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Analysis of Lifetime Performance Index with Compound Rayleigh Distribution in Presence of Progressive First-Failure-Censoring Scheme

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Abstract: In service (or manufacturing) industries the lifetime performance assessment is important, hence, the lifetime performance index C_L is used to measure the potential and performance of a process, where L is the lower specification limit. In this paper, we study different estimators of C_L under the compound Rayleigh distribution with general censored scheme called progressively first-failure-censored scheme, which is quite useful in many practical situations. The results in the cases of first-failure censoring, progressive Type II censoring, Type II censoring and complete sample are a special cases. The study will apply data transformation technology to constructs a maximum likelihood (ML) and Bayes estimators of C_L under the compound Rayleigh distribution based on the progressively first-failure-censored sample with assuming the conjugate prior distribution and squared-error loss function. ML and Bayes estimators of C_L are utilized to develop the new hypothesis testing algorithmic procedure in the condition of known L. Finally, we give two examples and the Monte Carlo simulation to assess the behavior of confidence interval for the lifetime performance index C_L under given confidence level γ.

Keywords: Compound Rayleigh distribution; Lifetime performance index; Progressive first-failure-censoring; Maximum likelihood estimator; Bayes estimators: Monte Carlo simulation.

1 Introduction

Process capability analysis is an effective means of measuring process performance and potential capability. In the service (or manufacturing) industry, process capability indices are utilized to assess whether product quality meets the required level. In practice, lifetime performance index C_L is used as a means of measuring business performance, where L is the lower specification limit. The purchasers can then employ the testing procedure to determine whether the lifetime of electronic components adheres to the required level. Manufacturers can also utilize this procedure to enhance process capability. For more details see for example Montgomery [1] and Kane [2] proposed the process capability index C_L for evaluating the lifetime performance of electronic components. Hong et al. [3,4] discussed the performance assessment of lifetime index of Pareto lifetime businesses based on confidence interval and service industries, Lee et al. [5] assessed the lifetime performance index of Rayleigh products based on the Bayesian estimation under progressive Type II right censored samples. The theoretical and practical results on the the process capability index relationships in industrial and economic systems during the last decades are collected and digested in Tong et al. [6], Hong et al. [7], Lee et al. [8] and Chen et al. [9]. Therefore, some incomplete data could be collected, such as progressive type I censoring (see Wu and Lin [10,11]). Wei et al. [12] estimated Multi-parameter in semitransparent graded-index media based on coupled optical and thermal information.

There are many situations in life-testing and reliability studies in which the experimenter may be unable to obtain complete information on failure times of all experimental items. There are also situations wherein the removal of items prior to failure is pre-planned in order to reduce the cost and time associated with testing. The most common censoring schemes are Type-I and Type-II censoring, but the conventional Type-I and Type-II censoring schemes do not have the flexibility of allowing removal of items at points other than the terminal point of the experiment. A generalization of

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Type-II censoring is the progressive Type-II censoring which allows for units to be removed from the test at points other than the final termination point. Inference, sampling design and generalization based on progressively censored samples were studied by Balakrishnan and Aggarwala [13], Balakrishnan et al. [14], Asgharzadeh [15], Wu et al. [16] and Wu et al. [17]. Johnson [18] described a life test in which the experimenter might decide to group the test units into several sets, each as an assembly of test units, and then run all the test units simultaneously until occurrence the first failure in each group. Such a censoring scheme is called a first-failure censoring scheme. If an experimenter desires to remove some sets of test units before observing the first failures in these sets this life test plan is called a progressive first-failure-censoring scheme (first-failure censoring scheme is combined with progressive censoring scheme) which introduced by Wu and Kuş [19].

Suppose that n independent groups with k items within each group are put in a life test, R_1 groups and the group in which the first failure is observed are randomly removed from the test as soon as the first failure (say $X^{\mathbf{R}}_{1:m:n:k}$) has occurred, R_2 groups and the group in which the second failure is observed are randomly removed from the test as soon as the second failure (say $X^{\mathbf{R}}_{2:m:n:k}$) has occurred, and finally R_m $(m \le n)$ groups and the group in which the $m-th$ failure is observed are randomly removed from the test when the $m-th$ failure (say $X^{\mathbf{R}}_{m:m:n:k}$) has occurred. The observations $X^{\mathbf{R}}_{1:m:n:k} < X^{\mathbf{R}}_{2:m:n:k} < \ldots < X^{\mathbf{R}}_{m:m:n:k}$ are called progressively first-failure-censored order statistics with progressive censoring scheme $\mathbf{R} = (R_1, R_2, \ldots, R_m)$. It is clear that m is the number of the first failure observed $(1 < m \le n)$ and $n = m + R_1 + R_2 + \ldots + R_m$. If the failure times of the $n \times k$ items originally in the test are from a continuous population with distribution function $F(x)$ and probability density function $f(x)$, the joint probability density function for $X^{\mathbf{R}}_{1:m:n:k}, X^{\mathbf{R}}_{2:m:n:k}, \ldots, X^{\mathbf{R}}_{m:m:n:k}$ is given by

$$f_{1,2,\ldots,m}(x^{\mathbf{R}}_{1:m:n:k}, x^{\mathbf{R}}_{2:m:n:k}, \ldots, x^{\mathbf{R}}_{m:m:n:k}) = Ck^m \prod_{j=1}^{m} f(x^{\mathbf{R}}_{j:m:n:k})(1 - F(x^{\mathbf{R}}_{j:m:n:k}))^{k(R_j+1)-1} \tag{1}$$

$$0 < x^{\mathbf{R}}_{1:m:n:k} < x^{\mathbf{R}}_{2:m:n:k} < \ldots < x^{\mathbf{R}}_{m:m:n:k} < \infty,$$

where

$$C = n(n - R_1 - 1)(n - R_1 - R_2 - 1)\ldots(n - R_1 - R_2 - \ldots R_{m-1} - m + 1). \tag{2}$$

Special cases

It is clear from (1) that the progressive first-failure censored scheme containing the following censoring schemes as special cases:

1.The first-failure censored scheme when $\mathbf{R} = (0, 0, \ldots, 0)$.
2.The progressive type II censored order statistics if $k = 1$.
3.Usually type II censored order statistics when $k = 1$ and $\mathbf{R} = (0, 0, \ldots, n - m)$.
4.The complete sample case when $k = 1$ and $\mathbf{R} = (0, 0, \ldots, 0)$.

