# Single Folding Cluster Potential for $P+{ }^{12} C$ Elastic Scattering 

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#### Abstract

The proton scattering from carbon has been analyzed within the framework, using the single folding optical model with a Gaussian shape of the effective alpha-nucleon interaction. In addition, the angular distributions of the differential cross-sections of the proton elastic scattering from ${ }^{12} C$ were analyzed, using the alpha-cluster structure of ${ }^{12} C$, where carbon has three atoms of helium and oxygen has four atoms of helium. Furthermore, we analyzed the $P+{ }^{12} C$ elastic scattering at twenty energies, ranging from 7 to 494 MeV . The Gaussian shape of the effective alpha-nucleon interaction was used at two values for the depth at 36.4 MeV with the range of $0.265 \mathrm{fm}^{-2}$ and 47.3 MeV with the range of $0.189 \mathrm{fm}^{-2}$. Thus, each of the two values for the depth succeeded to describe the proton scattering from carbon.


Keywords: Single Folding Model, Elastic Scattering, Nuclear Reactions, Proton Carbon Reactions

## 1 Introduction

We used the optical model to study the proton scattering, where the optical model is the most successful one of all nuclear models which were applied in order to understand the nucleus-nucleus interactions through the analysis of elastic scattering [1]. In this regard, a large number of studies have analyzed the proton scattering, such as M. A.Allam(2012) [2] and D. Abriolaet al. (2011) [3] for the proton elastic scattering.
We used the alpha-cluster model to generate the alpha-nucleus and alpha-nucleus single folding cluster potential, based on the alpha-nucleus interaction. In this model, we consider a nucleus of mass number $B$ composed of an integral number (m) of alpha-particles, i.e., $B=A m$. This model was successful to describe the angular distributions of the differential cross-sections of the proton elastic scattering from carbon, where we notice a consensus between the practical and the theoretical results. On the other hand, El-Azab Faridet al. (2001, 2006) [4,5], and recently Karakoe and Boztosun(2006) [6], have all employed the alpha-cluster structure of the interacting nuclei in the folding formalism, in order to generate the alpha-particle single folding optical potentials based on an appropriate alpha-interaction.

The proton scattering has been analyzed using the optical model with the Gaussian shape of the effective alpha-nucleon interaction. Thus, some people used the depth of 36.4 MeV for the effective alpha-nucleon interaction with a range of $0.265 \mathrm{fm}^{-2}$, and other people used the depth of 47.3 MeV for the effective alpha-nucleon interaction with a range $0.189 \mathrm{fm}^{-2}$; in this framework, we compared between the two values of the depth.

## 2 Theoretical Formalism

### 2.1 The Single Folding Cluster Model

When the nucleon (such as proton) collides with the nucleus, the nucleus-nucleus interaction could be written in the following form:

$$
\begin{equation*}
U(R)=-V(r)-i W(r)+V_{c}(r) \tag{1}
\end{equation*}
$$

where the $\mathrm{V}(\mathrm{r})$ and $\mathrm{W}(\mathrm{r})$ are the real and imaginary parts in the optical potential respectively; and $V_{c}(r)$ is the repulsive coulomb potential, where this potential is represented by the interaction of the charge of the

[^0]incident projectile with the charge distribution of protons in the target nucleus, and (r) is the separation between the centers of the two colliding nuclei.
When we analyze the nucleon-nucleus scattering by using the single folding model, we can study the Single Folding Optical (SFO) Potential in this model by folding the nuclear matter density of the target with the effective nucleon-nucleon interaction, see Fig. [1], the general form of this potential is given by:
\[

$$
\begin{equation*}
V_{o p t}(r, E)=\int d^{3} r_{1} \rho_{1}\left(r_{1}\right) V_{N-N}\left(s, \rho_{1}, E\right) \tag{2}
\end{equation*}
$$

\]

Where $\mathbf{S}=\mathbf{r}-\mathbf{r}_{1}, \rho_{1}(r)$ is the matter density distribution of the target and $V_{N-N}\left(S, \rho_{1}, E\right)$ is the effective nucleon? nucleon interaction. In this work, we used the cluster


