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A New Lifetime Distribution: Some of its Statistical Properties and Application

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Abstract: In the present paper, a new lifetime distribution has been introduced by the use of Minimum Guarantee transformation as suggested by Kumar et al. (2017). For the purpose, Lindley distribution is considered as a baseline distribution. Some of the statistical properties of this distribution has been studied and classical estimators like maximum likelihood estimator (MLE), least square estimator (LSE) and maximum product of spacing estimator (MPSE) has been obtained and their performance is carried out through simulation study. Further, a real data has been taken to show its application in the real scenario.

Keywords: Life Time Distribution, Reliability Analysis and Hazard Rate Function.

1 Introduction

Survival Analysis is a branch of statistics for analyzing data where the outcome variable is the time until the occurrence of a biological organism or failure of any electronic device. It can be measured in days, weeks, years etc. In statistical literature, for modeling any lifetime data, a number of continuous distributions such as exponential, gamma, log-normal and Weibull distributions etc are available. It is to be recalled here that exponential distribution is applicable to the data having constant failure rate pattern and its application to the data having non-constant failure rate pattern may mislead the result. Log normal and Weibull distributions are extensively used for modeling skewed data (see, Kundu and Manglick (2004)). Gamma distribution also fit such type of data (see, Kundu and Gupta (2004)). The applicability of a distribution can also be identified from the nature of their hazard rate function. Thus any distribution has been widely used and suitable for the data having increasing hazard rate function. It was introduced by Lindley (1958). The probability density function (pdf) and cumulative distribution function (cdf) of Lindley distribution with the shape parameter θ are given by,

$$f(x) = \frac{\theta^2}{\theta + 1} (1 + x) e^{-\theta x}; \quad x > 0, \quad \theta > 0$$

$$\tag{1}$$

and

$$F(x) = 1 - \left(\frac{\theta x}{\theta + 1}\right)e^{-\theta x}; \quad x > 0, \quad \theta > 0$$
⁽²⁾

respectively.

Ghitany et al. (2008(b)) have studied various statistical properties of Lindley distribution. Its different forms have also been studied by several authors such as Ghitany et al. (2008(a)) proposed zero-truncated Poisson-Lindley distribution, Deniz and Ojeda (2011) proposed discrete Lindley distribution, Nadarajah et al. (2011) proposed two parameter Lindley distribution using the concept of exponentiated generalization of distribution as proposed by Gupta et al. (1998). Recently, Rashid and Jan (2016) proposed Lindley power series distribution.

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As mentioned by Kumar et al. (2017), that it is the era of generalizing or transforming any available distribution with a hope to get more flexible distribution as compared to the considered distribution and other distributions too. In order to achieve this goal, they have suggested minimum guarantee (MG) transformation and derived, studied and showed application of a new lifetime distribution by considering exponential distribution as a baseline distribution.

In the present paper, we have derived a new lifetime distribution using MG transformation and the considered baseline distribution is Lindley distribution having pdf (1). We will use the abbreviation $MG_L(\theta)$ distribution to denote this new distribution. The cdf, pdf and hazard rate function of $MG_L(\theta)$ distribution are given by;

$$G(x) = \exp^{\frac{-\left(1+\frac{\theta_x}{\theta+1}\right)\exp^{-\theta_x}}{1-\left(1+\frac{\theta_x}{\theta+1}\right)\exp^{-\theta_x}}}; \quad x > 0, \quad \theta > 0$$
(3)

$$g(x) = \frac{\exp^{\frac{-\left(1+\frac{\theta_x}{\theta+1}\right)\exp^{-\theta_x}}{\left(1-\left(1+\frac{\theta_x}{\theta+1}\right)\exp^{-\theta_x}\right)^2}}}{\left(1-\left(1+\frac{\theta_x}{\theta+1}\right)\exp^{-\theta_x}\right)^2}\frac{\theta^2}{\theta+1}(1+x)\exp^{-\theta_x}; \quad x > 0, \quad \theta > 0$$
(4)

and,

$$h(x) = \frac{\exp^{\frac{-\left(1+\frac{\theta_x}{\theta+1}\right)\exp^{-\theta_x}}{1-\left(1+\frac{\theta_x}{\theta+1}\right)\exp^{-\theta_x}}}{\frac{\theta^2}{\theta+1}(1+x)\exp^{-\theta_x}}}{\left(1-\left(1+\frac{\theta_x}{\theta+1}\right)\exp^{-\theta_x}}; \quad x > 0, \quad \theta > 0$$
(5)

respectively,

The shapes of the cdf, pdf and hazard rate function of $MG_L(\theta)$ has been shown in Figures 1, 2 and 3 respectively, for different value of θ . Figure 3 shows that $MG_L(\theta)$ distribution has non monotonic hazard rate function.