Also, It should be noted that $X^{\mathbf{R}}_{1;m,n,k}, X^{\mathbf{R}}_{2;m,n,k}, \ldots, X^{\mathbf{R}}_{m;m,n,k}$ can be viewed as a progressive type II censored sample from a population with distribution function $1 - (1 - F(x))^k$. For this reason, results for progressive type II censored order statistics can be extend to progressive first-failure censored order statistics easily. Also, the progressive first-failure-censored plan has advantages in terms of reducing the test time, in which more items are used, but only m of $n \times k$ items are failures.

For more application about progressive-first-failure censoring data the readers may refer to Soliman et al. [20], Soliman et al. [21,22,23], Modhesh [24], Modhesh and Abd-Elmoudod [25] and Ahmadi et al. [26].

The two-parameter compound Rayleigh distribution (which is denoted by CRD(α, β) provides a population model which is useful in several areas of statistics, including life testing and reliability. The probability density function *pdf* and the cumulative distribution function *cdf* of CRD(α, β) are given, respectively, by

$$f(x) = 2\alpha\beta^{\alpha} x(\beta + x^2)^{-(\alpha+1)},\ x > 0,\ (\beta,\ \alpha > 0), \tag{3}$$

$$F(x) = 1 - (1 + \frac{x^2}{\beta})^{-\alpha},\ x > 0, \tag{4}$$

and the reliability and failure rate functions, at some t, are

$$S(t) = (1 + \frac{t^2}{\beta})^{-\alpha},\ t > 0, \tag{5}$$

$$H(t) = \frac{2\alpha t}{\beta + t^2},\ t > 0, \tag{6}$$

where α and β are the shape and scale parameter respectively.

The main aim of this paper is to construct a ML and Bayes estimator of C_L under the compound Rayleigh distribution with progressively first-failure-censored sample. These estimators of C_L is then utilized to develop a confidence and credible intervals of C_L. These intervals can be employed by managers to assess whether the product performance adheres to the required level in the condition of known L. In addition, a Bayesian test (also see Casella and Berger [27] and Lanping [28]) is also used to determine whether the product performance adheres to the required level, the organization of this paper is as follows. In Section 2, we introduce some properties of the lifetime performance index C_L when the lifetime of the products is coming from the compound Rayleigh distribution and we discussed the relationship between the lifetime performance index C_L and the conforming rate (the ratio of conforming products). In Section 3, we investigated the ML and Bayes estimators of the lifetime performance index and its statistical properties. Section 4 develops a lower bound for the lifetime performance index C_L. Two illustrative examples are analyzed in Section 5. In Section 6, Sensitivity study via a Monte Carlo method are conducted. Some concluding remarks are finally made in Section 7.

2 The Lifetime Performance Index

Process capability analysis is utilized to assess the non-normal quality data under a specific non-normal distribution. Hence, the lifetime performance index (or larger-the-better process capability index) C_L is also utilized to measure product quality with the CRD(α,β). Let X denote the lifetime of such a product and X has the CRD(α,β) with the *pdf* is given as (3). Clearly, a longer lifetime implies a better product quality. Hence, the lifetime is a larger-the-better type quality characteristic. The lifetime is generally required to exceed L unit times to both be economically profitable and satisfy customers. Montgomery [1] developed a capability index C_L for properly measuring the larger-the-better quality characteristic. C_L is defined as follows:

$$C_{L_x} = \frac{\mu - L}{\sigma}, \tag{7}$$

where the process mean μ, the process standard deviation σ, and L is the lower specification limit.

To assess the lifetime performance of products, C_{L_x} can be defines as the lifetime performance index. Under X has the CRD(α,β) and the data transformation $Y = \log(1+\frac{X^2}{\beta})$, $\beta > 0$, the distribution of Y is a exponential distribution. Hence, the *pdf* and *cdf* of Y are

$$f_Y(y;\alpha) = \alpha\exp(-\alpha y), \quad y > 0, \alpha > 0, \tag{8}$$

and

$$F_Y(y;\alpha) = 1 - \exp(-\alpha y), \quad y > 0, \alpha > 0, \tag{9}$$

respectively. Moreover, there are several important properties, as follows:

The lifetime performance index C_L can be rewritten as

$$C_L = \frac{\mu - L}{\sigma} = \frac{\frac{1}{\alpha} - L}{\frac{1}{\alpha}} = 1 - \alpha L, \quad -\infty < C_{L_y} < 1, \tag{10}$$

where the process mean $\mu = E(Y) = 1l\alpha$, the process standard deviation $\sigma = \sqrt{Var(Y)} = 1l\alpha$, and L is the lower specification limit.