Fig. 1: The Schematic representation for the SFC interaction.
model where this model considering the nucleus of the mass number B is consisted of an integral number of alpha-particles, i.e. $(B=4 \alpha)$, (such as $\left.{ }^{12} C=3 \alpha\right)$; therefore, we fold the effective a-nucleon interaction with $\alpha$-cluster distribution density of the target, in order to obtain the Single Folding Cluster (SFC) Potential; in this case, the potential take of the form is as follows:

$$
\begin{equation*}
V_{S F C}(r, E)=\int d^{3} r_{1} \rho_{c}\left(r_{1}\right) V_{\alpha-N}\left(S, \rho_{c}, E\right) \tag{3}
\end{equation*}
$$

Where $\rho_{c}\left(r_{1}\right)$ is the $\alpha$-cluster distribution density and $V_{\alpha-N}\left(S, \rho_{c}, E\right)$ is the effective alpha? nucleon interaction and it is given in a Gaussian form:

$$
\begin{equation*}
V_{\alpha-N}(S)=v_{0 \alpha N} \exp ^{-k s^{2}} \tag{4}
\end{equation*}
$$

Where $v_{0 \alpha N}$ is the depth and $k$ is the range parameter. In this framework the Gaussian shape of the effective $\alpha-\mathrm{N}$ interaction used at two values of the depth, describes each of the two values of the depth as shown in Table [1].

### 2.2 Alpha-Cluster Densities

In this work, we used the alpha ? cluster model to study the proton scattering from carbon, thus, we consider that

Table 1: The Parameters of the $\alpha-N$ Effective Interaction using the SFC Optical Potential.

| $v_{0 \alpha N} \mathrm{MeV}$ | $k\left(\mathrm{fm}^{-2}\right)$ | Ref |
| :--- | :--- | :--- |
| 36.4 | 0.2657 | $[7]$ |
| 47.3 | 0.1892 | $[8]$ |

Table 2: the Density parameters used in Eqs.(6),(7)and the root mean square radii in Eq.(9) where $\rho_{0}(M, \alpha, c) \mathrm{fm}^{-3}, \mathrm{w}(\gamma) \mathrm{fm}^{-2}$, $\mathrm{B}(\lambda)(\xi) \mathrm{fm}^{-2}$ and $\mathrm{Rms}(\mathrm{fm})$.

| NUCLEUS | $\rho_{0}(M, \alpha, c)$ | $\mathrm{w}(\gamma)$ | $\mathrm{B}(\lambda)(\xi)$ | Rms | Ref |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\rho_{M}$ for carbon | 0.1644 | 0.4988 | 2.407 | 2.407 | $[10]$ |
| $\rho_{c}$ for carbon | -1.644 | -1.7852 | 0.8003 | 1.912 | $[10]$ |
| $\rho_{M}$ for oxygen | 0.1317 | 0.6457 | 0.3228 | 2.64 | $[11]$ |
| $\rho_{c}$ for oxygen | -0.1286 | -1.4249 | 0.5973 | 2.199 | $[11]$ |
| $\rho_{\alpha}$ | 0.4229 | 0 | 0.7024 | 1.46 | $[11]$ |

carbon consists of $3 \alpha$ nucleuses. If the $\alpha$-cluster distribution density of the target is $\rho_{c}\left(r_{1}\right)$ and the alpha density is $\rho_{\alpha}(r)$ then, we can write the nuclear matter density distribution of the target nucleus in the following form:

$$
\begin{equation*}
\rho_{M}(r)=\int \rho_{c}\left(r_{1}\right) \rho_{\alpha}\left(\mathbf{r}-\mathbf{r}_{1}\right) d \mathbf{r}_{1} \tag{5}
\end{equation*}
$$

In addition, we used the Gaussian form for the target density and the a-density, thus, we can write them as follows:

$$
\begin{gather*}
\rho_{M}(r)=\rho_{0 M}\left(1+w r^{2}\right) \exp ^{-\beta r^{2}}  \tag{6}\\
\rho_{\alpha}(r)=\rho_{0 \alpha} \exp ^{-\lambda r^{2}} \tag{7}
\end{gather*}
$$