Fig. 1: Plots of Cumulative distribution function G(x) of $MG_L(\theta)$ distribution for different values of θ



Fig. 2: Plots of Probability density function g(x) of $MG_L(\theta)$ distribution for different values of θ



Fig. 3: Plots of Hazard Rate Function h(x) of $MG_L(\theta)$ distribution for different values of θ

The rest of the paper is organized as follows: The statistical properties of $MG_L(\theta)$ distribution such as mean, median, raw moments, skewness and kurtosis have been derived in section 2. Estimation of the parameter θ of $MG_L(\theta)$ distribution is carried out in section 3. Section 4 deals with the simulation study. A Real data application is carried out in section 5 and conclusion has been drawn in section 6.

2 Some Statistical properties

2.1 Mean

If μ is the mean of MG_L(θ) distribution, we have

$$\mu = E(X) = \int_0^\infty xg(x)dx$$

$$\mu = \int_0^\infty x \frac{\exp\left(\frac{-\left(1 + \frac{\theta_x}{\theta + 1}\right)\exp^{-\theta x}}{1 - \left(1 + \frac{\theta_x}{\theta + 1}\right)\exp^{-\theta x}}\right)^2}{\left(1 - \left(1 + \frac{\theta_x}{\theta + 1}\right)\exp^{-\theta x}\right)^2} \frac{\theta^2}{\theta + 1}(1 + x)\exp^{-\theta x}dx$$
(6)

The above integral is not solvable analytically. To solve it numerically for any given value of θ , one have to use some numerical integration technique such as Gauss-Lagurre quadrature formula or Monte-carlo integration or some other methods may be used.

2.2 Median

If M is the median of the $MG_L(\theta)$ distribution, we have

$$\int_{0}^{M} g(x)dx = \frac{1}{2}$$
$$\int_{0}^{M} \frac{\exp^{\frac{-\left(1+\frac{\theta x}{\theta+1}\right)\exp^{-\theta x}}{1-\left(1+\frac{\theta x}{\theta+1}\right)\exp^{-\theta x}}}{\left(1-\left(1+\frac{\theta x}{\theta+1}\right)\exp^{-\theta x}\right)^{2}}\frac{\theta^{2}}{\theta+1}(1+x)\exp^{-\theta x}dx = \frac{1}{2}$$

After simplification, it reduces to,

$$\exp^{\frac{-\left(1+\frac{\theta M}{\theta+1}\right)\exp^{-\theta M}}{1-\left(1+\frac{\theta M}{\theta+1}\right)\exp^{-\theta M}}} = \frac{1}{2}$$
(7)

which is not solvable analytically, some numerical iteration technique will be used for its numerical solution for any given value of θ .

2.3 Raw Moments

The r^{th} moments about origin μ'_r (raw moments) of MG_L(θ) distribution is obtained as follows;

$$\mu_r' = \frac{\theta^2}{\theta + 1} \int_0^\infty x^r \frac{\exp^{\frac{-(1 + \frac{\theta x}{\theta + 1}})\exp^{-\theta x}}}{\left(1 - \left(1 + \frac{\theta x}{\theta + 1}\right)\exp^{-\theta x}\right)^2} (1 + x) \exp^{-\theta x} dx \tag{8}$$

2.4 Skewness and Kurtosis

The coefficient of Skewness is a measure of the degree of symmetry of the distribution (see, Sheskin (2011)). It come in the form of negative skewness or positive skewness, depending on whether data points are skewed to the left or to the right of the data average. Similarly, the coefficient of Kurtosis is a measure for the degree of tailed-ness in the distribution (see, Westfall (2014)). There are three categories of kurtosis that can be displayed by set of data and they are mesokurtic, leptokurtic and platykurtic.