The failure rate function $H_Y(y)$ is defined by

$$H_Y(y) = \frac{f_Y(y,\alpha)}{1 - F_Y(y,\alpha)} = \frac{\alpha\exp(-\alpha y)}{1 - [1 - \exp(-\alpha y)]} = \alpha, \quad \alpha > 0. \tag{11}$$

The important properties can be determined by using logarithmically transformed data $Y = \log(1+\frac{X^2}{\beta})$, $\beta > 0$. Since, the logarithmic transformation $Y = \log(1+\frac{X^2}{\beta})$, $\beta > 0$ is one-to-one and strictly increasing, so data set of X and transformed data set of Y have the same effect in assessing the business performance of businesses. Moreover, the logarithmic transformation $Y = \log(1+\frac{X^2}{\beta})$, $\beta > 0$ enables the calculation of important properties to be easy. When the mean $1l\alpha$ $(> L)$, then the lifetime performance index $C_L > 0$. From Eqs. (7) and (8), we can see that the larger the mean $1l\alpha$, the smaller the failure rate and the lager the lifetime performance index C_L. Therefore, the lifetime performance index C_{L_y} reasonably and accurately represents the business performance of new businesses.

By the transformation $Y = \log(1+\frac{X^2}{\beta})$, $\beta > 0$, and the distribution of Y has a one-parameter exponential distribution

Table 1: The lifetime performance index C_{L_y} versus the conforming rate P_r for $\beta = 0.37$

C_L	P_r	C_L	P_r	C_L	P_r	C_L	P_r	C_L	P_r
$-\infty$	0.00000	-1.7	0.06721	-1.0	0.13534	-0.3	0.27253	0.4	0.54881
-2.3	0.03688	-1.6	0.07427	-0.9	0.14957	-0.2	0.30119	0.5	0.60653
-2.2	0.04076	-1.5	0.08208	-0.8	0.16530	-0.1	0.33287	0.6	0.67032
-2.1	0.04505	-1.4	0.09072	-0.7	0.18268	0.0	0.36788	0.7	0.74082
-2.0	0.04979	-1.3	0.10026	-0.6	0.20190	0.1	0.40657	0.8	0.81873
-1.9	0.05502	-1.2	0.11080	-0.5	0.22313	0.2	0.44933	0.9	0.90484
-1.8	0.06081	-1.1	0.12246	-0.4	0.24660	0.3	0.49659	1.0	1.00000

with the *pdf* Eq. (8) and cumulative distribution function *cdf* as (9). If the new lifetime of a product exceeds the lower specification limit (i.e. $Y \geq L$) then the product is labeled as a conforming product. Otherwise, the product is labeled as a non-conforming product. The conforming rate can be defined as

$$P_r = P(Y \geq L) = \exp(-\alpha L) = \exp(C_L - 1), \quad -\infty < C_L < 1. \tag{12}$$

Obviously, a strictly increasing relationship exists between conforming rate P_r and the lifetime performance index C_L. Thus, the larger the index value C_L, the larger conforming rate P_r, for given $\beta > 0$. For example, if $\beta = 0.37$, then Table 1 lists various values of C_{L_y} and the corresponding conforming rates P_r. For given $\beta = 0.37$ and the C_{L_y} values which are not listed in Table 1, the conforming rate P_r can be calculated by Eq. (12).

3 Estimation of Lifetime Performance Index

In this section, we first estimate the parameter by considering the maximum likelihood (ML) methods, and then we describe how to obtain the Bayes estimates and the corresponding credible intervals of parameter α when β is known.

3.1 Maximum likelihood estimator of lifetime performance index

Let $X^{\mathbf{R}}_{i:m:n:k}$, $i = 1, 2, \cdots, m$ be the progressive first-failure censored sample from a continuous population with (pdf) and (cdf) given by $f(.)$ and $F(.)$, respectively. Following upon substituting (3) and (4) into (1), and x_i is used instead of $X^{\mathbf{R}}_{i:m:n:k}$. The likelihood function may then be written as

$$L(\alpha, \beta|\mathbf{x}) = Ck^m \alpha^m \prod_{i=1}^{m} x_i \beta^{\alpha k(R_i+1)} (\beta + x_i^2)^{-\alpha k(R_i+1)-1}. \tag{13}$$

For known β, the ML estimator of α is readily derived from (13) as

$$\hat{\alpha} = \frac{m}{W}, \tag{14}$$

where

$$W = k \sum_{i=1}^{m} (R_i + 1) \log(1 + \frac{x_i^2}{\beta}). \tag{15}$$

By using the invariance property of ML estimators, the ML estimator of C_L is given by

$$\hat{C}_L = 1 - \hat{\alpha} L = 1 - \frac{mL}{W}. \tag{16}$$

Theorem 1. Let $X^{\mathbf{R}}_{i;m,n}$, $i = 1, 2, ..., m$ be an progressive first-failure censored order statistic from two-parameter compound Rayleigh distribution (3) with censored scheme $\mathbf{R}$. Then

$$2\alpha W \sim \chi^2_{(2m)}, \tag{17}$$

where W given by (15)

Proof. let Let $Z_i = \alpha \log(1+\frac{X_i^2}{\beta})$ It can be seen that $Z^{\mathbf{R}}_{1;m,n,k} < Z^{\mathbf{R}}_{2;m,n,k} < ... < Z^{\mathbf{R}}_{m;m,n,k}$ is progressive first-failure-censored order statistic from an standard exponential distribution. Consider the following transformations

$$\begin{cases} \varpi_1 = nZ^{\mathbf{R}}_{1;m,n,k}, \\ \varpi_2 = (n-R_1-1)\left(Z^{\mathbf{R}}_{2;m,n,k} - Z^{\mathbf{R}}_{1;m,n,k}\right), \\ \vdots \\ \varpi_m = (n-R_1-R_2-...-R_{m-1}-m+1)\left(Z^{\mathbf{R}}_{m;m,n,k} - Z^{\mathbf{R}}_{m-1;m,n,k}\right) \end{cases} \tag{18}$$

