_i} < ... < t_{im_i:m_i:n_i}$, $i = 1,2,...,k$.

Out of the CDF of PGW distribution in (2) and the assumption of the linear cumulative exposure model, the CDF of test units under progressive-stress $S_i(t)$ is defined by

$$Q_i(t) = F_i(\Delta(t)), i = 1,...,k, \tag{5}$$

where $\Delta(t) = \int_0^t \frac{du}{\sigma_i(u)} = \frac{a\beta_i^b t^{b+1}}{b+1}$, and $F_i(.)$ is the CDF under the stress level $S_i(t)$ with a scale parameter equal to one. Hence,

$$Q_i(t) = 1 - \exp\left\{1 - \left(1 + \left(\frac{a\beta_i^b t^{b+1}}{b+1}\right)^\nu\right)^\gamma\right\}, \qquad t > 0. \tag{6}$$

The PDF of (6) is given by

$$q_i(t) = a\gamma\nu\beta_i^b t^b \left(\frac{a\beta_i^b t^{b+1}}{b+1}\right)^{\nu-1} \left(1 + \left(\frac{a\beta_i^b t^{b+1}}{b+1}\right)^\nu\right)^{\gamma-1} \exp\left\{1 - \left(1 + \left(\frac{a\beta_i^b t^{b+1}}{b+1}\right)^\nu\right)^\gamma\right\}, \qquad t > 0. \tag{7}$$

3 Estimation by maximum likelihood method

In this section, MLEs of the model parameters γ, a, ν and b are obtained under progressive type-II censoring. Assuming that $t_{ij:m_i:n_i} = t_{ij}$, where $i = 1,2,\cdots,k$ and $j = 1,2,\cdots,m_i$, is the observed data of the lifetime under the progressive-stress level $S_i(t)$. The likelihood function of the four parameters γ, a, ν and b is given by

$$L(\gamma,a,\nu,b) = \prod_{i=1}^{k} C_i \prod_{j=1}^{m_i} q_i(t_{ij}) \left[1 - Q_i(t_{ij})\right]^{R_{ij}}, \tag{8}$$

where $C_i = n_i\,(n_i-1-R_{i1})\,(n_i-2-R_{i1}-R_{i2})\cdots\left(n_i-m_i+1-\sum_{j=1}^{m_i-1}R_{ij}\right)$.

From (6) and (7) in (8), we obtain the following equation

$$L(\gamma,a,\nu,b)=\prod_{i=1}^{k}C_i\prod_{j=1}^{m_i}a\gamma\nu\beta_i^{b}t_{ij}^{b}(\upsilon(t_{ij}))^{\nu-1}(1+(\upsilon(t_{ij}))^{\nu})^{\gamma-1}\exp\left\{(R_{ij}+1)\,(1-(1+(\upsilon(t_{ij}))^{\nu})^{\gamma})\right\}, \tag{9}$$

where $\upsilon(t_{ij})=\left(\frac{a\beta_i^{b}t_{ij}^{b+1}}{b+1}\right)$.

So, the log-likelihood function is written as

$$\begin{aligned}\ell(\gamma,a,\nu,b)=&\sum_{i=1}^{k}\log C_i+(\log\gamma+\log\nu+\log a)\sum_{i=1}^{k}m_i+b\sum_{i=1}^{k}m_i\log\beta_i\\&+b\sum_{i=1}^{k}\sum_{j=1}^{m_i}\log t_{ij}+(\nu-1)\sum_{i=1}^{k}\sum_{j=1}^{m_i}\log(\upsilon(t_{ij}))+(\gamma-1)\sum_{i=1}^{k}\sum_{j=1}^{m_i}\log\left(1+(\upsilon(t_{ij}))^{\nu}\right)\\&+\sum_{i=1}^{k}\sum_{j=1}^{m_i}(R_{ij}+1)\,(1-(1+(\upsilon(t_{ij}))^{\nu})^{\gamma}),\end{aligned} \tag{10}$$

the likelihood equations of γ, a, ν and b are respectively

$$\begin{aligned}\frac{\partial\ell}{\partial\gamma}=&\frac{\sum_{i=1}^{k}m_i}{\gamma}+\sum_{i=1}^{k}\sum_{j=1}^{m_i}\log\left(1+(\upsilon(t_{ij}))^{\nu}\right)\\&-\sum_{i=1}^{k}\sum_{j=1}^{m_i}(R_{ij}+1)\,(1+(\upsilon(t_{ij}))^{\nu})^{\gamma}\log(1+(\upsilon(t_{ij}))^{\nu}),\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{\partial\ell}{\partial a}=&\frac{\nu\sum_{i=1}^{k}m_i}{a}+\frac{(\gamma-1)\nu}{b+1}\sum_{i=1}^{k}\sum_{j=1}^{m_i}\frac{\beta_i^{b}t_{ij}^{b+1}(\upsilon(t_{ij}))^{\nu-1}}{(1+(\upsilon(t_{ij}))^{\nu})}\\&-\frac{\gamma\nu}{b+1}\sum_{i=1}^{k}\sum_{j=1}^{m_i}(R_{ij}+1)(\beta_i^{b}t_{ij}^{b+1})\,(\upsilon(t_{ij}))^{\nu-1}\,(1+(\upsilon(t_{ij}))^{\nu})^{\gamma-1},\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{\partial\ell}{\partial\nu}=&\frac{\sum_{i=1}^{k}m_i}{\nu}+\sum_{i=1}^{k}\sum_{j=1}^{m_i}\log(\upsilon(t_{ij}))+(\gamma-1)\sum_{i=1}^{k}\sum_{j=1}^{m_i}\frac{(\upsilon(t_{ij}))^{\nu}\log(\upsilon(t_{ij}))}{(1+(\upsilon(t_{ij}))^{\nu})}\\&-\gamma\sum_{i=1}^{k}\sum_{j=1}^{m_i}(R_{ij}+1)\,(1+(\upsilon(t_{ij}))^{\nu})^{\gamma-1}\,(\upsilon(t_{ij}))^{\nu}\log(\upsilon(t_{ij})),\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial\ell}{\partial b}=&\sum_{i=1}^{k}m_i\log\beta_i+\sum_{i=1}^{k}\sum_{j=1}^{m_i}\log t_{ij}+\frac{\nu-1}{b+1}\sum_{i=1}^{k}\sum_{j=1}^{m_i}(-1+(b+1)(\log t_{ij}+\log\beta_i))\\&+\frac{(\gamma-1)\nu}{b+1}\sum_{i=1}^{k}\sum_{j=1}^{m_i}\frac{(-1+(b+1)(\log t_{ij}+\log\beta_i))\,(\upsilon(t_{ij}))^{\nu}}{(1+(\upsilon(t_{ij}))^{\nu})}\\&-\frac{\gamma\nu}{b+1}\sum_{i=1}^{k}\sum_{j=1}^{m_i}(R_{ij}+1)\,(-1+(b+1)(\log t_{ij}+\log\beta_i))\,(1+(\upsilon(t_{ij}))^{\nu})^{\gamma-1}\,(\upsilon(t_{ij}))^{\nu}.\end{aligned} \tag{14}$$