The measure of skewness (β_1) and measure of kurtosis (β_2) can be calculated using the following expressions,

$$\beta_1 = \frac{(\mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3)^2}{(\mu'_2 - (\mu'_1)^2)^3} \tag{9}$$

$$\beta_2 = \frac{(\mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4)}{(\mu_2' - (\mu_1')^2)^2} \tag{10}$$

The values of β_1 and β_2 are calculated for $MG_L(\theta)$ distribution for different values of θ and for all considered values of θ , we get $\beta_1 > 0$ and $\beta_2 > 3$. Thus we may conclude that $MG_L(\theta)$ distribution is positively skewed and leptokurtic. The graphs of values of β_1 and β_2 for different values of θ are shown in Figures 4 and 5 respectively.



Fig. 4: Plots of the values of β_1 for different values of θ



Fig. 5: Plots of the values of β_2 for different values of θ

3 Different methods of estimation

In this section, we have derived some classical estimators of the parameter θ of MG_L(θ) distribution for complete sample taken from this distribution. The estimators derived are maximum likelihood estimator (MLE), least square estimator (LSE) and maximum product of spacing estimator (MPSE).

3.1 Maximum likelihood estimator

Let $\underline{X} = (X_1, X_2, \dots, X_n)$ is a random sample of size n from $MG_L(\theta)$ distribution. Then the likelihood function for \underline{X} is given by, $L = \prod_{j=1}^{n} (g(x_j))$

$$L = \frac{\prod_{j=1}^{n} \exp^{\frac{-\left(1 + \frac{\theta x_j}{\theta + 1}\right) \exp^{-\theta x_j}}{1 - \left(1 + \frac{\theta x_j}{\theta + 1}\right) \exp^{-\theta x_j}}}{\prod_{j=1}^{n} \left(1 - \left(1 + \frac{\theta x_j}{\theta + 1}\right) \exp^{-\theta x_j}\right)^2} \left(\frac{\theta^2}{\theta + 1}\right)^n \prod_{j=1}^{n} \left\{(1 + x_j) \exp^{-\theta x_j}\right\}$$
(11)

and hence the log likelihood function is,

$$\ln L = n \ln \left(\frac{\theta^2}{\theta+1}\right) - \sum_{j=1}^n \frac{\left(1 + \frac{\theta x_j}{\theta+1}\right) \exp^{-\theta x_j}}{1 - \left(1 + \frac{\theta x_j}{\theta+1}\right) \exp^{-\theta x_j}} - \theta \sum x_j + \sum_{j=1}^n \ln(1+x_j) - 2\sum_{j=1}^n \ln\left[1 - \left(1 + \frac{\theta x_j}{\theta+1}\right) \exp^{-\theta x_j}\right]$$
(12)

Now, the log likelihood equation for estimating θ is given by,

$$\frac{\partial \ln L}{\partial \theta} = 0 \tag{13}$$

which is not solvable analytically for θ and we have used Newton-Raphson method to solve it numerically.

3.2 Least square estimator

Least square method of estimation was introduced by Swain et al. (1988) to estimate the parameter of Beta distribution. The expression of empirical cdf for $\underline{X} = (X_1, X_2, \dots, X_n)$ is given by ,

$$E[G(x_j)] = \frac{j}{n+1}, j = 1, 2, ..., n$$
(14)

The least square estimator of the parameter of θ of $MG_L(\theta)$ distribution is obtained by minimizing $P(\theta)$ with respect to θ , where

$$P(\theta) = \sum_{j=1}^{n} \left((G(x_j; \theta)) - \frac{j}{n+1} \right)^2$$
(15)

and the normal equation for estimating θ is given by

$$\frac{\partial P(\theta)}{\partial \theta} = \sum_{j=1}^{n} G'_{\theta}(x_j; \theta) \left((G(x_j; \theta)) - \frac{j}{n+1} \right) = 0$$
(16)

which is not solvable for θ analytically, we have again used Newton-Raphson method for its numerical solution.