Where $w, \beta$ and $\lambda$ parameters are listed in Table [2]. $\rho_{0 M}$ and $\rho_{0 \alpha}$ can be determined by using the normalization condition as follows:

$$
\begin{equation*}
\int \rho(r) r^{2} d r=\frac{A}{4 \pi} \tag{8}
\end{equation*}
$$

In order to calculate the $\rho_{c}\left(r_{1}\right)$ from Eq. (4) we used the Fourier transform [9] , using Equations (5) and (6) in order to obtain it as follows:

$$
\begin{equation*}
\rho_{c}\left(r_{1}\right)=\rho_{0 c}\left(1+\gamma r_{1}^{2}\right) \exp ^{-\xi r_{1}^{2}} \tag{9}
\end{equation*}
$$

Where

$$
\begin{align*}
& \gamma=\frac{2 w \lambda^{2}}{[\eta(2 \eta-3 w)]}  \tag{10}\\
& \xi=\frac{\beta \lambda}{\eta}, \eta=\lambda-\beta \tag{11}
\end{align*}
$$

And $\rho_{0 c}$ is determined by using the normalization condition in Eq. (7) where $A=B$.

## 3 Procedure

We can study the Single Folding Cluster (SFC) Potential by analyzing the elastic scattering proton from carbon.

First, we calculated the SFC potential analytically by using the Eqs. (3),(4) and (9).We used the Fortran program for the result from the calculations; and the results from the Fortran program were checked by recalculating the potential numerically by the computer code DOLFIN [12],using Eq.(3) directly; and the parameters of the cluster density for the target and the effective $\alpha-\mathrm{N}$ interaction were taken from Tables [1] and [2]. Each of the two ways yielded identical results. Then, we fed the resulted potentials to the computer code HIOPTIM-94[3]; and through the HIOPTIM code, we obtained the differential cross-sections for the proton elastic scattering from carbon and oxygen; and the best fits can be obtained by minimizing the chi-square $x^{2}$ value, where:

$$
\begin{equation*}
\chi^{2}=\frac{1}{N} \sum_{i=1}^{N_{\sigma}}\left[\left[\frac{\sigma_{t h}\left(\theta_{i}\right)-\sigma_{e x}\left(\theta_{i}\right)}{\Delta \sigma_{e x}\left(\theta_{i}\right)}\right]\right]^{2} \tag{12}
\end{equation*}
$$

Where $\sigma_{e x}\left(\theta_{i}\right), \sigma_{t h}\left(\theta_{i}\right)$ are the experimental and theoretical differential cross sections, respectively, at the angle $\theta_{i}$, $\Delta \sigma_{e x}\left(\theta_{i}\right)$ is the error associated with $\sigma_{e x}\left(\theta_{i}\right)$ and $N_{\sigma}$ is the total number of $\theta_{i}$.

## 4 Results and Discussion

## 4.1 $P+{ }^{12} C$ Elastic Scattering

The differential cross-sections of proton from ${ }^{12} C$ have been measured over a wide range of energies, as $E=7$ [14], 14 [15], 17.8 [16], 21.1 [17], 22 [18], 30 [19], 30.4 [20], 40 [19], 50 [21], 59.5 [17], 69.5 [17],79.8 [17], 83.5 [17], 96 [22], 122 [23], 156 [24], 182.8 [25], 250 [26], 300 [27] and 494 [28] MeV. The Gaussian shape of $\alpha-\mathrm{N}$ effective interaction was used at the two values of the depth at 36.4 MeV with a range of $0.256 \mathrm{fm}^{-2}$ and 47.3 MeV with a range of $0.189 \mathrm{fm}^{-2}$, thus, we measured the differential cross-sections at these two values of the depth. The obtained values of the real normalization factor NR, the parameters of the imaginary parts of the SFC optical potential, the volume integrals, and the resulted reaction cross-sections are all listed in Tables [3] and [4] for the depth of 36.4 MeV of the $\alpha-\mathrm{N}$ effective interaction; in addition, Tables [5] and [6] display the same parameters, but for the depth of 47.3 MeV of the $\alpha-\mathrm{N}$ effective interaction.
The differential cross-sections are calculated by fitting our calculations with the experimental data; thus, Figures [2] and [3] display the best fit of the differential cross-sections for the protons scattered from ${ }^{12} \mathrm{C}$. Out of Fig.[2], we can see that the differential cross-sections give a good agreement at all energies, as $\mathrm{E}=7,14,17.8$. 21.1, $22,30,30.4,40,50$ and 59.5 MeV , with the experimental data; and they also give a good agreement between the two values of the depth of the effective $\alpha-\mathrm{N}$ interactions. This fitting is found at the large angels, where it reaches the following: $\theta_{C M}=170^{\circ}$. Fig.[3] also shows the best fit between the elastic scattering cross-sections for the two


