Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/100640

The Cauchy Problem for the Hierarchy of Quantum Kinetic Equations for Correlation Matrices with Coulomb Potential

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Received: 2 Aug. 2016, Revised: 12 Sep. 2016, Accepted: 19 Sep. 2016
Published online: 1 Nov. 2016

Abstract: The existence of a unique solution, in terms of initial data of the hierarchy of quantum kinetic equations for correlation matrices with Coulomb potential, has been proven. The proof is based on nonrelativistic quantum mechanics and application of semigroup theory methods

Keywords: BBGKY hierarchy; hierarchy of quantum kinetic equations for correlation matrices; Coulomb potential.

1 Introduction

The Bogolubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy of kinetic equations is an infinitely engaging system of the integro-diffierential equations, which is formulated in 1946 [1]. The BBGKY hierarchy is used to describe the evolution of non-equilibrium systems of interacting particles. There are two versions of the BBGKY's hierarchy of the kinetic equations: hierarchy of classical kinetic equations for distribution functions s and the quantum kinetic equations for density matrices. These hierarchies are the best relations connecting Liouville?s equationsfor many particle density matrices with the kinetic Boltzmann-Vlasov?s equations of for one particle. As known, the last equations are used to describe the evolution of many physical processes in the solid, gases, semiconductors and in plasma.

From the day of its formulation up to now, the BBGKY hierarchy remains an object of research both for physicists and mathematicians. One of the most important areas of BBGKY hierarchy application is the physics of plasma. As it known, plasma consists of charged particles interacting with the Coulomb potential. The evolution of plasma is described by a hierarchy of the kinetic equations for correlation functions, which is deduced from the BBGKY hierarchy. Difficulties in research of this field are due to the complexity of structure of a hierarchy and structure of the Coulomb potential, as well as the absence of a compact solution of this hierarchy. These difficulties were indicated by Bogolubov in his pioneer work on the kinetic equations [1]. Another problem of physics of non-equilibrium plasma is connected with the fact that up to now, the first 2-3 classical equations of the BBGKY hierarchy for correlation functions, which depend both on the position and velocities of the particles, have been used to describe the evolution of the charged 2-3 particles. The 2-3 particles system is usually an object of quantum physics and consequently it will be appropriate to investigate it applying the hierarchy of the quantum kinetic equations for correlation matrices. This follows from the uncertainty principle, which prohibits precise simultaneous specification of the position and velocity.

The present work is devoted to the solution of the hierarchy of quantum kinetic equations for correlation matrices, describing the dynamics of systems particles interacting through the Coulomb potential. Note, that, the hierarchy for correlation matrices is deduced from the BBGKY hierarchy of quantum kinetic equations for density matrices [1],[2], [3], [4]. In this paper, the system of finite many particles is considered in a finite region (vessel) Λ with volume $V = |\Lambda|$. Using the semigroup theory developed for the solution of the BBGKY hierarchy, both classical [5],[6] and quantum kinetic

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equations [7], [8] for the bounded potential, the existence and uniqueness of the solution of the hierarchy of quantum kinetic equations for correlation matrices [9],[10] is proved. At the same time, the theorem of a self-adjointness of the Hamiltonian with the Coulomb potential [11],[12] and the algebraic approach to problems of statistical physics in [13] are essentially used. It's worth noting, that the method applied to solve the hierarchy allows defining the kinetic equations for correlation matrices and their solution not only for 2-3 particles. The obtained solution is compact and can be used for calculation of physical values.

The Cauchy problem for the hierarchy of quantum kinetic equations with the Coulomb potential is formulated in the second section of the work. In the third section, the problem is solved by using the semigroup theory method. In the fourth section, the hierarchy of the quantum kinetic equations for the correlation matrices with the Coulomb potential is deduced and the solution of the hierarchy is defined. The last section contains some examples.