3.3 Maximum product of spacing estimator

Cheng and Amin (1983) have proposed maximum product of spacing method for estimating the unknown parameter and they have defined product of spacing as the geometric mean (G) of spacings D_i i.e.,

$$G = \sqrt[n+1]{\prod_{j=1}^{n+1} D_j}$$
(17)

where the differences D_i is defined as

$$D_j = \int_{x_{(j-1)}}^{x_{(j)}} g(x;\theta) dx; j = 1, 2, \dots, n+1$$
(18)

such that $G(x_{(0)}; \theta) = 0$ and $G(x_{(n+1)}; \theta) = 1$.

The maximum product of spacing estimator (MPSE) is that value of θ which maximizes G and hence ln G.

Now taking log of both sides of (17), we get

$$lnG = \frac{1}{n+1} \sum_{j=1}^{n+1} ln \left[(G(x_{(j)}; \theta)) - (G(x_{(j-1)}; \theta)) \right]$$
(19)

and the normal equation for estimating θ is

$$\overline{\partial \theta} = 0$$

$$\frac{\partial \ln G}{\partial \theta} = \frac{1}{n+1} \sum_{j=1}^{n+1} \left[\frac{G'_{\theta}(x_{(j)};\theta) - G'_{\theta}(x_{(j-1)};\theta)}{G(x_{(j)};\theta) - G(x_{(j-1)};\theta)} \right] = 0$$
(20)

which is not solvable analytically, to solve it numerically, some numerical iteration technique will be used, particularly, we have used Newton-Raphson method for its numerical solution.

∂lnG

4 Simulation Study

In this section, the simulation study is carried out to assess the performance of the proposed estimators of θ in terms of their MSEs with varying sample sizes. The value of θ is arbitrarily chosen as $\theta=1$ and the different considered values of n are 5,10,15,...,35. The process is repeated 2000 times and MLEs, LSEs and MPSEs have been computed based on each generated samples and consequently their mean squared errors (MSEs) have been calculated and reported in Table 1. Also approximate 95% confidence interval (CIs) and coverage probabilities (CPs) of the estimators are noted in this table.

We have used the symbols $\hat{\theta}_{ML}$, $\hat{\theta}_{LS}$ and $\hat{\theta}_{MPS}$ to denote MLE, LSE and MPSE of θ respectively. The formula used to calculate 95% CI for θ is $[\hat{\theta} - 1.96\sqrt{Var(\hat{\theta})}, \hat{\theta} + 1.96\sqrt{Var(\hat{\theta})}]$, where T is the considered estimator of θ .

$\begin{array}{ c c c c c c c c c c c c c c c c c c c$					
MSE 0.1598 0.042 0.0717 5 Confidence Interval coverage probability (0.2520,1.0639) (0.5493,1.2495) (0.5234,1.0511) 5 MSE 0.1237 0.0192 (0.329 10 Confidence Interval coverage probability 26.95 99.8 79.25 20 Confidence Interval coverage probability 0.0922 0.0093 0.0135 20 Confidence Interval coverage probability 0.44 99.5 86.8 MSE 0.0664 0.0055 0.0067 40 Confidence Interval coverage probability 50.5 99.9 88.9 60 MSE 0.0621 0.0045 0.0046 60 Confidence Interval coverage probability 50.5 99.9 88.9 60 Confidence Interval coverage probability 0.0621 0.0045 0.0046 60 Confidence Interval coverage probability 49.15 100 92.05 80 MSE 0.0485 0.0032 0.0032 80 Confidence Interval coverage proba	n		$\hat{ heta}_{ML}$	$\hat{ heta}_{LS}$	$\hat{ heta}_{MPS}$
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coverage probability 50.5 99.9 88.9 60 MSE 0.0621 0.0045 0.0046 60 Confidence Interval coverage probability (0.4675,1.2051) (0.8607,1.2200) (0.8509,1.0778) 60 MSE 0.0485 0.0032 0.0032 60 Confidence Interval (0.5167,1.2517) (0.8866,1.1116) (0.8749,1.0770)		MSE	0.0664	0.0055	0.0067
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80 Confidence Interval (0.5167,1.2517) (0.8866,1.1116) (0.8749,1.0770)		coverage probability	49.15	100	92.05
		MSE	0.0485	0.0032	0.0032
coverage probability 62.8 100 91.45	80	Confidence Interval	(0.5167,1.2517)	(0.8866,1.1116)	(0.8749,1.0770)
		coverage probability	62.8	100	91.45

Table 1: MSEs, CIs and CPs of MLEs, LSEs and MPSEs for $\theta = 1$ with varying n

From above table, it is clear that MSEs of all estimators decreases as n increases and MSE of $\hat{\theta}_{LS}$ is least as compared to that of $\hat{\theta}_{ML}$ and $\hat{\theta}_{MPS}$.