Fig. 2: A Comparison between the differential cross sections of $P+{ }^{12} C$ for the two values of the depth of the $\alpha-\mathrm{N}$ effective interaction with the experimental data at $\mathrm{E}=7,14,17.8,21.1,22$, $30,30.4,40,50$ and 59.5 MeV .


Fig. 3: A Comparison between the differential cross sections of $P+{ }^{12} C$ for the two values of the depth of the $\alpha-\mathrm{N}$ effective interaction with the experimental data at $\mathrm{E}=69.5,79.8,83.8,96$, $122,156,182.8,250,300$ and 494 MeV .
values of the depth of the effective $\alpha-\mathrm{N}$ interactions, with the experimental data at $\mathrm{E}=69.5,79.8,83.5,96,122$, $156,182.8,250,300$ and 494 MeV . In addition, this fitting is also found even at the large angels reaching $\theta_{c M}=150^{\circ}$. From these results, we notice that when the $\alpha-\mathrm{N}$ effective interaction increased by $30 \%$ from 36.4 MeV to 47.3 MeV , and the range decreased by $20 \%$ from $0.265 \mathrm{fm}^{-2}$ to $0.189 \mathrm{fm}^{-2}$, each of the two values of the depth of the effective $\alpha-\mathrm{N}$ interaction gives a good agreement with the experimental data Figure [4] shows the relation between the reaction cross-section OR and the incident energy. Out of this figure, we see that each of the two values of the depth of the a-N effective interaction gives the same shape; and the obtained $\sigma_{R}$ shall be close to each other in the two cases. The dependence of $\sigma_{R}$ on the incident energy is shown in this figure. Figure [5] demonstrates the change between the imaginary volume

Table 3: The Real normalization factor NR, the parameters of the imaginary parts of the optical potential, the volume integral, the parameters of the spin ? orbit W-S potential and the reaction cross sections for $P+{ }^{12} C$ at the depth of the $\alpha-\mathrm{N}$ effective interaction equal 36.4 MeV , these experimental values are taken from Refs.[29, 30], where $J_{I}\left(\mathrm{MeV} \mathrm{fm}{ }^{3}\right)$ and $J_{S . O}\left(\mathrm{MeV} \mathrm{fm}{ }^{3}\right)$.