2 Formulation of the Problem

We consider the hierarchy BBGKY of quantum kinetic equations, which describes the evolution of a system of identical particles with mass m and charge q interacting via a Coulomb potential [1],[14] $\phi(x_i,x_j)=q^2/|x_i-x_j|$, which depends on the distance between particles $|x_i-x_j|$ and charges q. We assume that the charge is a real constant.
In the present section, the Cauchy problem is formulated for a quantum system of a finite number particles contained in the finite region (vessel) with volume $V=|\Lambda|$ [15]. The BBGKY's hierarchy is given by [2],[3]

$$i\frac{\partial\rho_s^{\Lambda}(t,x_1,\ldots,x_s;x_1',\ldots,x_s')}{\partial t}=$$

$$[H_s^{\Lambda},\rho_s^{\Lambda}](t,x_1,\ldots,x_s;x_1',\ldots,x_s')+$$

$$+\frac{N}{V}(1-\frac{s}{N})Tr_{x_{s+1}}\sum_{1\le i\le s}(\phi_{i,s+1}(|x_i-x_{s+1}|)-$$

$$\phi_{i,s+1}(|x_i'-x_{s+1}|))\rho_{s+1}^{\Lambda}(t,x_1,\ldots,x_s,x_{s+1};x_1',\ldots,x_s',x_{s+1}), \tag{1}$$

with the initial condition

$$\rho_s^{\Lambda}(t,x_1,\ldots,x_s;x_1',\ldots,x_s')|_{t=0}=\rho_s^{\Lambda}(0,x_1,\ldots,x_s;x_1',\ldots,x_s'). \tag{2}$$

In the problem given by equation (1) and (2) the vector represented by x_i gives the position of ith particle in the 3-dimensional Euclidean space R^3, $x_i=(x_i^1,x_i^2,x_i^3)$, $i=1,2,\ldots.,s$, and $x_i^{\alpha},\alpha=1,2,3$ are coordinates of a vector x_i. The length of the vector x_i is denoted by

$$|x_i|=((x_i^1)^2+(x_i^2)^2+(x_i^3)^2)^{\frac{1}{2}}.$$

In (1) $\hbar=1$ is the Planck constant and $[,]$ denotes the Poisson bracket.

The reduced statistical operator of s particles is $\rho_s^{\Lambda}(x_1,..,x_s;x_1',..,x_s')$ related to the positive symmetric density matrix D of N particles by [2],[3]

$$\rho_s^{\Lambda}(x_1,.,x_s;x_1',.,x_s')=$$

$$V^sTr_{x_{s+1},.,x_N}D_N^{\Lambda}(x_1,.,x_s,x_{s+1},.,x_N;x_1',.,x_s',x_{s+1},.,x_N),$$

where $s\in N$, N is the number of particles, and V the volume of the system of particles. The trace is defined in terms of the kernel $\rho^{\Lambda}(x,x')$ by the formula

$$Tr_x\rho^{\Lambda}=\int_{\Lambda}\rho^{\Lambda}(x,x)dx.$$

The Hamiltonian of system is defined as

$$H_s^{\Lambda}(x_1,\ldots,x_s)=\sum_{1\le i\le s}\left(-\frac{1}{2m}\triangle_{x_i}+u^{\Lambda}(x_i)\right)+$$

$$+\sum_{1\le i<j\le s}\phi_{i,j}(|x_i-x_j|),$$

where $\triangle_i$ is the Laplacian

$$\triangle_i=\frac{\partial^2}{\partial(x_i^1)^2}+\frac{\partial^2}{\partial(x_i^2)^2}+\frac{\partial^2}{\partial(x_i^3)^2},$$

$$\phi_{i,j}(|x_i-x_j|)=\frac{q^2}{|x_i-x_j|},$$

and $u^{\Lambda}(x)$ is an external field which keeps the system in the region Λ ($u^{\Lambda}(x)=0$ if $x\in\Lambda$ and $u^{\Lambda}(x)=+\infty$ if $x\notin\Lambda$). Here $\phi_{i,j}(|x_i-x_j|)$ is symmetric.

3 Solution of the Cauchy Problem for the BBGKY Hierarchy of Quantum Kinetic Equations with Coulomb Potential

To obtain the solution of the Cauchy problem defined by (1) and (2) we use a semigroup method [5],[6],[7],[8], [12], [16],[17].