5 Real Data Application

In this section, a real data set has been considered for checking suitability and superiority of the proposed distribution over some existing distribution such as DUS Exponential distribution (DUSED) and Transmuted Inverse Rayleigh Distribution (TIRD). The data set has been extracted from Lee and Wang (2003) and shows the remission times (in months) of a random sample of 128 bladder cancer patients. The data is as shown below:

 $\underline{X} = (0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 3.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 9.22, 13.80, 25.74, 0.50, 2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 25.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 2.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 7.39, 10.34, 14.83, 34.26, 0.90, 2.69, 4.18, 5.34, 7.59, 10.66, 15.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 1.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 5.49, 7.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 1.40, 3.02, 4.34, 5.71, 7.93, 1.46, 18.10, 11.79, 4.40, 5.85, 8.26, 11.98, 19.13, 1.76, 3.25, 4.50, 6.25, 8.37, 12.02, 2.02, 13.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 12.07, 6.76, 21.73, 2.07, 3.36, 6.93, 8.65, 12.63, 22.69).$

Kumar et al.(2017) had considered this data set and showed that DUSED fit better than TIRD in terms of Akaike information criterion (AIC), Bayesian information criterion (BIC), Kolmogorov-Smirnov test (K-S) statistics criterions. These criterions are defined as follows,

$$AIC = -2\ln(k) + 2p$$
$$BIC = -2\ln(k) + p * \ln(n)$$
$$D_n = sup|F(x) - F_n(x)|$$

where n is the sample size, p is the no. of unknown parameters in the model, k is the maximized value of the likelihood function and $F_n(x)$ is empirical distribution function.

We have computed MLE of the parameter θ of MG_L(θ)-distribution having pdf (4) for the considered data set and found it as 0.3569373 and corresponding to it, AIC, BIC and K-S test statistics values have been calculated and are shown in

Table 2. Also, the values of AIC, BIC and K-S test statistics for this data set for DUSED and TIRD have been extracted from Kumar et al. (2017) and shown in Table 2.

Tuble 2. The, bie, bie and it blest subsides value for bladder calleer patients dat								
Distributions	AIC	BIC	-LL	K-S test statistics value				
$MG_L(\theta)$ -distribution	555.32	563.02	276.65	0.216				
DUSED	834.04	836.89	416.02	0.418				
TIRD	1424.4	1424.6	710.2	0.676				

Table 2: AIC, BIC, -LL and K-S test statistics value for bladder cancer patients data

From table 2, we observed that $MG_L(\theta)$ -distribution fits better as compared to DUSED and TIRD to the data set of remission times of 128 bladder cancer patients, in terms of AIC, BIC and K-S test values, as the criterion values are least for $MG_L(\theta)$ -distribution as compared to those for DUSED and TIRD.

6 Conclusions

A single parameter lifetime distribution $MG_L(\theta)$ distribution has been introduced by the use of MG transformation as suggested by Kumar et al. (2017). Some of its statistical properties such as mean, median, raw moments, skewness and kurtosis have been discussed. A real dataset has been considered and it is found that $MG_L(\theta)$ distribution fits better as compared to DUSED and TIRD in terms of AIC, BIC and K-S test values. The classical estimators MLE, LSE and MPSE of the parameter θ has been obtained. Simulation study is also carried out to see the performance of MLE, LSE and MPSE for their long run use and it is found that for all considered values of n, LSE out performs the other two estimators MLE and MPSE as its MSE is always least as compared to those of MPSE and MLE. Thus, we may recommend $MG_L(\theta)$ distribution for its further use in medical science with a hope to get better model for exact prediction of disease and related problems.

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