| E MeV | 7.0 | 14 | 17.8 | 21.1 | 22 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $V_{R} \mathrm{MeV}$ | 46.19 | 52.92 | 53.8 | 47.86 | 52.12 |
| $W_{S} \mathrm{MeV}$ | 0 | 0 | 8.082 | 6.131 | 5.237 |
| $W_{D} \mathrm{MeV}$ | 0.221 | 26.47 | 0 | 0 | 0 |
| $R_{I} \mathrm{fm}$ | 1.6 | 1.361 | 1.358 | 1.514 | 1.687 |
| $A_{I} \mathrm{fm}$ | 0.583 | 0.131 | 0.019 | 0.439 | 0.217 |
| USP MeV | 9.405 | 5.311 | 18.24 | 9.716 | 11.14 |
| WSP MeV | 0 | 0 | 0 | 0 | 0 |
| RSP fm | 0.868 | 1.038 | 1.206 | 1.182 | 1.175 |
| ASP fm | 0.300 | 0.174 | 0.115 | 0.348 | 0.187 |
| $R_{C} \mathrm{fm}$ | 1.25 | 1.25 | 1.2 | 1.25 | 1.25 |
| $\chi^{2}$ | 6.93 | 1.74 | 10.46 | 0.89 | 0.12 |
| $J_{I}$ | 7.85 | 142.5 | 547.0 | 103.3 | 108.6 |
| $J_{S . O}$ | 39.02 | 26.4 | 105.3 | 55 | 62.75 |
| $\sigma_{R} \mathrm{mb}$ | 139.2 | 469.3 | 401.8 | 452 | 439.6 |
| $\sigma_{\text {exp }} \mathrm{mb}$ |  |  |  |  |  |
| $J_{R} / V_{R}$ | 10.16 | 10.16 | 10.16 | 10.16 | 10.16 |
| E MeV | 30 | 30.4 | 40 | 50 | 59.5 |
| $V_{R} \mathrm{MeV}$ | 43.27 | 48.08 | 40.29 | 38.98 | 35.05 |
| $W_{S} \mathrm{MeV}$ | 7.605 | 6.991 | 0 | 0 | 2.629 |
| $W_{D} \mathrm{MeV}$ | 0 | 0 | 7.316 | 4.014 | 8.517 |
| $R_{I} \mathrm{fm}$ | 1.249 | 1.624 | 1.504 | 1.37 | 1.496 |
| $A_{I} \mathrm{fm}$ | 0.8 | 0.162 | 0.344 | 0.626 | 0.195 |
| USP MeV | 0.229 | 7.084 | 5.83 | 5.462 | 7.626 |
| WSP MeV | 0 | 0 | 0 | 0 | 0 |
| RSP fm | 0.75 | 1.257 | 0.95 | 0.996 | 0.743 |
| ASP fm | 0.36 | 0.653 | 0.4 | 0.425 | 0.39 |
| $R_{C} \mathrm{fm}$ | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| $\chi^{2}$ | 1.51 | 2.69 | 3.5 | 1.23 | 0.43 |
| $J_{I}$ | 110.4 | 127.8 | 129.4 | 117.3 | 356.0 |
| $J_{S . O}$ | 0.82 | 42.75 | 26.54 | 26.07 | 27.22 |
| $\sigma_{R} \mathrm{mb}$ | 413.9 | 411.4 | 372.8 | 343.9 | 294.9 |
| $\sigma_{\text {exp }} \mathrm{mb}$ | $412.2 \pm 10$ |  | $370 \pm 10$ | 342 | $293 \pm 12$ |
| $J_{R} / V_{R}$ | 10.61 | 10.16 | 10.16 | 10.16 | 10.16 |
|  |  |  |  |  |  |



Fig. 4: The Relation between the reaction cross section and incident energy for $P+{ }^{12} C$.

Table 4: The same parameters in Table [3] but at energies $\mathrm{E}=69.5$ to 494 MeV , these experimental values are taken from Refs.[29, 30].