Let $L_2^s(\Lambda)$ be the Hilbert space of functions $\psi_s^{\Lambda}(x_1,\ldots,x_s)$, $x_i\in R^3(\Lambda)$, and B_s^{Λ} be the Banach space of positive-definite, self adjoint nuclear operators $\rho_s^{\Lambda}(x_1,\ldots,x_s;x_1',\ldots,x_s')$ on $L_2^s(\Lambda)$

$$(\rho_s^{\Lambda}\psi_s^{\Lambda})(x_1,\ldots,x_s)=\int_{\Lambda}\rho_s^{\Lambda}(x_1,\ldots,x_s;x_1',\ldots,x_s')\times$$

$$\times\psi_s^{\Lambda}(x_1',\ldots,x_s')dx_1'\ldots dx_s',$$

with norm

$$|\rho_s^{\Lambda}|_1=sup\sum_{1\le i\le\infty}|(\rho_s^{\Lambda}\psi_i^s,\varphi_i^s)|,$$

where the upper bound is taken over all orthonormalied systems of finite, twice differentiable functions with compact support $\{\psi_i^s\}$ and $\{\varphi_i^s\}$ in $L_2^s(\Lambda)$, $s \geq 1$. We'll suppose that the operators ρ_s^Λ and H_s^Λ act in the space $L_2^s(\Lambda)$ with zero boundary conditions.

Let B^Λ be the Banach space of sequences of nuclear operators

$$\rho^\Lambda = \{\rho_0^\Lambda, \rho_1^\Lambda(x_1;x_1'), ..., \rho_s^\Lambda(x_1,...,x_s;x_1',...,x_s'), ...\},$$

where ρ_0^Λ are complex numbers, $|\rho_0^\Lambda|_1 = |\rho_0^\Lambda|$ and $\rho_s^\Lambda \subset B_s^\Lambda$,

$$\rho_s^\Lambda(x_1,...,x_s;x_1',...,x_s') = 0, \qquad when \qquad s > s_0,$$

where s_0 is finite and the norm is

$$|\rho^\Lambda|_1 = \sum_{s=0}^{\infty} |\rho_s^\Lambda|_1.$$

The Coulomb potential $\phi_{i,j} = \frac{q^2}{|r_{i,j}|}$ can be represented as [12]

$$\phi_{i,j} = \phi_{i,j}^1 + \phi_{i,j}^2,$$

where

$$\phi_{i,j}^1 = \frac{q^2}{|r_{i,j}|}\left(\frac{1}{1+|r_{i,j}|}\right) \subset L_2(R^3),$$

$$\phi_{i,j}^2 = \frac{q^2}{1+|r_{i,j}|} \subset L_\infty(R^3),$$

$$r_{i,j} = \left((x_i^1 - x_j^1)^2 + (x_i^2 - x_j^2)^2 + (x_i^3 - x_j^3)^2\right)^{1/2}.$$

Therefore the Coulomb potential H_s^Λ satisfies the conditions of Theorem X.15 [12] and

$$H_s^\Lambda(x_1,..,x_i,..,x_j,..x_s) = -\sum_{1\leq i\leq s}\frac{1}{2}\triangle_{x_i} + \sum_{1\leq i<j\leq s}\frac{q^2}{|x_i - x_j|}$$

is self-adjoint operator on the set $D(-\triangle)$.

Let $\tilde{B}_s^\Lambda$ be a dense set of "good" elements of B_s^Λ of type $B_s^\Lambda \cap D(H_s^\Lambda) \bigotimes D(H_s^\Lambda)$, where $D(H_s^\Lambda)$ is the domain of the operator H_s^Λ [11] and $\bigotimes$ denote the algebraic tensor product.

We introduce the operators $\omega^\Lambda(t)$, $\Omega(\Lambda)$ and $U^\Lambda(t)$ on the space B^Λ by

$$(\omega^\Lambda(t)\rho^\Lambda)_s(x_1,..,x_s;x_1',..,x_s') =$$

$$= (e^{-iH_s^\Lambda t}\rho^\Lambda e^{iH_s^\Lambda t})_s(x_1,..,x_s;x_1',..,x_s'),$$

$$(\Omega(\Lambda)\rho^\Lambda)_s(x_1,..,x_s;x_1',..,x_s') = \frac{N}{V}(1-\frac{s}{N})\times$$

$$\times\int_\Lambda \sum_i \rho_{s+1}^\Lambda(x_1,..,x_s,x_{s+1};x_1',..,x_s',x_{s+1})\times$$