Hence, four nonlinear equations in four unknowns γ, a, ν and b are obtained. It is very difficult to obtain a closed form solution for these equations. Thus, an iterative method such as Newton-Raphson can be used to obtain numerical solutions for the four nonlinear equations in (11), (12), (13) and (14).

4 Bayes estimation

In this section, Bayesian inference of the model parameters γ, a, ν and b under progressive type-II censoring are calculated by using the square error (SE) loss function and linear exponential loss function (LINEX). Assuming that the model parameters γ, ν, a and b are independent. The two parameters γ and ν have gamma prior. While, the two parameters a and b are noninformative prior.

$$\pi_1(\gamma) \propto \gamma^{\mu_1-1} e^{\frac{-\gamma}{\lambda_1}}, \qquad \gamma > 0,\ \mu_1,\ \lambda_1 > 0, \tag{15}$$

$$\pi_2(a) \propto \frac{1}{a}, \qquad a > 0, \tag{16}$$

$$\pi_3(\nu) \propto \nu^{\mu_2-1} e^{\frac{-\nu}{\lambda_2}}, \qquad \nu > 0,\ \mu_2,\ \lambda_2 > 0, \tag{17}$$

$$\pi_4(b) \propto \frac{1}{b}, \qquad b > 0. \tag{18}$$

From (15), (16), (17) and (18), the joint prior of the parameters γ, a, ν and b is given by:

$$\pi(\gamma,a,\nu,b) \propto \frac{\gamma^{\mu_1-1}\nu^{\mu_2-1}}{ab} e^{-\left(\frac{\gamma}{\lambda_1}+\frac{\nu}{\lambda_2}\right)}, \qquad \gamma,a,\nu,b > 0. \tag{19}$$

The joint posterior density function of the parameters γ, a, ν and b can be written from (9) and (19) as follows:

$$\begin{aligned}\pi^*(\gamma,a,\nu,b) &\propto L(\gamma,a,\nu,b)\ \pi(\gamma,a,\nu,b)\\ &\propto \frac{\gamma^{(\mu_1-1)+\sum_{i=1}^k m_i}\nu^{(\mu_2-1)+\sum_{i=1}^k m_i} a^{-1+\sum_{i=1}^k m_i} e^{-(\frac{\gamma}{\lambda_1}+\frac{\nu}{\lambda_2})}}{b}\times\\ &\prod_{i=1}^{k}\prod_{j=1}^{m_i}\beta_i^b t_{ij}^b(\upsilon(t_{ij}))^{\nu-1}(1+(\upsilon(t_{ij}))^{\nu})^{\gamma-1}\exp\left\{(R_{ij}+1)\left(1-(1+(\upsilon(t_{ij}))^{\nu})^{\gamma}\right)\right\}.\end{aligned} \tag{20}$$

By using the SE and LINEX loss functions, the Bayes estimator of the function of parameters $U(\vartheta) = U(\gamma,a,\nu,b)$ can be obtained, respectively, as follows :

$$\widetilde{U}_{SE} = \int_{\vartheta} U\pi^*(\vartheta)d\vartheta, \tag{21}$$

and

$$\widetilde{U}_{LINEX} = -\frac{1}{c}\log\left[\int_{\vartheta} e^{-cU}\pi^*(\vartheta)d\vartheta\right], \tag{22}$$

note that $c \neq 0$ represents the shape parameter of LINEX loss function. The integrations in equations (21) and (22) cannot be calculated analytically. So, these integrals can be approximated by using Markov chain Monte Carlo (MCMC) method.

4.1 *Bayesian inference by MCMC approach*

In this subsection, MCMC approach is used to obtain samples from the posterior distribution and then the BEs of γ, a, ν and b are computed.

Out of the joint posterior density function in (20), the conditional posterior distributions of γ, a, ν and b are given respectively by:

$$\begin{aligned}P_1(\gamma|a,\nu,b) \propto \gamma^{(\mu_1-1)+\sum_{i=1}^k m_i} e^{\frac{-\gamma}{\lambda_1}} \prod_{i=1}^{k}\prod_{j=1}^{m_i}(1+(\upsilon(t_{ij}))^{\nu})^{\gamma}\times\\ \exp\left\{-(R_{ij}+1)\,(1+(\upsilon(t_{ij}))^{\nu})^{\gamma}\right\},\end{aligned} \tag{23}$$

$$P_2(a|\gamma,\nu,b) \propto a^{-1+\sum_{i=1}^{k} m_i} \times \prod_{i=1}^{k}\prod_{j=1}^{m_i}(\upsilon(t_{ij}))^{\nu-1}(1+(\upsilon(t_{ij}))^{\nu})^{\gamma-1}\exp\left\{-(R_{ij}+1)\left(1+(\upsilon(t_{ij}))^{\nu}\right)^{\gamma}\right\}, \tag{24}$$