| E MeV | 69.5 | 79.8 | 83.8 | 96 | 122 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $V_{R} \mathrm{MeV}$ | 34.398 | 38.3 | 32.942 | 33.597 | 25.88 |
| $W_{S} \mathrm{MeV}$ | 7.55 | 0 | 12.41 | 4.867 | 19.346 |
| $W_{D} \mathrm{MeV}$ | 0 | 8.83 | 0.261 | 8.014 | 0 |
| $R_{I} \mathrm{fm}$ | 1.503 | 1.08 | 1.08 | 0.892 | 1.019 |
| $A_{I} \mathrm{fm}$ | 0.517 | 0.57 | 0.75 | 0.54 | 0.557 |
| USP MeV | 7.987 | 6.57 | 3.788 | 3.052 | 2.005 |
| WSP MeV | 0 | 0 | 0 | 0 | 0 |
| RSP fm | 0.75 | 0.87 | 0.897 | 0.861 | 0.93 |
| ASP fm | 0.36 | 0.449 | 0.33 | 0.276 | 0.429 |
| $R_{C} \mathrm{fm}$ | 1.25 | 1.25 | 1.25 | 1.51 | 1.51 |
| $\chi^{2}$ | 2.49 | 3.002 | 0.84 | 5.68 | 15.01 |
| $J_{I}$ | 131.44 | 389.7 | 131.91 | 117.51 | 134.3 |
| $J_{S . O}$ | 28.73 | 27.7 | 16.28 | 12.63 | 8.94 |
| $\sigma_{R} \mathrm{mb}$ | 304.3 | 325.4 | 284.6 | 232.2 | 220 |
| $\sigma_{\text {exp }} \mathrm{mb}$ |  |  |  | 232.5 |  |
| $J_{R} / V_{R}$ | 10.157 | 10.16 | 10.158 | 10.16 | 10.16 |
| E MeV | 156 | 182.8 | 250 | 300 | 494 |
| $V_{R} \mathrm{MeV}$ | 18.3 | 19.146 | 17.726 | 26.353 | 36.254 |
| $W_{S} \mathrm{MeV}$ | 9.15 | 0 | 15 | 5.172 | 16.291 |
| $W_{D} \mathrm{MeV}$ | 0 | 7.09 | 0 | 0 | 0 |
| $R_{I} \mathrm{fm}$ | 1.383 | 1.074 | 1.3229 | 1.332 | 1.285 |
| $A_{I} \mathrm{fm}$ | 0.572 | 0.562 | 0.546 | 0.77 | 0.511 |
| USP MeV | 3.96 | 2.509 | 2.614 | 1.25 | 1.525 |
| WSP MeV | 0 | 0 | -3.071 | -1.659 | 0 |
| RSP fm | 0.945 | 0.983 | 0.989 | 1.11 | 1.23 |
| ASP fm | 0.406 | 0.411 | 0.475 | 0.429 | 0.292 |
| $R_{C} \mathrm{fm}$ | 1.51 | 1.25 | 1.51 | 1.51 | 1.51 |
| $\chi^{2}$ | 5.82 | 6.48 | 2.81 | 3.79 | 1.73 |
| $J_{I}$ | 186.1 | 118.23 | 192.26 | 83.59 | 188.24 |
| $J_{S . O}$ | 17.9 | 11.81 | 12.41 | 6.65 | 9.02 |
| $\sigma_{R} \mathrm{mb}$ | 217.1 | 186.6 | 220 | 107.8 | 174 |
| $\sigma_{\text {exp }} \mathrm{mb}$ |  |  |  | 220 |  |
| $J_{R} / V_{R}$ | 10.16 | 10.16 | 10.16 | 10.16 | 10.16 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

integrals with the incident energy for both two depths. Out of this Figure, we can see that each of the two values of the depth gives the same behaviour with the incident energy. Figure [6] shows the relation between the real depths $V_{R}$ of the SFC optical potential with the incident energy. At a low energy down to 200 MeV , we see that the $V_{R}$ decreased with the increased energy; and at a high energy up to 494 MeV , we see the $V_{R}$ increased with the increased energy. In addition, each of the two cases of the depth gives the same behaviour. As for the two cases of the depth, we see that the relation between the spin-orbit volume integral with the energy gives the same fitting as shown in Figure [7].

## 5 Conclusions

In this current study, we used the single folding cluster model to calculate the proton scattering from ${ }^{12} C$. This

Table 5: The real normalization factor NR, the parameters of the imaginary parts of the optical potential, the volume integral, the parameters of the spin ? orbit W-S potential and the reaction cross sections for $\mathrm{P}+{ }^{12} \mathrm{C}$ at the depth of the $\alpha-\mathrm{N}$ effective interaction equal 47.3 MeV , these experimental values are taken from Refs. [29,30].

