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Thermal Stresses in Thermoelastic Half-Space Without Energy Dissipation Subjected to Rotation and Magnetic Field

S. M. Abo-Dahab[1,2,*], *A. M. Abd-Alla*[1,3] *and E. E. Mahmoud*[1,3]

[1] Department of Mathematics, Faculty of Science, Taif University, Taif 888, Saudi Arabia.
[2] Department of Mathematics, Faculty of Science, South Valley University, Qena 83523, Egypt.
[3] Department of Mathematics, Faculty of Science, Sohag University, Sohag 82524, Egypt.

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Abstract: The present paper is concerned with the investigation of disturbances in a homogeneous, isotropic and thermoelastic rotating medium withmagnetic fieldand a time-dependent heat source effect due to thermomechanical source. The formulation is without energy dissipation subjected to thermomechanical source. The normal mode analysisand eigenvalue approach techniques are applied to solve the problem. The expressions of displacement, mean value of normal stress, dilatation and temperature are obtained in the domain. Numerical simulated results are depicted graphically to show the effect of magnetic field and rotation on resulting quantities. The results indicate that the effect of magnetic field, rotation, frequency, wave number and time are very pronounced.

Keywords: Thermal stresses; Thermoelasticity; Energy dissipation;Magnetic field; Half-Space, Rotation.

1 Introduction

During the past few decades, wide spread attention has been given to thermoelasticity theories that admit a finite speed for the propagation of thermal signals. In contrast to the conventional theories based on parabolic-type heat equation, these theories are referred to as generalized theories. Because of the experimental evidence in support of the finiteness of the speed of propagation of a heat wave, generalized thermoelasticity theories are more realistic than conventional thermoelasticity theories in dealing with practical problems involving very short time intervals and high heat flux such as those occurring in last units, energy channels, nuclear reactors, etc. The phenomenon of coupling between the thermomechanical The investigator [1,7] studied the propagation of plane harmonic waves in homogeneous isotropc heat – conducting elastic materials. The wave propagation in the two temperature theory of thermoelasticity was investigated by Warren and Chen [8]. Green and Naghdi [9] postulated postulated a new concept in thermoelasticity theories and proposed three models which are subsequently referred to as GN-I, II and III models. The linearized version of model-I corresponds to the classical thermoelastic model (based on Fourier's law). The version of the model-II and III permit the propagation of thermal waves at finite speed Green-Nagdahi's second model (GN-II), in particular exhibits a feature that is not present in other established thermoelastic models as it does not sustain dissipation of thermal energy [10]. In this model the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. Green and Nagdahi [11] included the derivation of a complete set of governing equations of a linearized version of the theory of homogeneous and isotropic materials in terms of the displacement and boundary value problem. Quintanilla [12] investigated thermoelasticity without energy dissipation of materials with microstructure. Kumar and Devi [13] discussed Magneto thermoelastic with and without energy dissipation Half-Space in contact with Vacuum. Abd-Alla, et al. [14] investigated the propagation of Rayleigh waves in magneto-thermo-elastic half-space of a homogeneous orthotropic material under the effect of rotation, initial stress and gravity field.

* Corresponding author e-mail: sdahb@yahoo.com

Magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylinder under the hyperbolic heat conduction model has been investigated by Abd-Alla and Mahmoud [15]. Abo-Dahab and Mohamed [16] investigated the influence of magnetic field and hydrostatic initial stress on reflection phenomena of P and SV waves from a generalized thermoelastic solid half-space. Abd-Alla and Abo-Dahab [17] found the time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation. Abd-Alla, et al. [18] investigated the generalized magneto-thermoelastic Rayleigh waves in a granular medium under the influence of a gravity field and initial stress. Abd-Alla, et al. [19] investigated the thermal stresses in a non-homogeneous orthotropic elastic multilayered cylinder. Abd-Alla and Abo-Dahab [20] studied the effect of rotation and initial stress on an infinite generalized magneto-thermoelastic diffuse body with a spherical cavity. Zhu, et al. [21] studied the steady-state response of the thermoplastic half - plane with voids subjected to a surface harmonic force and a thermal source. Chirta [22] discussed the thermoelastic surface waves on an exponentially graded half-space. Yu, et al. [23] investigated the domain-independent-Integrals for the force and couple stress intensity factor evaluations of a crack in the micropolar thermoplastic medium. Singh and Chakraborty [24] presented the reflection of a plane magneto-thermoelastic wave of the boundary of a solid half-space in presence of initial stress. Effect of hydrostatic initial stress on a wave in a thermoelastic solid half-pace studied by Singh [25]. Gusarov, et al. [26] discussed the thermoelastic residual stresses and deformations at laser treatment. A three-dimensional thermoelastic problem for a half-space without energy dissipation investigated by Sarkar and Lahiri [27]. Deswal and Kalkal [28] studied the plane waves in a fractional order micropolar magneto-thermoelastic half-space. Wave propagation in an initially stressed transversely isotropic thermoelastic solid half-space presented by Singh [29]. Leguillon, et al. [30] found the applications of the coupled stress-energy criterion predict the fracture behavior of layered ceramics designed with internal compressive stress. Das, et al. [31] found the reflection generalized thermoelastic waves form isothermal and insulated boundaries of a half-space. Generalized thermoelastic diffusion in a thick circular plate, including heat source investigated by Tripathi, et al. [32]. Abo-Dahab and Singh [33] studied the rotational and the food's effect on the reflection of P-waves from stress-free surface on elastic half-space under the magnetic field and initial stress without energy dissipation. Sherief and Saleh [34] studied A half-space problem in the theory of generalized thermoelastic diffusion. Energy-based delamination theory for pixel loading in the presence of thermal stress discussed by McCartney, et al. [35]. Kumar and Deswal [36] investigated the steady-state response of a micropolar generalized thermoelastic half-space to the moving mechanical/thermal loads. Recently, Abd-Alla, et al. [37] presented the propagation of a thermoelastic wave in a half-space of a homogeneous isotropic material subjected to the effect of gravity field. Abd-Alla, et al. [38] investigated the rotation effect on thermoelastic Stoneley, Love and Rayleigh waves in fibre-reinforced anisotropic general viscoelastic media of higher order.