$$P_3(\nu|\gamma,a,b) \propto \nu^{(\mu_2-1)+\sum_{i=1}^{k} m_i} e^{\frac{-\nu}{\lambda_2}} \times \prod_{i=1}^{k}\prod_{j=1}^{m_i}(\upsilon(t_{ij}))^{\nu}(1+(\upsilon(t_{ij}))^{\nu})^{\gamma-1}\exp\left\{-(R_{ij}+1)\left(1+(\upsilon(t_{ij}))^{\nu}\right)^{\gamma}\right\}, \tag{25}$$

$$P_4(b|\gamma,a,\nu) \propto \frac{1}{b}\prod_{i=1}^{k}\prod_{j=1}^{m_i}\beta_i^b t_{ij}^b(\upsilon(t_{ij}))^{\nu-1}(1+(\upsilon(t_{ij}))^{\nu})^{\gamma-1} \exp\left\{-(R_{ij}+1)\left(1+(\upsilon(t_{ij}))^{\nu}\right)^{\gamma}\right\}. \tag{26}$$

It is clear that the conditional posterior distributions of γ, a, ν and b are very difficult to reduce analytically to known distributions. So, Metropolis-Hasting algorithm is considered to generate random samples from these distributions; see Upadhyay and Gupta [31].

The BEs of $U = U(\gamma,a,\nu,b)$ under SE and LINEX loss functions are calculated by the following algorithm :

Algorithm(1)

1.Start with initial guess point of (γ,a,ν,b) say $(\gamma^{(0)},a^{(0)},\nu^{(0)},b^{(0)})$.
2.Set $i = 1$.
3.Generate $\gamma^{(i)}$, $a^{(i)}$, $\nu^{(i)}$ and $b^{(i)}$ from equations (23), (24), (25) and (26) respectively.
4.Set $i = i+1$.
5.Repeat steps ((2)-(4)) N times.
6.The approximate means of U and e^{-cU} are given respectively by

$$E(U) = \frac{1}{N-M}\sum_{i=M+1}^{N} U(\gamma^{(i)},a^{(i)},\nu^{(i)},b^{(i)}), \tag{27}$$

$$E(e^{-cU}) = \frac{1}{N-M}\sum_{i=M+1}^{N} \exp\{-cU(\gamma^{(i)},a^{(i)},\nu^{(i)},b^{(i)})\}, \tag{28}$$

where M is the burn-in period.

5 Confidence intervals

In this section, the approximate and credible CIs of the parameters $\gamma,\ a,\ \nu$ and b are obtained.

5.1 Normal approximation CI

In this subsection, the asymptotic distributions of the MLEs of the unknown parameters is considered to obtain the approximate CIs of the four parameters $\vartheta = (\gamma,a,\nu,b)$. This asymptotic distribution of the MLEs of ϑ was introduced by Miller [32].

$$\left((\hat{\gamma}-\gamma),(\hat{a}-a),(\hat{\nu}-\nu),(\hat{b}-b)\right) \sim \mathbf{N}\left(0,\sum\right),$$

where $\sum = \sigma_{ij},\ i,j = 1,2,3,4$, represents the variance-covariance matrix of the unknown parameters $\vartheta = (\gamma,a,\nu,b)$. The approximate $100\ (1-\omega)\%$ two sided CI of ϑ is given by:

$$\left(\hat{\vartheta}_{iL},\ \hat{\vartheta}_{iU}\right) = \hat{\vartheta}_i \pm Z_{1-\omega/2}\sqrt{\sigma_{ii}}, \qquad i = 1,2,3,4, \tag{29}$$

where $\hat{\vartheta}_1 \equiv \hat{\gamma}$, $\hat{\vartheta}_2 \equiv \hat{a}$, $\hat{\vartheta}_3 \equiv \hat{\nu}$, $\hat{\vartheta}_4 \equiv \hat{b}$ and Z_q is the $100q-th$ percentile of a standard normal distribution.

5.2 Credible CI

A $100(1-\omega)\%$ posterior interval for a random quantity ϑ is the interval which has the posterior probability $(1-\omega)$, that ϑ exists in the interval where

$$p(L \leq \vartheta \leq U) = \int_L^U \pi^*(\vartheta|\mathbf{t})d\vartheta = 1-\omega.$$

The credible CI of γ, a, ν and b is obtained by the following algorithm :

Algorithm (2)

1. Do steps $((1)-(6))$ in algorithm (1).
2. Sort the posterior sample $\left\{\vartheta^{(i)}, i = M+1,...,N\right\}$ to obtain the ordered values as $\left\{\vartheta^{[1]}, \vartheta^{[2]},...,\vartheta^{[N-M]}\right\}$ where M is the burn-in period. Then, the the 100 $(1-\omega)\%$ credible CI of ϑ is given by

$\left(\hat{\vartheta}_i, \hat{\vartheta}_u\right) = \left(\vartheta^{[\omega(N-M)/2]}, \vartheta^{[(1-\omega/2)(N-M)]}\right)$, where ϑ is γ, a, ν or b.

6 Application

In this section, the proposed producers in Section 3, 4 and 5 are demonstrated with a real data example.

Data in Table (5.1) of Chapter 5 of Zhu [33] were collected from ramp-voltage tests of miniature light bulbs with voltage equal 2 V. In this test, 62 miniature light bulbs were tested by ramp-rate 2.01 V/h. Also, 61 miniature light bulbs were tested by ramp-rate 2.015 V/h. The lifetime data resulted from the test are represented in Table (1).

To check the fitting of PGW distribution with the data in Table (1) for every ramp-stress $S_i(t), i = 1,2$. We compute Kolmogorov-Smirnov (K-S) distance between the empirical distribution function and the fitted distribution function when the parameters are obtained by MLE. The values of K-S distance and the corresponding P-values for each stress level are presented in Table (6.2). Since all resulted P-values are greater than 0.05, the PGW distribution provides a good fit to to the given data.