The generalized spacings $\varpi_1, \varpi_2, ..., \varpi_m$ are independent and identically distributed as standard exponential distribution, see Tomas and Wilson [29]. Hence, $2(\varpi_1+\varpi_2+...+\varpi_m) \sim \chi^2_{(2m)}$,where

$$2(\varpi_1+\varpi_2+...+\varpi_m) = 2\alpha k\sum_{i=1}^{m}(R_i+1)\log(1+\frac{x_i^2}{\beta}) = 2\alpha W, \tag{19}$$

Remark 1. The expectation of $\hat{C}_L$ can be derived as follows

$$\mathrm{E}(\hat{C}_L) = \mathrm{E}\left(1-\frac{mL}{W}\right) = 1-2\alpha mLE\left(\frac{1}{2\alpha W}\right) = 1-\frac{\alpha mL}{m-1} \tag{20}$$

The ML estimator $\hat{C}_L$ is not an unbiased estimator of C_L. But when $m \longrightarrow \infty$, $\mathrm{E}(\hat{C}_L) \longrightarrow C_L$, so the ML estimator $\hat{C}_L$ is asymptotically unbiased estimator. Moreover, we also show that $\hat{C}_L$ is consistent.

3.2 Bayes estimator of lifetime performance index

Bayesian approach has received large attention for analyzing failure data and other time-to-event data, and has been often proposed as a valid alternative to traditional statistical perspectives. In the Bayesian estimation unknown parameters are assumed to behave as random variables with distributions commonly known as prior probability distributions.This section deals with finding Bayes estimates for unknown parameter α. Here we considered α is a random variable having the conjugate gamma prior distribution with the *pdf*

$$\pi(\alpha|a,b) = \begin{cases} \dfrac{b^a}{\Gamma(a)}\alpha^{a-1}e^{-b\alpha} & \text{if } \alpha > 0 \\ 0 & \text{if } \alpha \le 0. \end{cases} \tag{21}$$

Then the posterior distribution of α is obtained as using (13) and (21),

$$\pi(\alpha) = \frac{(W^*)^{m+a}}{\Gamma(m+a)}\alpha^{m+a-1}\exp(-\alpha W^*), \tag{22}$$

where

$$W^* = k\sum_{i=1}^{m}(R_i+1)\log(1+\frac{x_i^2}{\beta})+b. \tag{23}$$

By consider a squared-error loss function , $\phi(\alpha,\tilde{\alpha}) = (\alpha-\tilde{\alpha})^2$, then the Bayes estimator of α is the posterior mean

$$\tilde{\alpha} = \mathrm{E}(\alpha|\pi(\alpha)) = \frac{m+a}{W^*}, \tag{24}$$

Hence, the Bayes estimator $\tilde{C}_L$ of C_L can be written by using (10) and (22) as

$$\tilde{C}_L = \mathrm{E}(C_L|\pi(\alpha)) = 1-\frac{(m+a)L}{W^*}. \tag{25}$$

Theorem 2. Let $X^{\mathbf{R}}_{i;m,n}$, $i = 1, 2, ..., m$ be an progressive first-failure censored order statistic from two-parameter compound Rayleigh distribution (3) with censored scheme $\mathbf{R}$. Then

$$2\alpha W^* \sim \chi^2_{2(m+a)}, \tag{26}$$

where W^* given by (23)
Proof. Let $Y = 2\alpha W^*$, where W^* as in (23), by using the change of variables (see Casella and Berger [27], pp. 184–185), then we obtain that the *pdf.* of Y is given by

$$g_Y(y) = \pi(\frac{y}{2W^*})||I_y|| = \frac{y^{\frac{2(m+a)}{2}-1}}{2^{\frac{2(m+a)}{2}}\Gamma\left(\frac{2(m+a)}{2}\right)}\exp\left(-\frac{y}{2}\right), \tag{27}$$

hence, $2\alpha W^* \sim \chi^2_{2(m+a)}$.
Remark 2. The expectation of $\tilde{C}_L$ can be derived as follows

$$\mathrm{E}(\hat{C}_L) = 1 - 2(m+a)\alpha L\mathrm{E}(\frac{1}{2\alpha W^*}) = 1 - \frac{(m+a)\alpha L}{(m+a)-1}. \tag{28}$$

The Beyas estimator $\tilde{C}_L$ is not an unbiased estimator of C_L. But when $m \longrightarrow \infty$, $\mathrm{E}(\tilde{C}_L) \longrightarrow C_L$, so the Beyas estimator $\tilde{C}_L$ is asymptotically unbiased estimator. Moreover, we also show that $\tilde{C}_L$ is consistent.

4 Confidence Interval for C_L

In this section, we construct a statistical testing procedure to assess whether the lifetime performance index adheres to the required level. A $100(1-\gamma)\%$ lower bound for C_L is obtained by using the ML and Bayes estimator given by (16) and (25) then, based on this lower bound, a hypothesis testing procedure is developed in order to determine whether the lifetime performance index of products meets the predetermined level. To this end, let c denote the lower bound of C_L. Notice that C_L of products must be larger than c. The null hypothesis $H_0 : C_L \leq c$ (the product is unreliable) against the alternative $H_1 : C_L > c$ (the product is reliable) are constructed.
Theorem 3. Let $X^{\mathbf{R}}_{i;m,n}$, $i = 1, 2, \ldots, m$ be an progressive first-failure censored order statistic from two-parameter compound Rayleigh distribution (3) with censored scheme $\mathbf{R}$, $\hat{C}_L$ is asymptotically unbiased ML estimator of C_L, then $100(1-\gamma)\%$ lower bound for C_L is

$$\underline{\mathrm{LB}}_{\mathrm{MLE}} = 1 + \frac{(\hat{C}_L - 1)\chi^2_{1-\gamma,2m}}{2m}. \tag{29}$$