The problem of response of thermomechanical sources in isotropic solid with rotation and magnetic field effect and in contact with the vacuum in the context Green Naghdi theories of type- II has not been analyzed earlier and is considered for the first time in this paper. The effect of the magnetic field, frequency, wave number and time on the displacements, temperature distribution, the mean value of principle stresses and the dilatation have been shown graphically. The normal mode analysis and eigenvalue approach techniques are used to solve the resulting non-dimensional coupled equations. The results obtained have also been compared and reduce to those available in the literature at appropriate stages of this work. The theoretical development has been verified numerically and illustrated graphically.

2 Formulation of the problem

We considera medium is a perfect electric conductor, we take the linearized Maxwell equations governing the electromagnetic field, taking into account absence of the displacement current SI as [29]:

$$\begin{aligned}
&\vec{J} = \operatorname{curl} \vec{h},\\
&-\mu_e \frac{\partial \vec{h}}{\partial t} = \operatorname{curl} \vec{E},\\
&\operatorname{div} \vec{h} = 0,\\
&\operatorname{div} \vec{E} = 0,\\
&\vec{E} = -\mu_e \left(\frac{\partial \vec{u}}{\partial t} \times \vec{H}_0\right),\\
&\vec{h} = \operatorname{curl}(\vec{u} \times \vec{H}_0).
\end{aligned} \tag{1a}$$

Applying an initial magnetic field vector $\vec{H} = \vec{H}(0, H_0, 0)$ in Cartesian coordinates (x, y, z) to the equation (1a) we have

$$\vec{u} = \vec{u}(u, 0, w), \quad \vec{h} = (0, -H_0(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}), 0), \tag{1b}$$

$$\vec{j} = (H_0(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2}), 0, -H_0(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z})). \tag{1c}$$

Maxwell stress components are given by:

$$\tau_{ij} = \mu_e(H_i h_j + H_j h_i - H_k h_k \delta_{ij}) \tag{1d}$$

where $\vec{h}$ is the perturbed magnetic field over the primary magnetic field, $\vec{E}$ is the electric intensity, $\vec{J}$ is the electric current density, μ_e is the magnetic permeability, H_0 is the constant primary magnetic field, δ_{ij} is the the Kronecker delta and $\vec{u}$ is the displacement vector.

Equation of motion for isotropic thermoelastic medium rotating uniformly with an angular velocity $\overrightarrow{\Omega} = \Omega \underline{n}$ has centripetal acceleration $\overrightarrow{\Omega} \times (\overrightarrow{\Omega} \times \overrightarrow{u})$ due to time varying motion only, where $\underline{n}$ is a unit vector representing the direction of the axis of rotation and taking into account Lorentz force $\overrightarrow{F}$ is:

$$\tau_{ij} + \overrightarrow{F} = \rho(\vec{u}_i + (\overrightarrow{\Omega}x(\overrightarrow{\Omega}x\overrightarrow{u}). \tag{2}$$

For a homogeneous, isotropic elastic solid, the basic equations of the linear generalized theory of thermoelasticity without energy dissipation in the presence of a magnetic field are:

(i)The equations of motion in terms of displacement in a rotating medium with magnetic fields as follows:

$$\rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u\right] = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z}\right) + (\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 w}{\partial x \partial z} - \gamma \frac{\partial T}{\partial x}, \tag{3}$$

$$\rho\left[\frac{\partial^2 w}{\partial t^2} - \Omega^2 w\right] = \mu_e H_0^2 \left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2}\right) + (\lambda + 2\mu)\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial x \partial z} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial z} - \gamma \frac{\partial T}{\partial z}. \tag{4}$$

(ii) The modified heat conduction equation is

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \rho C_E \frac{\partial^2 T}{\partial t^2} + \gamma T_0 \frac{\partial^2 e}{\partial t^2} \tag{5}$$

(ii)The stress-displacement-temperature relations with incremental isotropy are given by:

$$\begin{aligned} \sigma_{xx} &= 2\mu \frac{\partial u}{\partial x} + \lambda e - \gamma T, \\ \sigma_{zz} &= 2\mu \frac{\partial w}{\partial z} + \lambda e - \gamma T, \\ \tau_{xz} &= \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \end{aligned} \tag{6}$$

(iii)The components of magnetic stresses are

$$\begin{aligned} \tau_{xx} &= \frac{\beta \mu_e H_0^2}{\mu}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right), \\ \tau_{zz} &= \frac{\beta \mu_e H_0^2}{\mu}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) \end{aligned} \tag{7}$$

where, λ and μ are Lame's elastic constants, ρ is the density, $\sigma_{xx}, \sigma_{zz}, \tau_{xz}$ are the components of the stress, u, w are the components of the displacement, t is the time, T is the temperature, $\gamma = (3\lambda + 2\mu)\alpha_T$ where α_T is the coefficient of linear thermal expansion, K is the thermal diffusivity, C_E isthe specific heat at constant strain, T_0 is the temperature of the medium in its natural state, assumed to be such that $\left|\frac{T - T_0}{T_0}\right| << 1$, where, $e = \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)$.

Introducing the following non-dimensional variables

$$\begin{aligned} &(x', z') = \tfrac{1}{l}(x, z),\ t' = \tfrac{C_1 t}{l},\ \ (u', w') = \tfrac{(\lambda+2\mu)}{\gamma T_0 l}(u, w),\ \Omega' = \tfrac{l}{C_1}\Omega \\ &T' = \tfrac{T}{T_0},\ \ \sigma'_{ij} = \tfrac{\sigma_{ij}}{\gamma T_0},\ \ C_T^2 = \tfrac{K}{\rho C_E C_1^2},\ \ \varepsilon_T = \tfrac{\gamma^2 T_0}{\rho C_E(\lambda+2\mu)}, \end{aligned} \tag{8}$$

where, l is the length and $C_T = \frac{C_3}{C_1}, C_1^2 = \frac{\lambda+2\mu}{\rho}, C_3^2 = \frac{\mu}{\rho}$. Here $C_1$is the longitudinal wave velocity, C_3 is the shear wave velocity, C_T is the non dimensional thermal wave speed in TEWOED and ε_T is a the thermoelastic coupling parameter.