Table (6.3) represents the MLEs and BEs of the four parameters γ, a, ν and b. Basyian analysis is constructed in case of NIPs (when $\mu_1, \mu_2 \to 0$ and $\lambda_1, \lambda_2 \to \infty$).

Table (6.4) represents the lengths of 99% and 95% approximate and credible CIs of the model parameters.

It is clear that the BEs of the model parameters γ, a, ν and b give more accurate results than the MLEs through the length of the CIs.

Table 1 *The lifetime data from ramp-voltage tests*

Ramp-Rate 2.01 V/h						Ramp-Rate 2.015 V/h					
No.	Failure Time	No.	Failure Time	No.	Failure Time	No.	Failure Time	No.	Failure Time	No.	Failure Time
1	13.57	22	72.33	43	42.06	1	19.3	22	49.65	43	31.00
2	19.92	23	72.60	44	47.88	2	23.28	23	51.42	44	34.81
3	23.3	24	75.43	45	54.21	3	23.50	24	51.27	45	36.03
4	27.81	25	75.85	46	54.55	4	26.50	25	53.25	46	43.08
5	31.16	26	76.20	47	55.85	5	27.42	26	54.25	47	45.63
6	31.56	27	77.78	48	56.43	6	28.32	27	55.47	48	46.03
7	34.00	28	79.13	49	58.86	7	28.62	28	56.83	49	46.33
8	46.26	29	80.65	50	60.60	8	30.62	29	56.17	50	49.62
9	46.41	30	82.65	51	62.48	9	34.42	30	8.85	51	49.86
10	50.60	31	90.33	52	62.81	10	35.30	31	11.31	52	50.66
11	56.76	32	14.51	53	63.41	11	35.48	32	11.83	53	50.93
12	56.85	33	15.61	54	63.76	12	38.30	33	14.50	54	51.03
13	60.13	34	15.85	55	64.18	13	40.52	34	14.83	55	51.73
14	65.00	35	17.73	56	66.15	14	43.83	35	17.73	56	51.95
15	65.86	36	19.65	57	66.41	15	43.00	36	19.35	57	52.36
16	66.20	37	21.05	58	69.91	16	43.00	37	25.50	58	54.78
17	66.40	38	21.20	59	71.73	17	43.12	38	26.15	59	55.58
18	66.80	39	24.21	60	72.46	18	44.43	39	27.45	60	55.83
19	66.93	40	24.85	61	73.78	19	45.32	40	27.61	61	57.13
20	68.25	41	31.18	62	78.91	20	47.58	41	28.05		
21	70.23	42	35.08			21	47.65	42	30.96		

Table 2 *K-S distances and the corresponding P-values of ramp-voltage tests*

Ramp-Rate	2.01 V/h	2.015 V/h
K-S distance	0.2419	0.1459
P-value	0.0527	0.1492

Table 3 *MLEs and BEs under SE (BSE) and LINEX (BLINEX) loss functions of γ, a, ν and b for ramp-voltage tests*

Θ	MLE	BSE	BLINEX		
			$c=-2$	$c=0.001$	$c=2$
$\hat{\gamma}$	3.2595	3.4894	3.3112	3.4894	3.4891
$\hat{a}$	0.2756	0.6245	0.7300	0.6245	0.5993
$\hat{\nu}$	0.9587	1.0198	1.0153	1.0198	1.0056
$\hat{b}$	1.3136	1.1258	1.2754	1.1258	1.1043

Table 4 *Lengths of 99% and 95% approximate and credible CIs for γ, a, ν and b for ramp-voltage tests*

Θ	99% CI		95% CI	
	Approximate CI	Credible CI	Approximate CI	Credible CI
$\hat{\gamma}$	3.2591	1.0712	3.0189	1.0703
$\hat{a}$	0.9648	0.3043	0.9564	0.1825
$\hat{\nu}$	0.9688	0.4932	0.9168	0.3102
$\hat{b}$	1.3137	1.0087	1.2317	1.0049

7 Simulation studies

In this section, Monte Carlo simulation studies are performed to illustrate the performance of the MLEs and BEs. MLEs and BEs are considered under SE and LINEX loss functions for different sample sizes $(n_i, m_i, i = 1,2,...,k)$ and CSs $(R_{ij}, j = 1,2,...,m_i)$. Also, the 95% asymptotic and credible CIs are obtained. The progressive censoring schemes which are used in the simulation studies are shown in Table (7.1). MSEs of MLEs and BEs in the case of informative priors of the model parameters are shown in Table (7.2). Table (7.3) introduces lengths and coverage probabilities of 95% (approximate and credible CIs) in the case of informative priors CIs.

The estimation procedure is performed according to the following algorithm.

Algorithm (3)

1. Determine the values of n_i, m_i, k, a, b, c and β_i, $i = 1,2,...,k$.
2. Generate γ from $\pi_1(\gamma)$ and ν from $\pi_2(\nu)$ by using the determined values of the prior parameters μ_1, μ_2, λ_1 and λ_2.
3. Generate k simple random samples of size m_i from Uniform$(0,1)$ distribution, $(U_{i1}, U_{i2},...,U_{im_i})$, $i = 1,2,...,k$.
4. Determine the values of the censored schemes, R_{ij}, $i = 1,2,...,k$, and $j = 1,2,...,m_i$, such that $\sum_{j=1}^{m_i} R_{ij} = n_i - m_i$.
5. Set $E_{ij} = U_{ij}^{1/(j+\sum_{d=m_i-j+1}^{m_i} R_{id})}$, $j = 1,2,...,m_i$ and $i = 1,2,...,k$.
6. Obtain the progressive type-II censored samples $(U^*_{i1}, U^*_{i2},...,U^*_{im_i})$, where $U^*_{ij} = 1 - \prod_{d=m_i-j+1}^{m_i} E_{id}$, $j = 1,2,...,m_i$, $i = 1,2,...,k$.
7. Use step 6, to generate random samples $(t_{i1}, t_{i2},...,t_{im_i})$, $i = 1,2,...,k$, from CDF in equation (6) as follows:

$$t_{ij} = \left(\left(\frac{b+1}{a\beta_i^b} \right)^{\nu} \left[\left(1 - \log(1 - U^*_{ij})\right)^{\frac{1}{\gamma}} - 1 \right] \right)^{\frac{1}{\nu(b+1)}}, \quad j = 1,2,...,m_i,\ i = 1,2,...,k.$$

8. Compute the MLEs of the model parameters by using the progressive censored data and solving the nonlinear system ((11)-(14)).
9. Compute the BEs of the model parameters relative to SE and LINEX loss functions, using algorithm (1), with $N = 11000$ and $M = 1000$.