Proof. In the non-Bayesian approach, for known β, given the specified significance level γ and using the fact that $(2\alpha W) \sim \chi^2_{(2m)}$ (Theorem 1), we have

$$\begin{aligned} 1-\gamma &= P\left(2\alpha W \leq \chi^2_{1-\gamma,2m}\right) \\ &= P\left(\alpha \leq \frac{\chi^2_{1-\gamma,2m}}{2W}\right) \\ &= P\left(C_L \geq 1 - \frac{L\chi^2_{1-\gamma,2m}}{2W}\right) \\ &= P\left(C_L \geq 1 + \frac{(\hat{C}_L - 1)\chi^2_{1-\gamma,2m}}{2m}\right). \end{aligned}$$

In the Bayesian approach, given the specified significance level γ, the level $100(1-\gamma)\%$ one-sided credible interval for C_L can be derived as following thereom
Theorem 4. Let $X^{\mathbf{R}}_{i;m,n}$, $i = 1, 2, \ldots, m$ be an progressive first-failure censored order statistic from two-parameter compound Rayleigh distribution (3) with censored scheme $\mathbf{R}$ and conjugate gamma prior distribution (21), $\tilde{C}_L$ is asymptotically unbiased ML estimator of C_L, then $100(1-\gamma)\%$ lower bound for C_L is

$$\underline{\mathrm{LB}}_{\mathrm{Bayes}} = 1 + \frac{(\tilde{C}_L - 1)\chi^2_{1-\gamma,2(m+a)}}{2(m+a)} \tag{30}$$

Proof. In the Bayesian approach, for known β, given the specified significance level γ and using the fact that $(2\alpha W^*) \sim \chi^2_{(2m)}$ (Theorem 2), we have.

$$\begin{aligned}1-\gamma &= P\left(2\alpha W^* \leq \chi^2_{1-\gamma,2(m+a)}\right) \\ &= P\left(\alpha \leq \frac{\chi^2_{1-\gamma,2(m+a)}}{2W^*}\right) \\ &= P\left(C_L \geq 1-\frac{L\chi^2_{1-\gamma,2(m+a)}}{2W^*}\right) \\ &= P\left(C_L \geq 1+\frac{(\tilde{C}_L-1)\chi^2_{1-\gamma,2(m+a)}}{2(m+a)}\right).\end{aligned}$$

The managers can then employ the one-sided hypothesis testing to determine whether the lifetime performance index adheres to the required level. The proposed testing procedure about C_L can be organized as follows:

Step 1.In order to estimate the shape parameter β in the CRD(α,β), the Gini statistic is suggested, see for instance, Gail and Gastwirth [30], Lee et. al. [5] and Ahmadi et al. [26], the Gini statistic is defined as

$$G_m = \frac{\sum_{i=1}^{m-1} iD_{i-1}}{(m-1)\sum_{i=1}^{m} D_i}, \tag{31}$$

where $D_i = (n-\sum_{i=1}^{m-1} R_i - m+1)(T_{i:m} - T_{i-1:m})$ for $i = 2,3,\cdots,m$ and $D_1 = nT_{1:m}$ while $T_{i:m} = \log(1+\frac{X_i^2}{\beta})$. For $m > 20$, $\sqrt{12(n-1)}(g_n - 0.5)$ tends to the standard normal distribution $N(0,1)$. Hence, the p-value $= P\left\{|Z| > |\sqrt{12(m-1)}(g_m - 0.5)|\right\}$, where g_m is the observed value of G_m and Z has an approximation of $N(0,1)$. So, by using the maximum p-value method, the optimum value of β is selected and then we suppose β is known.

Step 2.From the observed progressive first-failure-censoring data $(x^{\mathbf{R}}_{1:m:n:k}, x^{\mathbf{R}}_{2:m:n:k}, \ldots, x^{\mathbf{R}}_{m:m:n:k})$, we can obtained $y^{\mathbf{R}}_{1:m:n:k}$, $y^{\mathbf{R}}_{2:m:n:k}, \ldots, y^{\mathbf{R}}_{m:m:n:k}$, by using transformation $Y_i = \log(1+\frac{X_i^2}{\beta})$.

Step 3.Determine the lower lifetime limit $L = \log(1+\frac{L_X^2}{\beta})$ for products and performance index value c, then the testing null hypothesis H_0: $C_L < c$ and the alternative hypothesis $H_1 : C_L > c$ is constructed.

Step 4Specify a significance level γ.

Step 5.Calculate the value of test statistic $\hat{C}_L$ and $\tilde{C}_L$ using (15) and (21).

Step 6.Calculate the value of lower bound $\underline{\text{LB}}_{\text{MLE}}$ and $\underline{\text{LB}}_{\text{Bayes}}$ for C_L from (25) and (27).

Step 7.The decision rule of statistical test is provided as follows: If $c \notin [\underline{\text{LB}}_{\text{MLE}}, \infty)$ or $c \notin [\underline{\text{LB}}_{\text{Bayes}}, \infty)$, we reject the null hypothesis and it is concluded that the lifetime performance index of product meets the required level.

5 Illustrative examples

The application of the above testing procedures is presented to one practical data set and one simulated data set.