Substituting from equation (8) into Eqs. (3)-(7), the equations in dimensionless form (after dropping out the prime) become

$$\frac{\partial^2 u}{\partial t^2} - \Omega^2 u = (1+S)\frac{\partial^2 u}{\partial x^2} + \beta \frac{\partial^2 u}{\partial z^2} + (1+S-\beta)\frac{\partial^2 w}{\partial x \partial z} - \frac{\partial T}{\partial x}, \tag{9}$$

$$\frac{\partial^2 w}{\partial t^2} - \Omega^2 w = (1+S)\frac{\partial^2 w}{\partial z^2} + \beta \frac{\partial^2 w}{\partial x^2} + (1+S-\beta)\frac{\partial^2 u}{\partial x \partial z} - \frac{\partial T}{\partial z}, \tag{10}$$

$$C_T^2 \nabla^2 T = \frac{\partial^2 T}{\partial t^2} + \varepsilon_T \frac{\partial^2 e}{\partial t^2}, \tag{11}$$

$$\begin{aligned} \sigma_{xx} &= 2\beta \frac{\partial u}{\partial x} + (1-2\beta)e - T\ , \\ \sigma_{zz} &= 2\beta \frac{\partial w}{\partial z} + (1-2\beta)e - T, \\ \tau_{xz} &= \beta\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \end{aligned} \tag{12}$$

where

$$S = \frac{\mu_e H_0^2}{(\lambda+2\mu)},\quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2},\quad \beta = \frac{\mu}{(\lambda+2\mu)}.$$

From equations (9) and (10) by using the value of strain e, we get the following equations

$$\beta \nabla^2 \frac{\partial u}{\partial x} + (1+S-\beta)\frac{\partial^2 e}{\partial x^2} - \frac{\partial^2 T}{\partial x^2} = \frac{\partial^3 u}{\partial t^2 \partial x} - \Omega^2 \frac{\partial u}{\partial x}, \tag{13}$$

$$\beta \nabla^2 \frac{\partial w}{\partial z} + (1+S-\beta)\frac{\partial^2 e}{\partial z^2} - \frac{\partial^2 T}{\partial z^2} = \frac{\partial^3 w}{\partial t^2 \partial z} - \Omega^2 \frac{\partial w}{\partial z}. \tag{14}$$

From equations (13) and (14), we get

$$(1+S)\nabla^2 e - \nabla^2 T = \frac{\partial^2 e}{\partial t^2} - \Omega^2 e - 2\Omega \frac{\partial e}{\partial t} \tag{15}$$

The mean value of the normal stresses σ can be obtained as:

$$\sigma = \frac{\sigma_{xx} + \sigma_{zz}}{2} \tag{16}$$

From equations (12) and (16) by using the value of strain e, and the normal stresses σ, we get

$$\sigma = \alpha e - T \tag{17}$$

where

$$\alpha = \frac{(2-3\beta)}{2}.$$

Eliminating e from equations (11), (15) and (16), we obtain after simple calculations as follows:

$$(1+S)\nabla^2 \sigma + (1+S-\alpha)\nabla^2 T = \frac{\partial^2 T}{\partial t^2} + \frac{\partial^2 \sigma}{\partial t^2} - \frac{\Omega^2}{1+s}(T+\sigma) - 2\Omega \frac{\partial \sigma}{\partial t}, \tag{18}$$

$$\nabla^2 T = \left(\frac{\alpha + \varepsilon_T}{\alpha C_T^2}\right)\frac{\partial^2 T}{\partial t^2} + \frac{\varepsilon_T}{\alpha C_T^2}\frac{\partial^2 \sigma}{\partial t^2}. \tag{19}$$