$$g_i^1(x_{s+1})\tilde{g}_i^1(x_{s+1})dx_{s+1}, \qquad (3)$$

$$U^\Lambda(t)\rho_s^\Lambda(x_1,..,x_s;x_1',..,x_s') = (e^{\Omega(\Lambda)}e^{-iH^\Lambda t}e^{-\Omega(\Lambda)}\rho^\Lambda\times$$

$$e^{iH^\Lambda t})_s(x_1,..,x_s;x_1',..,x_s').$$

In (3) $g_i^1(x_{s+1})$ is a complete orthonormal system of vectors in the one-particle space $L_2(\Lambda)$.

Let

$$(\tilde{\mathscr{H}}^\Lambda\rho^\Lambda)_s(x_1,..,x_s;x_1',..,x_s') =$$

$$[H_s^\Lambda, \rho_s^\Lambda](x_1,..,x_s;x_1',..,x_s') +$$

$$\frac{N}{V}(1-\frac{s}{N})Tr_{x_{s+1}}\sum_{1\leq i\leq s}(\phi_{i,s+1}(|x_i - x_{s+1}|) -$$

$$-\phi_{i,s+1}(|x_i' - x_{s+1}|))\rho_{s+1}^\Lambda(x_1,..,x_{s+1};x_1',..,x_{s+1}).$$

Theorem 1. *Let potential $\phi(x_i,x_j) = q^2/|x_i - x_j|$ is Coulomb potential. The operator $U^\Lambda(t)$ generates a strongly continuous semigroup of bounded operators on $\tilde{B}^\Lambda$, whose generators coincide with the operator $-i\tilde{\mathscr{H}}^\Lambda$ on $\tilde{B}^\Lambda$ everywhere dense in B^Λ.*

Proof: According to the general theory of groups of bounded strongly continuous operators, there always exists an infinitesimal generator of the group $U^\Lambda(t)$ given by the formula $lim_{t\longrightarrow 0}\frac{U^\Lambda(t)\rho^\Lambda - \rho^\Lambda}{t}$ in the sense of convergence in norm in the space B^Λ for ρ^Λ that belong to a certain set $D(\tilde{\mathscr{H}}^\Lambda)$ everywhere dense in B^Λ [8]. Therefore, since $U^\Lambda(t)$ is a strongly continuous semigroup on B^Λ with generator $-i\tilde{\mathscr{H}}^\Lambda$ on the right-hand side of the BBGKY hierarchy of quantum kinetic equations on $\tilde{B}_s^\Lambda$ which is dense in B_s^Λ [15], the abstract Cauchy problem (1)-(2) has the unique solution

$$\rho_s^\Lambda(t,x_1,...,x_s;x_1',...,x_s') =$$

$$(U^\Lambda(t)\rho^\Lambda)_s(x_1,...,x_s;x_1',...,x_s')$$

$$= (e^{\Omega(\Lambda)}e^{-iH^\Lambda t}e^{-\Omega(\Lambda)}\rho^\Lambda e^{iH^\Lambda t})_s(x_1,...,x_s;x_1',...,x_s') \quad (4)$$

for each $\rho_s^\Lambda(x_1,...,x_s;x_1',...,x_s') \subset \tilde{B}_s^\Lambda$. For the initial data ρ_s^Λ belonging to a certain subset of B_s^Λ (to the domain of definition of $D(-i\tilde{\mathscr{H}}^\Lambda)$), which is everywhere dense in B_s^Λ, (5) is strong solution of Cauchy problem (1)-(2).

This proves the Theorem 1.