Example 1 (Real life data). We performed a real data analysis. The original data is a subset of data reported by Bekker et al. [31] and Stablein et al. [32], represent the survival times in years of a group of patients given chemotherapy treatment alone. The data consisting of 46 survival times (in years) for 46 patients are: 0.047, 0.115, 0.121, 0.132, 0.164, 0.197, 0.203, 0.260, 0.282, 0.296, 0.334, 0.395, 0.458, 0.466, 0.501, 0.507, 0.529, 0.534, 0.540, 0.570, 0.641, 0.644, 0.696, 0.841, 0.863, 1.099, 1.219, 1.271, 1.326, 1.447, 1.485, 1.553, 1.581, 1.589, 2.178, 2.343, 2.416, 2.444, 2.825, 2.830, 3.578, 3.658, 3.743, 3.978, 4.003, 4.033. Bekker et al. [31] and Stablein et al. [32] indicated that the Compound Rayleigh model is acceptable for these data. In order to estimate the shape parameter β in the CRD(α,β), the Gini statistic is suggested as defined in (28). For the data set which given above, the values of β and the corresponding p-values are shown in Table 2. Table 2 indicates that $\beta = 0.37$ is very close to the optimum value and the maximum p-value $= 0.98998$. So, we assume that the survival times in years of a grof patients follow a CRD(α,β) with the shape parameter $\beta = 0.37$.

The data are randomly grouped into 23 groups with $(k = 2)$ items within each group. The relief times of the groups are: $\{0.047,0.115\}$, $\{0.121,0.132\}$, $\{0.164,0.197\}$, $\{0.203,0.26\}$, $\{0.282,0.296\}$, $\{0.334,0.395\}$, $\{0.458,0.466\}$,

Table 2: Numerical values of p -values for ball bearing data set.

β	p-value	β	p-value	β	p-value	β	p-value
0.21	0.38334	0.32	0.83108	0.43	0.80646	0.54	0.55279
0.22	0.42427	0.33	0.86867	0.44	0.77917	0.55	0.53431
0.23	0.46569	0.34	0.90538	0.45	0.75279	0.56	0.51651
0.24	0.50738	0.35	0.94119	0.46	0.72730	0.57	0.49934
0.25	0.54914	0.36	0.97608	0.47	0.70267	0.58	0.48280
0.26	0.59079	**0.37**	**0.98998**	0.48	0.67889	0.59	0.46686
0.27	0.63217	0.38	0.95700	0.49	0.65594	0.60	0.45149
0.28	0.67317	0.39	0.92498	0.50	0.63380	0.61	0.43669
0.29	0.71364	0.40	0.89392	0.51	0.61243	0.62	0.42242
0.30	0.75351	0.41	0.86382	0.52	0.59182	0.63	0.40867
0.31	0.79268	0.42	0.83467	0.53	0.57195	0.64	0.39541

Table 3: Progressive first-failure censored sample for survival times (in years) for 46 patients with transformation $Y_{i:m:n:k} = \log(1+\frac{X^2_{i:m:n:k}}{0.37})$.

i	1	2	3	4	5	6	7
	8	9	10	11	12	13	14
$x_{i:m:n:k}$	0.047	0.121	0.164	0.203	0.282	0.334	0.458
	0.529	0.540	0.696	0.863	1.326	2.178	3.743
$y_{i:m:n:k}$	0.006	0.039	0.070	0.106	0.195	0.264	0.449
	0.563	0.581	0.837	1.103	1.750	2.626	3.660
R_i	1	1	1	1	1	1	1
	1	1	0	0	0	0	0

Table 4: Simulated progressive first-failure censored sample from CRD(1.5, 0.37 with transformation $Y_{i:m:n:k} = \log(1+\frac{X^2_{i:m:n:k}}{0.37})$.

i	1	2	3	4	5	6	7	8	9	10	11	12
	13	14	15	16	17	18	19	20	21	22	23	24
	25	26	27	28	29	30	31	32	33	34	35	
$x_{i:m:n:k}$	0.024	0.056	0.074	0.081	0.084	0.085	0.097	0.103	0.129	0.149	0.161	0.161
	0.162	0.176	0.187	0.206	0.212	0.216	0.217	0.239	0.242	0.248	0.254	0.255
	0.256	0.262	0.267	0.297	0.301	0.302	0.323	0.332	0.343	0.475	0.734	
$y_{i:m:n:k}$	0.002	0.008	0.015	0.018	0.019	0.020	0.025	0.028	0.044	0.058	0.068	0.069
	0.070	0.080	0.090	0.109	0.115	0.119	0.120	0.143	0.147	0.154	0.161	0.162
	0.163	0.170	0.176	0.214	0.219	0.220	0.248	0.261	0.27	0.476	0.899	
R_i	2	2	2	2	2	1	1	1	1	1	0	0
	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	

$\{0.501, 0.507\}$, $\{0.529, 0.534\}$, $\{0.54, 0.570\}$, $\{0.641, 0.644\}$, $\{0.696, 0.841\}$, $\{0.863, 1.099\}$, $\{1.219, 1.271\}$, $\{1.326, 1.447\}$, $\{1.485, 1.553\}$, $\{1.581, 1.589\}$, $\{2.178, 2.343\}$, $\{2.416, 2.444\}$, $\{2.825, 2.83\}$, $\{3.578, 3.658\}$, $\{3.743, 3.978\}$, $\{4.003, 4.033\}$. Suppose that the pre-determined progressively first-failure censoring plan is applied using progressive censoring scheme $R = \{1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0\}$. The progressively first-failure censored data of size ($m = 14$) out of 23 groups of patients were observed as in Taple 3

Using progressive first-failure censored with the progressive censoring scheme which are given in Table 3, The vector of it is transformation $Y_{i:m:n:k} = \log(1+\frac{X^2_{i:m:n:k}}{0.37})$ were presented in Table 3.

The lower lifetime limit L_x is assumed to be 0.335, hence $L_y = \log(1+\frac{L_x^2}{0.37}) = 0.2649$. To deal with the product managers' concerns regarding operational performance, the conforming rate Pr of operational performances is required to exceed 80%. Referring to Table 1, the C_L value operational performances are required to exceed 0.8187. Thus, the performance index value is set at $c = 0.80$. The testing hypothesis H_0: $C_L < 0.80$ against the alternative $H_1 : C_L > 0.80$ is constructed.