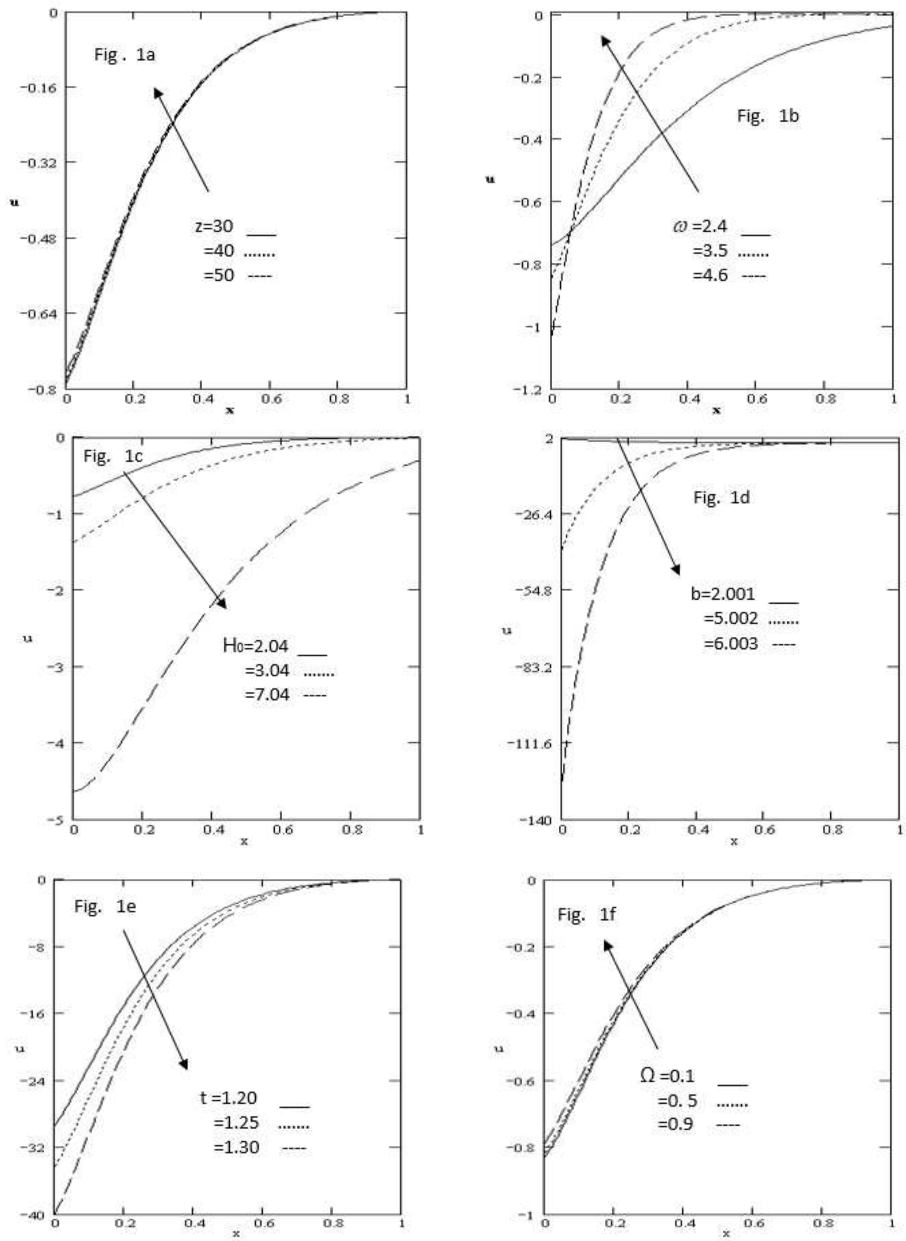

Fig. 1: Variations of z, H_0, b, t and Ω with respect to x on the displacement u

3 Normal mode analysis

Solution of the physical variables can be decomposed in terms of normal modes in the following form:

$$(u, w, e, T, \sigma_{ij})(x, z, t) = (u^*, w^*, e^*, T^*, \sigma_{ij}{}^*)(x)e^{(\omega t + ibz)} \tag{20}$$

where, $i = \sqrt{-1}$, ω is the angular frequency and b is the wave numbers in the z-direction.

Using equation (20), we can obtain the following equations from equations (18) and (19) respectively

$$\frac{d^2T^*}{dx^2} = C_1T^* + C_2\sigma^*, \tag{21}$$

$$\frac{d^2\sigma^*}{dx^2} = D_1T^* + D_2\sigma^* \tag{22}$$

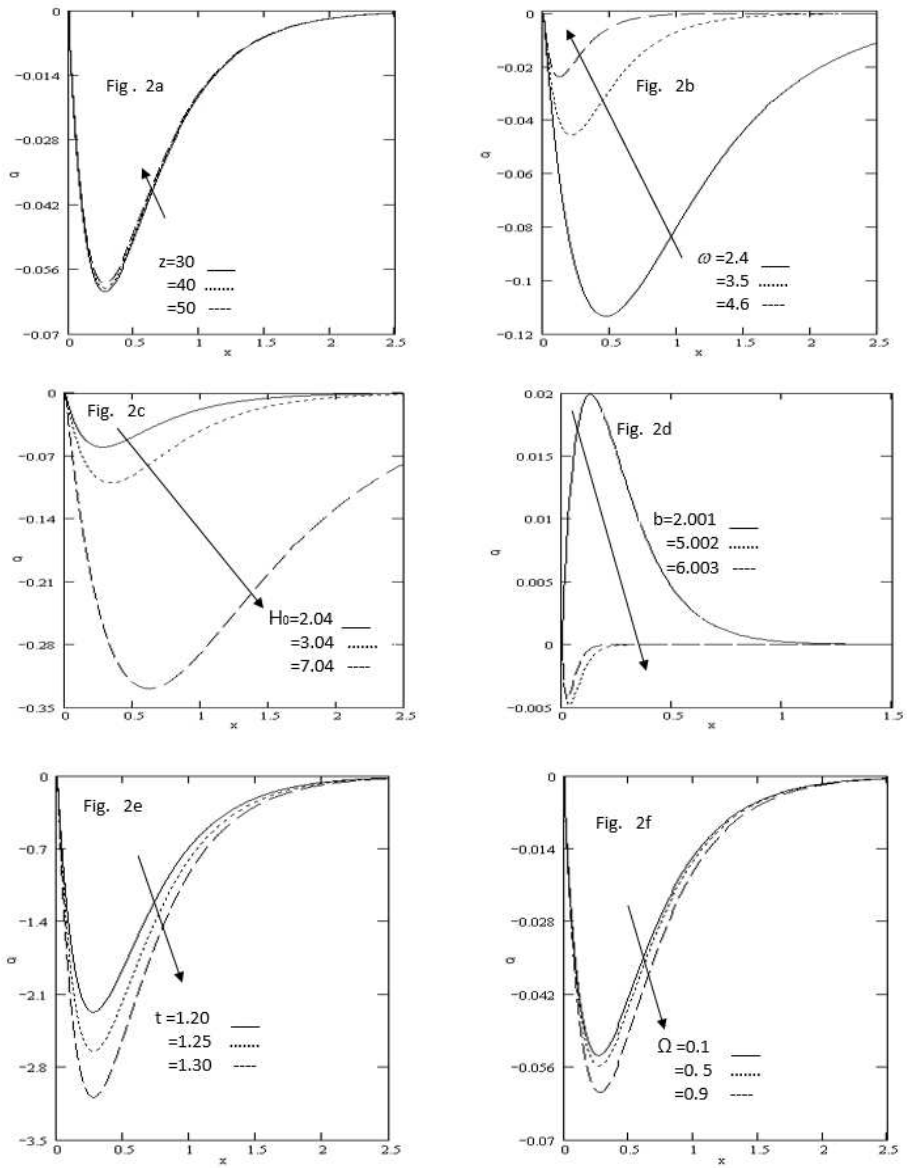

Fig. 2: Variations of z, ω, H_0, b, t and Ω with respect to x on the mean stresses σ