10.Compute the approximate confidence bounds with confidence level 95% for the model parameters γ, a, ν and b.
11.Compute 95% credible confidence intervals using algorithm (2) of the parameters γ, a, ν and b.
12.Replicate the steps $((3)-(11))$ 1000 times.
13.Compute the average values of the MSEs of the MLEs and BEs of the parameters.
14.Compute the average values of the length and the coverage probability of CIs of the parameters.
15.Repeat steps ((1)-(14)) with different values of n_i, m_i and R_{ij}, $j=1,2,...,m_i$, $i=1,2,...,k$.

Table 5 *The progressive censoring schemes used in the simulation studies*

$C.S$	$(R_{i1},\cdots,R_{im_i})$	$C.S$	$(R_{i1},\cdots,R_{im_i})$	$C.S$	$(R_{i1},\cdots,R_{im_i})$
[1]	$R_{ij}=\begin{cases}5 & i=1,j=1\\4 & i=2,j=1\\3 & i=3,j=1\\3 & i=4,j=1\\0 & other\ wise\end{cases}$	[2]	$R_{ij}=\begin{cases}5 & i=1,j=m_1\\4 & i=2,j=m_2\\3 & i=3,j=m_3\\3 & i=4,j=m_4\\0 & other\ wise\end{cases}$	[3]	$R_{ij}=\begin{cases}1 & i=1,j=17,\cdots,20\\1 & i=2,j=9,10,11\\1 & i=3,j=8,9\\1 & i=4,j=5\\0 & other\ wise\end{cases}$
[4]	$R_{ij}=\begin{cases}8 & i=1,j=1\\5 & i=2,j=1\\4 & i=3,j=1\\2 & i=4,j=1\\0 & other\ wise\end{cases}$	[5]	$R_{ij}=\begin{cases}8 & i=1,j=m_1\\5 & i=2,j=m_2\\4 & i=3,j=m_3\\2 & i=4,j=m_4\\0 & other\ wise\end{cases}$	[6]	$R_{ij}=\begin{cases}1 & i=1,j=16,\cdots,20\\1 & i=2,j=8,\cdots,12\\1 & i=3,j=7,8,9\\1 & i=4,j=5,6\\0 & other\ wise\end{cases}$
[7]	$R_{ij}=\begin{cases}6 & i=1,j=1\\5 & i=2,j=1\\4 & i=3,j=1\\3 & i=4,j=1\\0 & other\ wise\end{cases}$	[8]	$R_{ij}=\begin{cases}6 & i=1,j=m_1\\5 & i=2,j=m_2\\4 & i=3,j=m_3\\3 & i=4,j=m_4\\0 & other\ wise\end{cases}$	[9]	$R_{ij}=\begin{cases}1 & i=1,j=28,\cdots,35\\1 & i=2,j=14,\cdots,18\\1 & i=3,j=11,\cdots,14\\1 & i=4,j=6,7\\0 & other\ wise\end{cases}$

8 Conclusion

In this paper, we have considered a progressive-stress ALT model for PGW distribution under progressive type-II censoring. From simulation studies, MLEs and BEs in the case of informative priors of the model parameters γ, a, ν and b were calculated. Point estimation of the model parameters γ, a, ν and b was illustrated via maximum likelihood and Bayes methods. Moreover, approximate and credible CIs were established for the model parameters γ, a, ν and b. The calculations have been worked out based on different sample sizes and three different progressive CSs.

From the results in Tables (7.1)-(7.3), we observed the following:

1.The MSEs of MLEs and BEs of the parameters decrease as the sample size increases, except for few cases. This may be due to variation in data.
2.The BEs of γ, a, ν and b give more accurate results through the MSEs than MLEs, except for few cases.
3.The BEs of γ, a, ν and b under LINEX loss function $(c=2)$ have the smallest MSEs as compared with estimates under SE and LINEX $(c=-2)$ loss function.
4.The BEs of γ, a, ν and b under LINEX loss function $(c=.001)$ have the same MSEs as estimates under SE loss function.
5.The lengths of approximate and credible CIs decrease as the sample size increases, except for few cases. This may be due to variation in data.
6.The credible CIs of γ, a, ν and b give more accurate results than approximate CIs through lengths.
7.The coverage probability of credible CIs of γ, a, ν and b greater than the corresponding coverage probability of approximate CIs.