Since we do not have any prior information and to find the Bayes estimates, small values are given to the gamma hyper parameters to reflect vague prior information. Namely, we assumed that $a = b = 0,0001$. Hence the results in the Bayesian and non-Bayesian are conforming, hence from (27), the 95% lower bound for C_L is obtained as $\underline{\text{LB}}_{\text{Bayes}} = 0.8220$. Since $c = 0.80 \notin [0.8219, \infty)$, so the null hypothesis $H_0 : C_L \leq 0.8$ is rejected.

Example 2 (Simulated data set): A progressive first-failure censored sample with $n = 200$, $k = 4$, $m = 35$ and $(R_1 = \ldots = R_5 = 2, R_6 = \ldots = R_{10} = 1, R_{11} \ldots, R_{35} = 0)$ was generated from a CRD(α, β) with the (pdf). (8) and $(\alpha, \beta) = (1.5, 0.37)$. The observed data and it is transformation were reported in Table 4.

Based on the censored data in Table 4, we estimate the shape parameter α in the CRD(α, β) by usig the Gini statistic with maximum P-value method.

Table 5 shown that the values of $\beta = 0.38$ is very close to the optimum value and the maximum P-value $= 0.99380$ assuming $L_x = 0.12$, hence $L_y = \log(1+\frac{L_x^2}{0.37}) = 0.0382$. Suppose that the conforming rate Pr of operational performances

Table 5: Numerical values of p-values for simulated data set.

β	p-value	β	p-value	β	p-value	β	p-value
0.21	0.60995	0.32	0.88757	0.43	0.93454	0.54	0.81490
0.22	0.64076	0.33	0.90708	0.44	0.92171	0.55	0.80601
0.23	0.67033	0.34	0.92582	0.45	0.90933	0.56	0.79738
0.24	0.69869	0.35	0.94383	0.46	0.89738	0.57	0.78901
0.25	0.72590	0.36	0.96114	0.47	0.89738	0.58	0.78090
0.26	0.75199	0.37	0.97779	0.48	0.87467	0.59	0.77301
0.27	0.77701	**0.38**	**0.99380**	0.49	0.86387	0.60	0.76536
0.28	0.80100	0.39	0.99078	0.50	0.85343	0.61	0.75793
0.29	0.82400	0.40	0.97593	0.51	0.84333	0.62	0.75070
0.30	0.84607	0.41	0.96163	0.52	0.83355	0.63	0.74368
0.31	0.86724	0.42	0.94784	0.53	0.82408	0.64	0.73685

is required to exceed 80%. Referring to Table 1, the C_L value operational performances are required to exceed 0.8187. Thus, the performance index value is set at $c = 0.80$. The testing hypothesis $H_0 : C_L \leq 0.80$ against the alternative $H_1 : C_L > 0.80$ is constructed.

In non-Bayesian approach, the 95% lower bound for C_L is obtained as $\underline{\text{LB}}_{\text{MLE}} = 0.8692$. Since $c = 0.80 \notin [0.8692, \infty)$, so we reject the null hypothesis $H_0 : C_L < 0.80$ in favor of $H_1 : C_L > 0.80$.

In the Bayesian approach, for $\alpha = 1.5$ we choose the best hyperparameters $(a,\, b)$ to satisfies $\text{E}(\alpha) = \frac{a}{b} \cong$ actual population parameter, hence we put $a = 3$ and $b = 2$. The 95% lower bound for C_L is obtained as $\underline{\text{LB}}_{\text{Bayes}} = 0.8674$. Since $c = 0.80 \notin [0.8674, \infty)$, so we reject the null hypothesis $H_0 : C_L \leq 0.80$ in favor of $H_1 : C_L > 0.80$.

6 Monte Carlo Simulations

In this section, we report some numerical experiments performed to evaluate the behavior of the lifetime performance index C_L for different sample sizes n, different effective sample sizes m, different k, different values of $\beta = 0.2$, 0.4 and 1.3. We also consider $\alpha = 1.5$, $L_X = 0.25$ and three different sampling schemes. The samples were generated by using the algorithm described in Balakrishnan and Sandhu [33]. We take into consideration that the progressively first-failure censored order statistics $x^{\mathbf{R}}_{1:m:n:k} < x^{\mathbf{R}}_{2:m:n:k} < \ldots < x^{\mathbf{R}}_{m:m:n:k}$ is a progressively Type II censored sample from a population with distribution function $1-(1-F(x))^k$. We consider the following different sampling schemes:

Scheme I: $R_1 = n-m, R_i = 0$ for $i \neq 1$.

Scheme II: $R_{\frac{m}{2}} = n-m, R_i = 0$ for $i \neq \frac{m}{2}$.

Scheme III: $R_m = n-m, R_i = 0$ for $i \neq m$.

The simulation algorithm of $(1-\gamma)$% lower bound is given see Ahmadi et al. [26]. The results of the simulation study were reported in Tables 6 and 7.

7 Conclusions

Process capability indices are widely utilized by manufacturers to assess the performance and potential of their processes. In lifetime testing experiments in which a failure time of a product is recorded if it exceeds all preceding failure times. Therefore, censored samples may arise in practice. The progressive first-failure censored sampling plan has an advantage in terms of shorter test-time, a saving of resources, and in which a specific fraction of individuals at risk may be removed from the experiment at each of several ordered failure times. The familiar complete, Type II right censored, first-failure censored and progressively Type II right censored samples are special cases of the progressive first-failure censored sampling plan.