where

$$C_1 = \frac{1}{\alpha C_T^2}[\omega^2(\alpha+\varepsilon_T)+\alpha b^2 C_T^2],$$
$$C_2 = \frac{\omega^2\varepsilon_T}{\alpha C_T^2},$$
$$D_1 = \frac{\omega^2}{\alpha C_T^2(1+S)}[(1-\Omega^2+2\omega\Omega)\alpha C_T^2-(1+S-\alpha)(\alpha+\varepsilon_T)],$$
$$D_2 = \frac{1}{\alpha C_T^2(1+S)}[(1+S)(\alpha b^2 C_T^2-\omega^2\varepsilon_T)+\alpha\omega^2(C_T^2+\varepsilon_T)-(\Omega^2+2\Omega\omega)\alpha C_T^2]. \tag{23}$$

Equations (21) and (22) can be written in a vector-matrix differential equation as follows:

$$\frac{dV}{dx} = AV \tag{24}$$

where

$$\frac{d}{dx}\begin{pmatrix} T^* \\ \sigma^* \\ \frac{dT^*}{dx} \\ \frac{d\sigma^*}{dx} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ C_1 & C_2 & 0 & 0 \\ D_1 & D_2 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} T^* \\ \sigma^* \\ \frac{dT^*}{dx} \\ \frac{d\sigma^*}{dx} \end{pmatrix}$$

$$V = \begin{pmatrix} T^* \\ \sigma^* \\ \frac{dT^*}{dx} \\ \frac{d\sigma^*}{dx} \end{pmatrix}, \qquad A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ C_1 & C_2 & 0 & 0 \\ D_1 & D_2 & 0 & 0 \end{pmatrix} \tag{25}$$

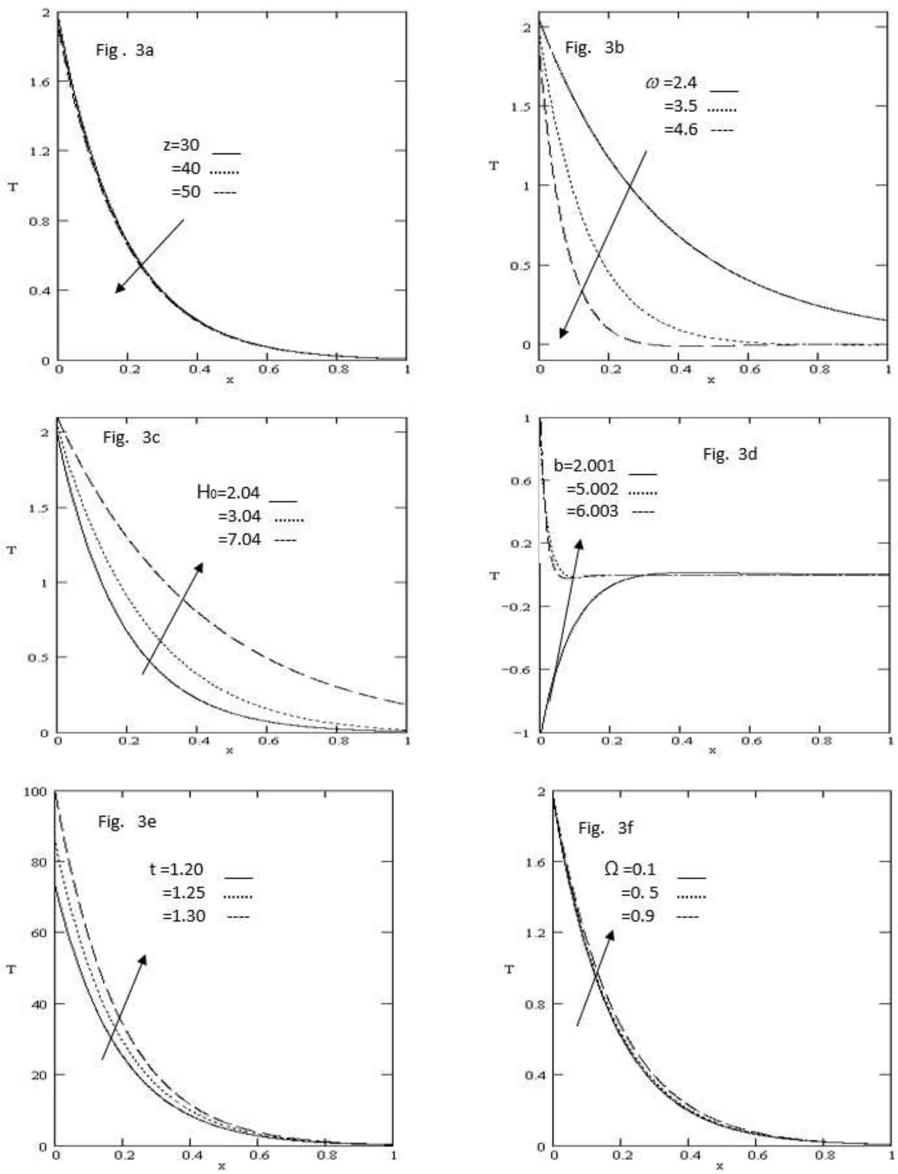

Fig. 3: Variations of z, ω, H_0, b, t and Ω with respect to x on the temperature T