Table 6 *MSEs of MLEs and BEs under SEL and LINEXL function of γ, a, ν and b with true values ($\gamma = 1.1,\ a = 0.4,\ \nu = 1.1$ and $b = 0.8$), values of the prior parameters ($\mu_1 = 12.1,\ \mu_2 = 12.1$ and $\lambda_1 = 0.0909,\ \lambda_2 = 0.0909$), $k = 4$, $\beta_1 = 4$, $\beta_2 = 8$, $\beta_3 = 12$ and $\beta_4 = 16$.*

n_i	m_i	CS	θ	ML	SEL	LINEXL $c=-2$	LINEXL $c=.001$	LINEXL $c=2$
$n_i = \begin{cases} 20 & i=1 \\ 16 & i=2 \\ 12 & i=3 \\ 10 & i=4 \end{cases}$	$m_i = \begin{cases} 15 & i=1 \\ 12 & i=2 \\ 9 & i=3 \\ 7 & i=4 \end{cases}$	1	γ	1.7486	0.0112	0.0244	0.0112	0.0141
			a	0.2365	0.0514	0.0937	0.0514	0.0344
			ν	0.1869	0.0360	0.0937	0.0360	0.0309
			b	0.3225	0.1563	0.3791	0.1563	0.1133
		2	γ	1.7571	0.0067	0.0242	0.0067	0.0088
			a	0.1471	0.0399	0.0561	0.0398	0.0233
			ν	0.1391	0.0299	0.0396	0.0299	0.0264
			b	0.3352	0.1435	0.4096	0.1434	0.0952
		3	γ	1.6506	0.0033	0.0285	0.0033	0.0016
			a	0.1716	0.0305	0.0740	0.0305	0.0250
			ν	0.1216	0.0207	0.0451	0.0207	0.0103
			b	0.3185	0.1336	0.3089	0.1336	0.0985
$n_i = \begin{cases} 20 & i=1 \\ 16 & i=2 \\ 12 & i=3 \\ 10 & i=4 \end{cases}$	$m_i = \begin{cases} 20 & i=1 \\ 16 & i=2 \\ 12 & i=3 \\ 10 & i=4 \end{cases}$		γ	1.3091	0.0034	0.0260	0.0034	0.0019
			a	0.1703	0.0212	0.0657	0.0212	0.0101
			ν	0.1426	0.0134	0.0411	0.0134	0.0105
			b	0.2359	0.1229	0.3366	0.1228	0.0861
$n_i = \begin{cases} 45 & i=1 \\ 25 & i=2 \\ 20 & i=3 \\ 10 & i=4 \end{cases}$	$m_i = \begin{cases} 37 & i=1 \\ 20 & i=2 \\ 16 & i=3 \\ 8 & i=4 \end{cases}$	4	γ	1.2809	0.0025	0.0135	0.0025	0.0012
			a	0.0614	0.0198	0.0417	0.0198	0.0087
			ν	0.1034	0.0121	0.0210	0.0121	0.0054
			b	0.1534	0.0993	0.1027	0.0993	0.0315
		5	γ	1.2766	0.0015	0.0311	0.0015	0.0009
			a	0.0456	0.0162	0.0387	0.0162	0.0010
			ν	0.0948	0.0173	0.0452	0.0173	0.0031
			b	0.1533	0.0965	0.1847	0.0965	0.0722
		6	γ	0.9813	0.0012	0.0293	0.0012	0.0008
			a	0.0108	0.0121	0.0302	0.0121	0.0088
			ν	0.0725	0.0126	0.0315	0.0126	0.0066
			b	0.1433	0.0765	0.2261	0.0765	0.0145
$n_i = \begin{cases} 45 & i=1 \\ 25 & i=2 \\ 20 & i=3 \\ 10 & i=4 \end{cases}$	$m_i = \begin{cases} 45 & i=1 \\ 25 & i=2 \\ 20 & i=3 \\ 10 & i=4 \end{cases}$		γ	0.6287	0.0018	0.0373	0.0018	0.0002
			a	0.0087	0.0106	0.0327	0.0106	0.0065
			ν	0.0672	0.0103	0.0265	0.0103	0.0026
			b	0.1193	0.0726	0.2045	0.0726	0.0066
$n_i = \begin{cases} 50 & i=1 \\ 30 & i=2 \\ 25 & i=3 \\ 15 & i=4 \end{cases}$	$m_i = \begin{cases} 44 & i=1 \\ 25 & i=2 \\ 21 & i=3 \\ 12 & i=4 \end{cases}$	7	γ	0.4527	0.0017	0.0487	0.0017	0.0005
			a	0.0040	0.0051	0.0319	0.0051	0.0019
			ν	0.0763	0.0089	0.0303	0.0089	0.0057
			b	0.0969	0.0618	0.1456	0.0618	0.0357
		8	γ	0.3821	0.0011	0.0331	0.0011	0.0006
			a	0.0072	0.0043	0.0361	0.0043	0.0015
			ν	0.0648	0.0074	0.0242	0.0074	0.0017
			b	0.0770	0.0531	0.1603	0.0531	0.0126
		9	γ	0.1050	0.0009	0.0295	0.0009	0.0005
			a	0.0055	0.0049	0.0344	0.0049	0.0009
			ν	0.0608	0.0065	0.0243	0.0065	0.0018
			b	0.0659	0.0401	0.1363	0.0401	0.0064
$n_i = \begin{cases} 50 & i=1 \\ 30 & i=2 \\ 25 & i=3 \\ 15 & i=4 \end{cases}$	$m_i = \begin{cases} 50 & i=1 \\ 30 & i=2 \\ 25 & i=3 \\ 15 & i=4 \end{cases}$		γ	0.0547	0.0007	0.0399	0.0007	0.0004
			a	0.0048	0.0021	0.0285	0.0021	0.0014
			ν	0.0607	0.0048	0.0278	0.0048	0.0019
			b	0.0590	0.0370	0.1188	0.0369	0.0093

Table 7 *Lengths and coverage probabilities of* 95% *approximate and credible CIs for* γ, a, ν *and* b *with true values* ($\gamma = 1.1,\ a = 0.4,\ \nu = 1.1$ *and* $b = 0.8$), *values of the prior parameters* ($\mu_1 = 12.1,\ \mu_2 = 12.1$ *and* $\lambda_1 = 0.0909,\ \lambda_2 = 0.0909$), $k = 4$, $\beta_1 = 4$, $\beta_2 = 8$, $\beta_3 = 12$ *and* $\beta_4 = 16$.