| E MeV | 7.0 | 14 | 17.8 | 21.1 | 22 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $V_{R} \mathrm{MeV}$ | 46.11 | 31.73 | 37.5 | 36.51 | 38.83 |
| $W_{S} \mathrm{MeV}$ | 1.57 | 6.029 | 4.803 | 5.24 | 0.076 |
| $W_{D} \mathrm{MeV}$ | 0 | 0 | 0 | 0 | 29.1 |
| $R_{I} \mathrm{fm}$ | 1.25 | 1.923 | 1.725 | 1.32 | 1.51 |
| $A_{I} \mathrm{fm}$ | 0.318 | 0.043 | 0.45 | 0.569 | 0.073 |
| USP MeV | 28.69 | 16.89 | 15.81 | 17.93 | 19.76 |
| WSP MeV | 0 | 0 | 0 | 0 | 0 |
| RSP fm | 0.762 | 0.654 | 1.262 | 1.33 | 1.249 |
| ASP fm | 0.561 | 0.579 | 0.12 | 0.136 | 0.233 |
| $R_{C} \mathrm{fm}$ | 1.25 | 1.25 | 1.2 | 1.25 | 1.25 |
| $c h i^{2}$ | 3.22 | 21.7 | 5.69 | 0.045 | 1.74 |
| $J_{I}$ | 14.76 | 179.7 | 634.6 | 68.17 | 657.0 |
| $J_{S . O}$ | 106.1 | 54.39 | 95.6 | 113.0 | 118.1 |
| $\sigma_{R} \mathrm{mb}$ | 304.9 | 688 | 532.8 | 383.9 | 404.5 |
| $\sigma_{\text {exp }} \mathrm{mb}$ |  |  |  |  |  |
| $J_{R} / V_{R}$ | 16.91 | 16.91 | 16.91 | 16.9 | 16.91 |
| E MeV | 30 | 30.4 | 40 | 50 | 59.5 |
| $V_{R} \mathrm{MeV}$ | 27.71 | 28.23 | 24.12 | 24.69 | 23.46 |
| $W_{S} \mathrm{MeV}$ | 7.17 | 10.86 | 10 | 0 | 0 |
| $W_{D} \mathrm{MeV}$ | 0 | 0 | 0 | 3.143 | 3.885 |
| $R_{I} \mathrm{fm}$ | 1.287 | 1.25 | 1.475 | 1.47 | 1.14 |
| $A_{I} \mathrm{fm}$ | 0.8 | 0.562 | 0.261 | 0.591 | 0.638 |
| USP MeV | 13.95 | 8.671 | 16.65 | 8.45 | 8.76 |
| WSP MeV | 0 | 0 | 0 | 2.341 | 0 |
| RSP fm | 0.493 | 0.631 | 0.75 | 0.589 | 0.511 |
| ASP fm | 0.36 | 0.019 | 0.36 | 0.335 | 0.362 |
| $R_{C} \mathrm{fm}$ | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| chi | 7.05 | 6.55 | 4.43 | 4.24 | 0.86 |
| $J_{I}$ | 110.8 | 122.7 | 142.4 | 417.5 | 396.5 |
| $J_{S . O}$ | 33.4 | 26.78 | 59.9 | 23.98 | 21.7 |
| $\sigma_{R} \mathrm{mb}$ | 419 | 412.2 | 363.4 | 303.2 | 244.4 |
| $\sigma_{\text {exp }} \mathrm{mb}$ | $412.2 \pm 10$ |  | $370.0 \pm 10$ | 342 | $293.0 \pm 12$ |
| $J_{R} / V_{R}$ | 16.92 | 16.91 | 16.91 | 16.9 | 16.92 |



Fig. 5: The Energy dependence of the Imaginary Volume Integral of the depth 36.4 MeV and 47.3 MeV .

Table 6: The same parameters in Table [5] at energies $\mathrm{E}=69.5$ to 494 MeV , these experimental values are taken from Refs.[29, 30].