4 Solution of the vector-matrix differential equation

Following the solution methodology through eigenvalue approach [31], we now proceed to solve the vector-matrix differential equation (24). The characteristic equation of a matrix A is

$$\lambda^4 - (C_1 + D_2)\lambda^2 + (C_1 D_2 - C_2 D_1) = 0. \qquad (26)$$

Let λ_1^2 and λ_2^2 are the roots of the above characteristic equation with positive real parts.Then all the four roots of the characteristic equation (26) which are also the eigenvalue of matrix A are of the form:

$$\lambda = \pm\lambda_1, \pm\lambda_2 \qquad (27)$$

where

$$\lambda_i = \frac{(C_1+D_2)+(-1)^{i+1}\sqrt{((C_1+D_2)^2-4(C_1D_2-C_2D_1))}}{2}, \qquad i = 1,2$$

The right eigenvector χ which it corresponds to the eigenvalue λ can be written as:

$$\chi = \begin{pmatrix} (\lambda^2 - D_2) \\ C_1 \\ \lambda(\lambda^2 - D_2) \\ \lambda C_1 \end{pmatrix} \qquad (28)$$

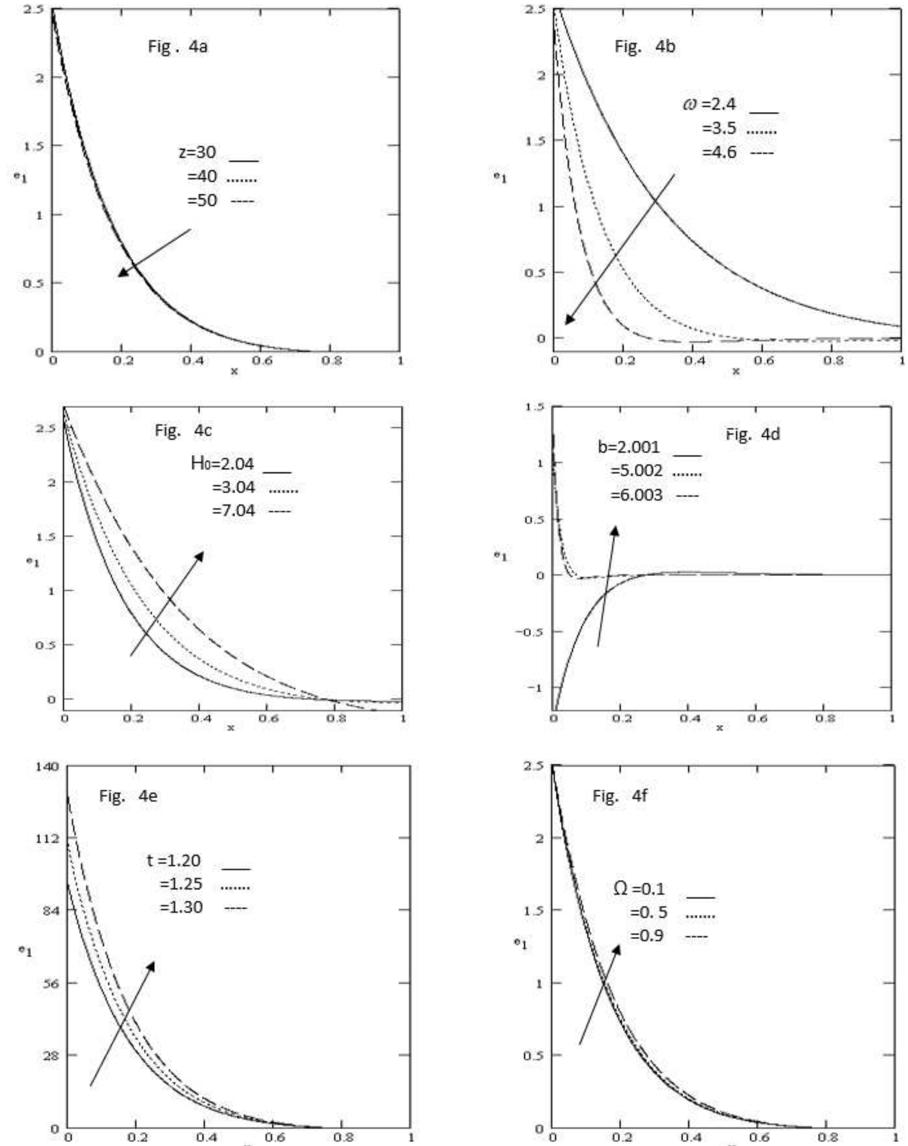

Fig. 4: Variations of z, ω, H_0, b, t and Ω with respect to x on the dilatational e_1

From Eq. (28) we can easily calculate the eigenvector $\chi_i (i = 1,2,3,4)$ corresponding to the eigenvalue $\pm\lambda_i$ $(i = 1, 2)$. For our further reference we shall use the following notations:

$$\chi_1 = [\lambda]_{\lambda=\lambda_1}, \quad \chi_2 = [\chi]_{\lambda=-\lambda_1}, \quad \chi_3 = [\chi]_{\lambda=\lambda_2}, \quad \chi_4 = [\chi]_{\lambda=-\lambda_2}. \tag{29}$$

The solution of equation (24) can be written as follows:

$$V = A_1\chi_2 e^{-\lambda_1 x} + A_2\chi_4 e^{-\lambda_2 x} \quad (x \geq 0), \tag{30}$$

where, the terms containing exponentials of growing naturally in the half-space variable x has been discarded due to the regularity condition of the solution at infinity and A_1, A_2 are constants to be determined by the boundary conditions of the problem.