By a similar argument, one can show that the infinitesimal generator of the group $U(t)$ coincides with the operator that defines the BBGKY chain

$$i\frac{\partial\rho_s(t,x_1,...,x_s;x_1',...,x_s')}{\partial t} =$$

$$[H_s, \rho_s](t,x_1,...,x_s;x_1',...,x_s') +$$

$$+\frac{1}{v}Tr_{x_{s+1}}\sum_{1\leq i\leq s}(\phi_{i,s+1}(|x_i - x_{s+1}|) -$$

$$\phi_{i,s+1}(|x_i' - x_{s+1}|))\rho_{s+1}(t,x_1,...,x_s,x_{s+1};x_1',...,x_s',x_{s+1}),$$

in the thermodynamic limit $N \longrightarrow \infty, V \longrightarrow \infty, v = \frac{V}{N} = const$, on an everywhere dense subset of B of finite sequences

$$\rho = \{\rho_0, \rho_1(x_1;x_1'), ..., \rho_s(x_1,...,x_s;x_1',...,x_s'), ...\},$$

where ρ_0, complex number, $s > s_0$, such that $[H_s, \rho_s]$ belongs to B_s together with ρ_s.

4 Derivation of Hierarchy of Kinetic Equations for Correlation Matrices with Coulomb Potential and its Solution

Introducing the notation

$$\left(\mathscr{H}^\Lambda \rho^\Lambda\right)_s(t,x_1,...,x_s;x_1',...,x_s') =$$

$$= \left[H_s^\Lambda, \rho_s^\Lambda\right](t,x_1,...,x_s;x_1',...,x_s');$$

$$\left(\mathscr{D}_{x_{s+1}}^\Lambda \rho^\Lambda\right)_s(x_1,\cdots,x_s;x_1',\cdots,x_s') =$$

$$\rho_{s+1}^\Lambda(x_1,\cdots x_s,x_{s+1};x_1',\cdots,x_s',x_{s+1});$$

$$(\mathscr{A}_{x_{s+1}}^\Lambda \rho^\Lambda)_s(t,x_1,...,x_s;x_1',...,x_s')$$

$$= \frac{N}{V}(1-\frac{s}{N})\sum_{1\le i\le s}(\phi_{i,s+1}(|x_i - x_{s+1}|) -$$

$$-\phi_{i,s+1}(|x_i' - x_{s+1}|))\rho_s^\Lambda(t,x_1,...,x_s;x_1',...,x_s');$$

$$\rho^\Lambda(t) = \{\rho_1^\Lambda(t,x_1;x_1'),...,\rho_s^\Lambda(t,x_1,...,x_s:x_1',...,x_s'),...\}, \tag{5}$$

$$s = 1,2,\cdots,$$

we can cast (1) and (2) in the form

$$i\frac{\partial}{\partial t}\rho_s^\Lambda(t,x_1,...,x_s;x_1',...,x_s') =$$

$$\left(\mathscr{H}^\Lambda \rho^\Lambda\right)_s(t,x_1,...,x_s;x_1',...,x_s')$$

$$+\int_\Lambda \left(\mathscr{A}_{x_{s+1}}^\Lambda \mathscr{D}_{x_{s+1}}^\Lambda \rho^\Lambda\right)_s(t,x_1,...,x_s;x_1',...,x_s')dx_{s+1},$$

$$\rho_s^\Lambda(t,x_1,...,x_s;x_1',...,x_s')|_{t=0} =$$

$$\equiv \rho_s^\Lambda(x_1,...,x_s;x_1',...,x_s').$$

For sequences (5) this problem can formulated as

$$i\frac{\partial}{\partial t}\rho^\Lambda(t) = \left(\mathscr{H}^\Lambda \rho^\Lambda\right)(t) + \int_\Lambda \mathscr{A}_x^\Lambda \mathscr{D}_x^\Lambda \rho^\Lambda(t)dx, \tag{6}$$

$$\rho^\Lambda(t)|_{t=0} = \rho^\Lambda(0). \tag{7}$$

Proposition For sequence of correlation matrices

$$\varphi = \{\varphi_0, \varphi_1(x_1;x_1'),...,\varphi_s(x_1,...,x_s;x_1',...,x_s'),...\},$$

the hierarchy of kinetic equations has the form:

$$i\frac{\partial}{\partial t}\varphi(t) = \mathscr{H}\varphi(t) + \frac{1}{2}\mathscr{W}(\varphi(t),\varphi(t)) +$$

$$\int_\Lambda \mathscr{A}_x\mathscr{D}_x\varphi(t)dx + \int_\Lambda (\mathscr{A}_x\varphi \star \mathscr{D}_x\varphi)(t)dx, \tag{8}$$