n_i	m_i	CS	θ	Length		Coverage Probability	
				Approximate CI	Credible CI	Approximate CI	Credible CI
$n_i = \begin{cases} 20 & i=1 \\ 16 & i=2 \\ 12 & i=3 \\ 10 & i=4 \end{cases}$	$m_i = \begin{cases} 15 & i=1 \\ 12 & i=2 \\ 9 & i=3 \\ 7 & i=4 \end{cases}$	1	γ	8.0009	1.1786	0.967	1
			a	1.8818	0.9703	0.825	0.958
			ν	1.5984	0.9919	0.95	0.975
			b	2.1435	1.6071	0.892	0.958
		2	γ	7.6616	1.1423	0.975	1
			a	1.7164	0.9038	0.858	0.975
			ν	1.4573	0.9840	0.967	1
			b	2.0622	1.6244	0.967	0.975
		3	γ	7.5078	1.1380	0.963	1
			a	1.6525	0.9010	0.825	0.983
			ν	1.3239	1.0067	0.975	1
			b	2.0825	1.5575	0.967	0.975
$n_i = \begin{cases} 20 & i=1 \\ 16 & i=2 \\ 12 & i=3 \\ 10 & i=4 \end{cases}$	$m_i = \begin{cases} 20 & i=1 \\ 16 & i=2 \\ 12 & i=3 \\ 10 & i=4 \end{cases}$		γ	6.1523	1.1091	0.98	1
			a	1.5521	0.8621	0.85	0.97
			ν	1.2860	0.9175	0.98	0.99
			b	1.7563	1.4605	0.96	0.96
$n_i = \begin{cases} 45 & i=1 \\ 25 & i=2 \\ 20 & i=3 \\ 10 & i=4 \end{cases}$	$m_i = \begin{cases} 37 & i=1 \\ 20 & i=2 \\ 16 & i=3 \\ 8 & i=4 \end{cases}$	4	γ	6.5284	1.1156	0.95	1
			a	1.4023	0.8025	0.87	0.96
			ν	1.2506	0.9091	0.97	0.99
			b	1.5209	1.3516	0.95	0.951
		5	γ	6.0251	1.0873	0.933	1
			a	1.3474	0.7810	0.883	0.967
			ν	1.2151	0.9067	0.967	0.992
			b	1.4314	1.2554	0.95	0.982
		6	γ	5.3377	1.0631	0.933	1
			a	1.3733	0.7381	0.833	0.975
			ν	1.1244	0.8463	0.958	0.983
			b	1.3540	1.2063	0.917	0.958
$n_i = \begin{cases} 45 & i=1 \\ 25 & i=2 \\ 20 & i=3 \\ 10 & i=4 \end{cases}$	$m_i = \begin{cases} 45 & i=1 \\ 25 & i=2 \\ 20 & i=3 \\ 10 & i=4 \end{cases}$		γ	4.2698	1.0514	0.968	1
			a	1.1845	0.6869	0.858	0.975
			ν	1.0451	0.8013	0.958	0.992
			b	1.3045	1.1654	0.95	0.951
$n_i = \begin{cases} 50 & i=1 \\ 30 & i=2 \\ 25 & i=3 \\ 15 & i=4 \end{cases}$	$m_i = \begin{cases} 44 & i=1 \\ 25 & i=2 \\ 21 & i=3 \\ 12 & i=4 \end{cases}$	7	γ	4.1178	1.0456	0.958	1
			a	1.1736	0.6687	0.85	0.951
			ν	1.0182	0.7712	0.975	1
			b	1.2284	1.1252	0.985	0.985
		8	γ	3.8391	1.0394	0.942	1
			a	1.1386	0.6249	0.883	1
			ν	1.0354	0.8022	0.967	1
			b	1.2406	1.1047	0.965	0.967
		9	γ	3.2952	1.0185	0.962	1
			a	1.0526	0.6172	0.858	0.958
			ν	1.0126	0.7203	0.975	1
			b	1.2150	1.0942	0.967	0.95
$n_i = \begin{cases} 50 & i=1 \\ 30 & i=2 \\ 25 & i=3 \\ 15 & i=4 \end{cases}$	$m_i = \begin{cases} 50 & i=1 \\ 30 & i=2 \\ 25 & i=3 \\ 15 & i=4 \end{cases}$		γ	3.0757	1.0120	0.962	1
			a	1.0432	0.6095	0.825	0.975
			ν	0.9221	0.7510	0.973	0.983
			b	1.1356	0.9734	0.967	0.967

References

[1] W. Nelson, (1990). Accelerated Testing: Statistical Models, Test Plans and Data Analysis, Wiley, New York.

[2] C. M. Kim and D. S. Bai, (2002). Analysis of acceleratedJ. Reliab. Qual. Saf. Eng., 9, 111-125.

[3] A. J. Watkins and A. M. John, (2008). On constant stress accelerated life tests terminated by type-II censoring at one of the stress levels, J. Stat. Plan. Inference, 138, 768-786.

[4] A. H. Abdel-Hamid, (2009). Constant-partially accelerated life tests for Burr type-XII distribution with progressive type-II censoring, Comput. Stat. Data Anal., 53, 2511-2523.

[5] Q. Guan, Y. Tang, J. Fu and A. Xu, (2014). Optimal multiple constant-stress accelerated life tests for generalized exponential distribution, Commun. Stat. Simul. Comput., 43, 1852-1865.