Thus the field variables can be written from the equation (25), for $x \geq 0$ us:

$$T^*(x) = A_1(\lambda_1^2 - D_2)e^{-\lambda_1 x} + A_2(\lambda_2^2 - D_2)e^{-\lambda_2 x}, \tag{31}$$

$$\sigma^*(x) = C_1(A_1 e^{-\lambda_1 x} + A_2 e^{-\lambda_2 x}). \tag{32}$$

Also from the equation (17) on using equations (31) and (32), we get

$$e^*(x) = \frac{1}{\alpha}[A_1(\lambda_1^2 + C_1 - D_2)e^{-\lambda_1 x} + A_2(\lambda_2^2 + C_1 - D_2)e^{-\lambda_2 x}] \tag{33}$$

5 Application

In order to determine the constants A_1, A_2, we need to consider the following boundary conditions at the surface $x=0$:

(a)Free surface traction:

$$\sigma(0,z,t)=\sigma_{xx}(0,z,t)=\sigma_{zz}(0,z,t)=0 \tag{34a}$$

which gives

$$\sigma^*(x)=\sigma^*_{xx}(x)=\sigma^*_{zz}(x)=0 \quad at \quad x=0 \tag{34b}$$

(b)The thermal boundary condition is

$$q_n+\nu T=Q_0 e^{\omega t+ibz} \tag{35}$$

where q_n are the normal components of the heat flux vector, ν is Biot's number, and $Q_0 e^{\omega t+ibz}$ represents the intensity of the applied heat sources. In order to use the thermal boundary condition, we use the generalized Fourier's law of heat conduction in the non-dimensional form, namely

$$q_n=-\frac{\partial T}{\partial n} \tag{36}$$

From Eqs. (34), (35) and (21), we get

$$\nu T^*-\frac{dT^*}{dx}=Q_0 \quad at \quad x=0 \tag{37}$$

Using the boundary conditions (34) and (37) in equations (31) and (32) respectively, we get

$$A_1(\lambda_1+\nu)(\lambda_1^2-D_2)+A_2(\lambda_2+\nu)(\lambda_2^2-D_2)=Q_0$$
$$A_1+A_2=0$$

Solving the above system of equations by using Cramer's rule we obtain

$$A_1=\frac{Q_0}{\Delta} \quad and \quad A_2=\frac{-Q_0}{\Delta} \tag{38a}$$

where

$$\Delta=(\lambda_1-\lambda_2)[(\lambda_1+\lambda_2)(\nu+\lambda_1+\lambda_2)-\lambda_1\lambda_2-D_2] \tag{38b}$$

To get the displacement, we will use equation (13) and we can write it after using equation (20) as follows:

$$\left[\frac{d^2}{dx^2}-\lambda_u^2\right]u^*=\eta_1 e^{-\lambda_1 x}+\eta_2 e^{-\lambda_2 x} \tag{39}$$

where, $\omega_\beta=\frac{\omega^2}{\beta}$, $\Omega_\beta=\frac{\Omega^2}{\beta}$,$\lambda_u^2=\left(b^2+\omega_\beta+\Omega_\beta\right)$
and

$$\eta_i=\left(\frac{A\lambda_i}{\alpha\beta}\right)[(1+S-\beta)C_1+(1+S-\alpha-\beta)(\lambda_i^2-D_2)] \quad i=1,2$$

The solution of the ordinary differential equation (39) takes the form

$$u^*(x)=A_3 e^{-\lambda_u x}+\frac{\eta_1}{(\lambda_1^2-\lambda_u^2)}e^{-\lambda_1 x}+\frac{\eta_2}{(\lambda_2^2-\lambda_u^2)}e^{-\lambda_2 x} \tag{40}$$

where $\lambda_1^2\neq\lambda_2^2\neq\lambda_u^2$ and A_3 is a constant to be determined from boundary conditions (34).
From equations (12) and (20) after using equation (24) we have

$$\sigma^*_{xx}=2\beta\frac{du^*(x)}{dx}+\frac{(1-2\beta)}{\alpha}\sigma^*(x)+\left(\frac{(1-2\beta-\alpha)}{\alpha}\right)T^*(x). \tag{41}$$

Using the boundary conditions (34) in the above equation, we get

$$\frac{du^*(x)}{dx}=\left(\frac{\alpha+2\beta-1}{\alpha}\right)T^*(x) \quad at \quad x=0 \tag{42}$$

and hence from equations (31), (40) and (42), we get

$$A_3=\left[\frac{Q_0(1-\alpha-2\beta)(\lambda_1^2-\lambda_2^2)}{\alpha\Delta\lambda_u}-\frac{\eta_1\lambda_1}{\lambda_u(\lambda_1^2-\lambda_u^2)}-\frac{\eta_2\lambda_2}{\lambda_u(\lambda_2^2-\lambda_u^2)}\right] \tag{43}$$

where Δ is given by (38a).
The solutions of equations (31)-(33) and (40) by using (20) can be written as

$$T(x,z,t)=e^{\omega t}\cos(bz)[A_1(\lambda_1^2-D_2)e^{-\lambda_1 x}+A_2(\lambda_2^2-D_2)e^{-\lambda_2 x}, \tag{44}$$

$$\sigma(x,z,t)=C_1 e^{\omega t}\cos(bz)[A_1 e^{-\lambda_1 x}+A_2 e^{-\lambda_2 x}], \tag{45}$$

$$e(x,z,t)=\frac{e^{\omega t}\cos(bz)}{\alpha}[A_1(\lambda_1^2+C_1-D_2)e^{-\lambda_1 x}+A_2(\lambda_2^2+C_1-D_2)e^{-\lambda_2 x}, \tag{46}$$

$$u(x,z,t)=e^{\omega t}\cos(bz)[A_3 e^{-\lambda_u x}+\frac{\eta_1}{(\lambda_1^2-\lambda_u^2)}e^{-\lambda_1 x}+\frac{\eta_2}{(\lambda_2^2-\lambda_u^2)}e^{-\lambda_2 x}. \tag{47}$$

6 Numerical results and discussion

In order to illustrate the theoretical results obtained in the preceding sections, we now present some numerical results. The material chosen for this purpose physical data for which is given below [12]:

$\varepsilon_T=0.0168$, $\alpha=0.67$, $\beta=0.25$, $\omega=2$, $a=1.2$, $b=1.3$, $\nu=50$, $Q_0=100$, $C_T=2$.