From empirical evidence in Tables 6 and 7, we have:

(i) The results obtained in this paper can be specialized to: (a) first-failure-censored order statistics by taking $\mathbf{R} = (0, \ldots, 0)$. (b) progressively type II censored statistics for $k = 1$. (c) usually type II censored order statistics for $k = 1$ and $R = (0, \ldots, n-m)$. (d) complete sample for $k = 1, n = m$; and $R = (0, \ldots, 0)$.

(ii) The lower bound for C_L is quite sensitive to the value of β. Also, when β is underestimated (< 0.37), the actual coverage probabilities of the confidence lower bound for C_L is less than the nominal level, while the overestimated value of β (> 0.37) leads to the larger values for these coverage probabilities relative to the nominal level. Thus, the exact determination of β seems very important.

(iii) When the effective sample proportion m/n increases, the mean of lower bounds of different Bayes estimators and MLEs are reduced, also the censoring scheme $R = (n-m, \ldots, 0)$ is most efficient for all choices, it seems to usually provide the smallest MSE for all estimators.

Table 6: The mean square errors of lower bounds and the coverage probabilities of the confidence intervals of C_L for the different values of β and the various progressive first-failure censoringschemes with $\alpha = 1.5$.

k	n	m	Scheme	β	MLE	Bayes		
						a=1, b=1	a=2, b=2	a=3, b=3
1	20	15	I	0.2	0.484(0.22713)	0.466(0.21654)	0.455(0.21895)	0.438(0.20804)
			II		0.445(0.25909)	0.422(0.25027)	0.437(0.24536)	0.424(0.24907)
			III		0.373(0.33525)	0.359(0.3217)	0.342(0.33158)	0.328(0.31826)
			I	0.4	0.925(0.00262)	0.918(0.00212)	0.906(0.00252)	0.896(0.00298)
			II		0.894(0.00481)	0.884(0.00462)	0.873(0.00648)	0.862(0.00435)
			III		0.854(0.01157)	0.849(0.01112)	0.843(0.01172)	0.832(0.01034)
			I	1.3	0.985(0.00223)	0.978(0.00219)	0.982(0.00228)	0.975(0.00217)
			II		0.977(0.00398)	0.980(0.00368)	0.991(0.00354)	0.983(0.00319)
			III		0.985(0.00425)	0.978(0.00420)	0.982(0.00429)	0.966(0.00399)
3			I	0.2	0.838(0.01219)	0.824(0.01126)	0.813(0.01214)	0.824(0.01031)
			II		0.801(0.02402)	0.783(0.02477)	0.769(0.02363)	0.754(0.02059)
			III		0.732(0.05931)	0.760(0.05918)	0.682(0.05155)	0.695(0.04387)
			I	0.4	0.976(0.00089)	0.971(0.00066)	0.966(0.00055)	0.962(0.00050)
			II		0.989(0.00264)	0.988(0.00159)	0.978(0.00152)	0.968(0.00152)
			III		0.956(0.00288)	0.965(0.00274)	0.975(0.00276)	0.974(0.00254)
			I	1.3	0.993(0.00561)	0.985(0.00489)	0.988(0.00501)	0.977(0.00478)
			II		0.971(0.00612)	0.977(0.00578)	0.976(0.00547)	0.968(0.00539)
			III		0.969(0.00674)	0.971(0.00542)	0.978(0.00612)	0.969(0.00619)

Table 7: The mean square errors of lower bounds and the coverage probabilities of the confidence intervals of $C_L$for the different values of β and the various progressive first-failure censoring schemes with $\alpha = 1.5$.

k	n	m	Scheme	β	MLE	Bayes		
						$a=1, b=1$	$a=2, b=2$	$a=3, b=3$
1	30	20	I	0.2	0.493(0.21106)	0.472(0.21587)	0.452(0.22004)	0.433(0.21322)
			II		0.372(0.23865)	0.354(0.2495)	0.335(0.24562)	0.312(0.22823)
			III		0.332(0.32328)	0.321(0.31896)	0.306(0.31814)	0.298(0.43109)
			I	0.4	0.926(0.00251)	0.918(0.00201)	0.912(0.00202)	0.906(0.00208)
			II		0.877(0.00644)	0.866(0.00421)	0.852(0.00386)	0.836(0.00367)
			III		0.875(0.00710)	0.866(0.00761)	0.857(0.00672)	0.845(0.00512)
			I	1.3	0.981(0.00251)	0.968(0.00240)	0.958(0.00241)	0.955(0.00233)
			II		0.972(0.00282)	0.969(0.00280)	0.966(0.00283)	0.985(0.00252)
			III		0.977(0.00330)	0.956(0.00312)	0.967(0.00323)	0.968(0.00303)
3			I	0.2	0.896(0.00381)	0.891(0.00451)	0.886(0.00403)	0.881(0.00609)
			II		0.813(0.01979)	0.799(0.01609)	0.789(0.01609)	0.772(0.01586)
			III		0.659(0.04673)	0.646(0.04392)	0.627(0.04331)	0.616(0.04294)
			I	0.4	0.977(0.00085)	0.976(0.00076)	0.974(0.00067)	0.971(0.00053)
			II		0.976(0.00218)	0.956(0.00215)	0.971(0.00211)	0.969(0.00195)
			III		0.972(0.00245)	0.965(0.00244)	0.964(0.00244)	0.955(0.00243)
			I	1.3	0.978(0.00244)	0.977(0.00241)	0.966(0.00239)	0.974(0.00232)
			II		0.980(0.00265)	0.959(0.00277)	0.972(0.00269)	0.976(0.00281)
			III		0.971(0.00379)	0.974(0.00358)	0.987(0.00344)	0.954(0.00322)

(iv)The mean square errors for one-sided credible interval based on Bayes estimates are smaller than the mean square errors for one-sided confidence interval based on the MLE.

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