[6] Z. F. Jaheen, H. M. Moustafa and G. H. Abd El-Monem, (2014). Bayes inference in constant partially accelerated life tests for the generalized exponential distribution with progressive censoring, Commun. Stat. Theor. M., 43, 2973-2988.
[7] M. M. Mohie El-Din, S. E. Abu-Youssef, N. S. A. Ali and A. M. Abd El-Raheem, (2016). Estimation in constant-stress accelerated life tests for extension of the exponential distribution under progressive censoring, METRON, 74, 253-273.
[8] M. M. Mohie El-Din, S. E. Abu-Youssef, N. S. A. Ali and A. M. Abd El-Raheem, (2017). Optimal plans of constant-stress accelerated life tests for the Lindley distribution, J. Test. Eval., 45(4), 1463-1475.
[9] M. M. Mohie El-Din, S. E. Abu-Youssef, N. S. A. Ali and A. M. Abd El-Raheem, (2018). Inference on constant-stress accelerated life testing based on geometric process for extension of the exponential distribution under type-II progressive censoring, Pak. J. Stat. Oper. Res., 14(2), 233-251.
[10] A. M. Abd El-Raheem, (2019). Optimal plans of constant-stress accelerated life tests for extension of the exponential distribution, J. Test. Eval., 47(2), (in press).
[11] A. M. Abd El-Raheem, (2019). Optimal plans and estimation of constant-stress accelerated life tests for the extension of the exponential distribution under type-I censoring, J. Test. Eval., 47(5), (in press).
[12] R. Miller and W. Nelson, (1983). Optimum simple step-stress plans for accelerated life testing, IEEE Trans. Reliab., 32, 59-65.
[13] E. Gouno, A. Sen and N. Balakrishnan, (2004). Optimal step-stress test under progressive type-I censoring, IEEE Trans. Reliab., 53, 388-393.
[14] N. Balakrishnan, D. Kundu, H. K. T. Ng and N. Kannan, (2007). Point and interval estimation for a simple step-stress model with type-II censoring, J. Qual. Technol., 39, 35-47.
[15] M. M. Mohie El-Din, S. E. Abu-Youssef, N. S. A. Ali and A. M. Abd El-Raheem, (2015). Estimation in step-stress accelerated life tests for power generalized Wiebull distribution with progressive censoring, Adv. Stat., 2015, 1-13.
[16] M. M. Mohie El-Din, S. E. Abu-Youssef, N. S. A. Ali and A. M. Abd El-Raheem, (2015). Estimation in step-stress accelerated life tests for Weibull distribution with progressive first-failure censoring, J. Stat. Appl. Pro., 3, 403-411.
[17] M. M. Mohie El-Din, S. E. Abu-Youssef, N. S. A. Ali and A. M. Abd El-Raheem, (2016). Parametric inference on step-stress accelerated life testing for the extension of exponential distribution under progressive type-II censoring, Commun. Stat. Appl. Methods, 23(4), 269-285.
[18] X. K. Yin and B. Z. Sheng, (1987). Some aspects of accelerated life testing by progressives tress, IEEE Trans Reliab, R-36:150-155.
[19] A. H. Abdel-Hamid and E. K. AL-Hussaini, (2011). Inference for a progressive stress model from Weibull distribution under progressive type-II censoring, J. Comput. Appl. Math., 235, 5259-5271.
[20] A. H. Abdel-Hamid and T. A. Abushal, (2015). Inference on progressive-stress model for the exponentiated exponential distribution under type-II progressive hybrid censoring, J. Stat. Comput. Simul., 85, 1165-1186.
[21] M. M. Mohie El-Din, S. E. Abu-Youssef, N. S. A. Ali and A. M. Abd El-Raheem, (2017). Classical and Bayesian inference on progressive-stress accelerated life testing for the extension of the exponential distribution under progressive type-II censoring. Qual. Reliab. Eng. Int., 33, 1-14.
[22] N. Balakrishnan and R. Aggarwala, (2000). Progressive Censoring: Theory, Methods, and Applications, Birkhauser, Boston.
[23] V. Bagdonavicius and M. Nikulin, (2002). Accelerated Life Models: Modeling and Statistical Analysis, Chapman and Hall/CRC, Florida.
[24] N. Nikulin and F. Haghighi, (2009). On the power generalized Weibull family: model for cancer censored data, Metron, LXVII, 75-86.
[25] M. Nikulin, F. Haghighi, (2006). *A chi-squared test for the generalized power Weibull family for the head-and-neck cancer censored data*, J. Math. Sci., 133, 1333-1341.
[26] N. Nikulin, F. Haghighi, (2009). *On the power generalized Weibull family: model for cancer censored data*, Metron, LXVII, 75-86.
[27] V. Bagdonavicius, M. Nikulin, (2011). *Chi-squared goodness of fit test for right censored data*, Int. J. Appl. Math. Stat., 24, 30-50.
[28] V. Voinov, N. Pya, N. Shapakov, Y. Voinov, (2013). *Goodness-of-fit tests for the power-generalized Weibull probability distribution*, Commun. Stat. Simul. Comput., 42, 1003-1012.
[29] D. Kumar and S. Dey, (2017). Power generalized Weibull distribution based on order statistics, J. of Stat. Res., 51, 61-78.
[30] S. Nadarajah and F. Haghighi, (2011). An extension of the exponential distribution, Statistics 45, 543-558,
[31] S. K. Upadhyay and A. Gupta, (2010). A Bayes analysis of modified Weibull distribution via Markov chain Monte Carlo simulation, J. Stat. Comput. Simul., 80, 241-254.
[32] R. Miller, (1981). Survival Analysis, Wiley, New York.
[33] Y. Zhu, (2010). Optimal Design and Equivalency of Accelerated Life Testing Plans, PhD thesis, The State University of New Jersey, United States.

Mostafa Mohamed Mohie El-Din is a professor of Mathematical Statistics, Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City (11884), Cairo, Egypt His research areas are order statistics, record values, generalized order statistics, accelerated life-testing and data analysis. He published several research papers in international peer-reviewed journals.

Abd El-Raheem Mohamed Abd El-Raheem received his Ph.D. in Statistics at Ain Shams University, Cairo (Egypt). His research areas are order statistics, accelerated life-testing, and data analysis. He published several research papers in international peer-reviewed journals. Currently, he is working as a lecturer of Mathematical Statistics, Department of Mathematics, Faculty of Education, Ain Shams University, Cairo, Egypt

Sara Omar Abd El-Azeem is a demonstrator of Basic Sciences, Faculty of Engineering, at MTI University, Cairo, Egypt Her area of interest is accelerated life-testing.