Taking into consideration, z =1.09, t=0.03, μ =4.25, H_0=2.0, λ=20, b=0.006, ω=3.08.
Fig. (1a-1f) shows the variations of the non-dimensional values for displacement component u with respect to the axial x with a wide range $(0\leq x\leq 1)$ of different values of the axial z, frequency ω, magnetic field H_0, wave number b, time t and rotation Ω. It is observed that the displacement component increases with the increasing of

axial z, frequency and rotation, while it decreases with increasing of magnetic field, wave number and time as well it increases with increasing of the axial x. It is clear that all quantities have a non zero value in a bounded region of the plane. These results obey the physical properties of thermoelasticity.
Fig. (2a-2f) shows the variations of the non-dimensional values of the mean value of normal stresses σ with respect to the axial x with a wide range $(0 \leq x \leq 1)$ has oscillatory behavior in the whole range of axial x for different values of the axial z, frequency ω, magnetic field H_0, wave number b, time t and rotation Ω. It is observed that the mean value of normal stress increases with the increasing of axis z and frequency, while it decreases with increasing of the magnetic field, wave number, time and rotation as well it satisfied the boundary conditions and physical meaning. It is clear that all quantities have a non zero value in a bounded region of the plane.
Fig. (3a-3b) shows the variations of the non-dimensional values of temperature T with respect to the axial x with a wide range $(0 \leq x \leq 1)$ for different values of the axial z, frequency ω, magnetic field H_0, wave number b, time t and rotation Ω. It is observed that the temperature increases with the increasing of magnetic field, wave number, time and rotation, while it decreases with increasing of axis$z$ and frequency, as well it is shifting from the negative into positive gradually with the axial x under influence of the wave number. It is clear that all quantities have a non zero value in a bounded region of the plane.
Fig. (4a-4f) shows the variations of the non-dimensional values of dilatation e_1 with respect to the axial x with a wide range $(0 \leq x \leq 1)$ for different values of the axial z, frequency ω, magnetic field H_0, wave number b, time t and rotation Ω. It is observed that the dilatation increases with increasing of magnetic field, wave number, time and rotation, while it decreases with increasing of axis $z$and frequency, while it decreases with increasing of the axial z and frequency, as well it is shifting from the negative into positive gradually with the axial x under influence of the wave number. It is clear that all quantities have a non zero value in a bounded region of the plane.

7 Conclusion

The analysis of graphs permits us some concluding remarks:
1.The medium deforms due to the application of normal/thermal point source or uniformly distributed force/thermal source with rotation and a magnetic field which indicates the magneto-thermoelastic coupled effects with vacuum on physical quantities.
2.The temperature T has significant effect on the resulting quantities. The theory of Green and Naghdi II (thermoelasticity without energy dissipation) of magneto-thermoelasticity describes the behavior of the particles of elastic body more real than the theory of classical thermoelasticity.
3.The magnetic field and rotation play a significant role in the distribution of all the physical quantities. The parameters of all the physical quantities vary (increase or decrease).
4.The displacement component, the mean value of the normal stresses σ and the dilatation e show increases and decreases with increasing of the physical quantities with respect to x due to presence of magnetic field and rotation. 5.The results are graphically described for the medium of copper. The present theoretical results may provide interesting information for experimental scientists/researchers /seismologists working on this subject. 6.The nature of variations of all the studied physical quantities in the Green–Naghdi model II is very much similar to the nature of the variations of the field variables in the Lord–Shulman model of thermoelasticity, see Ezzat and Youssef [31] for details.
7.All the physical quantities satisfy the boundary conditions.8-The result provides a motivation to investigate conducting thermo-electric materials as a new class of applications thermo-electric solids. The result provides a motivation to investigate conducting thermo-electric materials as a new class of applications thermo-electric solids. The results presented in this paper should prove useful for researchers in material science, designers of new materials, physicists as well as for those working on the development of magneto-thermo-elasticity and in practical situations as in geophysics, optics, acoustics, geomagnetic and oil prospecting etc. The used methods in the present article are applicable to a wide range of problems in thermodynamics and thermoelasticity.

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El-Sayed Mohamed Abo-Dahab Khedary, Professor in Applied Mathematics (Continuum Mechanics), he was Born in Egypt-Sohag-El-maragha-Ezbet Bani-Helal in 1973. He has got Master in Applied Mathematics in 2001 from SVU, Egypt. He has got PHD in 2005 from Assiut University, Egypt. In 2012 he has got Assistant Professor Degree in Applied Mathematics. In 2017 he has got Professor Degree in Applied Mathematics. He works in elasticity, thermoelasticity, fluid mechanics, fiber-reinforced, magnetic field. He is the author or co-author of over 150 scientific publications in Science, Engineering, Biology, Geology, Acoustics, Physics, Plasma,..., etc. He is a reviewer of 62 an international Journals in solid mechanics and applied mathematics. His research papers have been cited in many articles and textbooks. He authored many books in mathematics. He obtained a lot of local and international awards in Science and Technology.

Abdelmooty Mohamed Abd-Alla, Professor in Applied Mathematics (Continuum Mechanics), he Born in Egypt-Assuit in 1956. Now he is working as professor of Mathematics, in faculty of Science, Sohag University, Egypt. His research interests include theory of elasticity, vibration, thermoelasticity and fluid mechanics. He is the author or co-author of over 160 scientific publications, reviewer of many international Journals in Solid Mechanics and Applied Mathematics. His research papers have been cited in many articles and textbooks. He authored many books in mathematics.

Emad E. Mahmoud was born in Sohag, Egypt, in 1975. He received his B. Sc. Degree in mathematics from south valley university 2004 with very good with honors degree, and his M. Sc from Sohag University, Sohag, Egypt 2007. He received his Ph.D. in dynamical systems, Sohag University, Sohag, Egypt 2010. He has published over 45 papers in international refereed journals and two books in Lambert Academic Publishing. The areas of interest are synchronization and control of nonlinear dynamical systems (complex systems) and nonlinear differential equations.