| E MeV | 69.5 | 79.8 | 83.8 | 96 | 122 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $V_{R} \mathrm{MeV}$ | 24.359 | 22.656 | 17.1699 | 21.332 | 18.494 |
| $W_{S} \mathrm{MeV}$ | 10.04 | 9.394 | 0 | 9.215 | 12.666 |
| $W_{D} \mathrm{MeV}$ | 0 | 0 | 6.734 | 0 | 0 |
| $R_{I} \mathrm{fm}$ | 1.493 | 1.493 | 1.25 | 1.434 | 1.216 |
| $A_{I} \mathrm{fm}$ | 0.367 | 0.367 | 0.487 | 0.365 | 0.532 |
| USP MeV | 9.553 | 7.731 | 8.555 | 6.303 | 2.333 |
| WSP MeV | 0 | 0 | 0 | 0 | -2.454 |
| RSP fm | 0.864 | 0.869 | 1.024 | 0.835 | 0.944 |
| ASP fm | 0.512 | 0.464 | 0.554 | 0.37 | 0.473 |
| $R_{C} \mathrm{fm}$ | 1.25 | 1.25 | 1.25 | 1.25 | 1.51 |
| $\chi^{2}$ | 0.609 | 0.723 | 0.861 | 5.87 | 15.69 |
| $J_{I}$ | 155.89 | 145.85 | 123.36 | 127.79 | 129.89 |
| $J_{S . O}$ | 39.74 | 32.27 | 42.1 | 25.23 | 10.57 |
| $\sigma_{R} \mathrm{mb}$ | 323.8 | 293.6 | 269.7 | 245.8 | 218 |
| $\sigma_{\text {exp }} \mathrm{mb}$ |  |  |  | 232.5 |  |
| $J_{R} / V_{R}$ | 16.91 | 16.91 | 16.91 | 16.91 | 16.92 |
| E MeV | 156 | 182.8 | 250 | 300 | 494 |
| $V_{R} \mathrm{MeV}$ | 14.71 | 11.683 | 11.115 | 15.23 | 17.595 |
| $W_{S} \mathrm{MeV}$ | 10.04 | 14.385 | 21.682 | 12.506 | 23.045 |
| $W_{D} \mathrm{MeV}$ | 0 | 0 | 0 | 0 | 0 |
| $R_{I} \mathrm{fm}$ | 1.36 | 1.21 | 1.134 | 1.265 | 1.25 |
| $A_{I} \mathrm{fm}$ | 0.46 | 0.485 | 0.483 | 0.264 | 0.307 |
| USP MeV | 4.229 | 3.056 | 2.708 | 0.899 | 0.662 |
| WSP MeV | 0 | 0 | -2.299 | -2.667 | 0 |
| RSP fm | 0.93 | 0.93 | 0.914 | 0.953 | 1.073 |
| ASP fm | 0.492 | 0.429 | 0.463 | 0.471 | 0.202 |
| $R_{C} \mathrm{fm}$ | 1.51 | 1.51 | 1.51 | 1.51 | 1.51 |
| $\chi^{2}$ | 3.73 | 2.92 | 2.25 | 1.03 | 2.22 |
| $J_{I}$ | 128.83 | 139.27 | 177.95 | 114.86 | 210.25 |
| $J_{S . O}$ | 18.85 | 13.62 | 11.88 | 4.11 | 3.4 |
| $\sigma_{R} \mathrm{mb}$ | 203.5 | 195 | 195.8 | 124.2 | 171.5 |
| $\sigma_{\text {exp }} \mathrm{mb}$ | 220 |  |  |  |  |
| $J_{R} / V_{R}$ | 16.89 | 16.91 | 16.88 | 16.9 | 16.9 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |



Fig. 6: The Energy dependence of the real depths of the SFC optical potential.


Fig. 7: The Energy dependence of the spin-orbit volume integral of the SFC optical potential.
analysis was based on folding the cluster density distribution with the $\alpha-\mathrm{N}$ effective interaction. We used two values of the depth of the $\alpha$-N effective interaction as follows: 36.4 MeV with a range of $0.265 \mathrm{fm}^{-2}$, and 47.3 MeV with a range of $0.189 \mathrm{fm}^{-2}$. From these results, we notice that when the $\alpha$-N effective interaction increased by $30 \%$ from 36.4 MeV to 47.3 MeV , and the range decreased by $20 \%$ from $0,265 \mathrm{fm}^{-2}$ to $0.189 \mathrm{fm}^{-2}$, each of the two values of the depth of the effective $\alpha-\mathrm{N}$ interaction gave a good agreement with the experimental data. The predictions were successful similar to or some time better than those obtained by previous phenomenological and microscopic potential analyses. In addition, each of the two values of the depth gives a good agreement with the experimental data; and the resulted differential cross-sections are similar to or some time better than those obtained by previous works for all energies each of the reactions $P+{ }^{12} C$.

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