$$\varphi(t)|_{t=0} = \varphi(0). \tag{9}$$

Proof: To obtain (8), (9) we use relation between density matrices and correlation matrices [9],[10], [13]:

$$\rho(t) = \Gamma\varphi(t) = I + \varphi(t) +$$

$$+\frac{\varphi(t)\star\varphi(t)}{2!} + \cdots \frac{(\star\varphi(t))^s}{s!} + \cdots, \tag{10}$$

and

$$\Gamma^-(I+\varphi(t)) = I + \varphi(t) -$$

$$-\frac{\varphi(t)\star\varphi(t)}{2!} + \cdots \frac{(\star\varphi(t))^s}{s!} + \cdots,$$

$$\varphi \in B_+$$

where:

$$(\varphi\star\varphi)(X) = \sum_{YCX}\varphi(Y)\varphi(X\setminus Y),$$

$$I\star\varphi = \varphi, \quad (\star\varphi)^s = \underbrace{\varphi\star\varphi\star\cdots\star\varphi} \text{ s times;}$$

$$X = (x_1,\cdots,x_s;x_1',\cdots,x_s'),$$

$$Y = (x_1,\cdots,x_s;x_1',\cdots,x_{s'}'), \quad s' \in s, \quad s = 1,2,\cdots;$$

$$\mathscr{W}(\varphi,\varphi)(X) = \sum_{YCX}\mathscr{U}(Y;X\setminus Y)\varphi(Y)\varphi(X\setminus Y),$$

$$(\mathscr{U}\varphi)(X) = \left[\sum_{1\le i<j\le s}\phi(x_i - x_j), \varphi\right](X),$$

$\Gamma(-\varphi(t))\star\Gamma\varphi(t) = I$ and substitute (10) in (6),(7):

$$\frac{\partial}{\partial t}\Gamma\varphi(t) = \mathscr{H}\Gamma\varphi(t) + \int_\Lambda \mathscr{A}_x\mathscr{D}_x\Gamma\varphi(t)dx \tag{11}$$

$$\Gamma\varphi(t)|_{t=0} = \Gamma\varphi(0). \tag{12}$$

We have

$$\mathscr{D}_x\Gamma\varphi(t) = \mathscr{D}_x\varphi(t)\star\Gamma\varphi(t), \tag{13}$$

$$\mathscr{A}_x\Gamma\varphi(t) = \mathscr{A}_x\varphi(t)\star\Gamma\varphi(t), \tag{14}$$

$$\mathscr{A}_x\mathscr{D}_x\Gamma\varphi(t) = \mathscr{A}_x\mathscr{D}_x\varphi(t)\star\Gamma\varphi(t) +$$

$$\mathscr{A}_x\varphi(t)\star\mathscr{D}_x\varphi(t)\star\Gamma\varphi(t), \tag{15}$$

$$T\Gamma\varphi(t) = T\varphi(t)\star\Gamma\varphi(t), \tag{16}$$

$$\mathscr{U}\Gamma\varphi(t) = \mathscr{U}\varphi(t)\star\Gamma\varphi(t) + \frac{1}{2}\mathscr{W}(\varphi(t),\varphi(t)\star\Gamma\varphi(t)), \tag{17}$$

$$\frac{\partial}{\partial t}\Gamma\varphi(t)=\frac{\partial}{\partial t}\varphi(t)\star\Gamma\varphi(t). \quad (18)$$

Substituting $(13)-(18)$ in $(11),(12)$, multiplying both sides by $\Gamma(-\varphi(t))$, where

$$\Gamma^{-}(I+\varphi(t))=I+\varphi(t)-$$

$$-\frac{\varphi(t)\star\varphi(t)}{2!}+\cdots\frac{(\star\varphi(t))^{s}}{s!}+\cdots, \quad (19)$$

$$\varphi\in B_{+}$$

and $\Gamma(-\varphi(t))\star\Gamma\varphi(t)=I$, we obtain (8)-(9).

Here we shall call B_+ the subspace of B formed by the elements φ such that $\varphi(0)=0$. The power series expansion of the exponential yields a well-defined mapping Γ of B_+ onto $I+B_+$ by (19), where $\varphi\in B_+,\Gamma$ has inverse Γ^{-} (corresponding to the logarithm).

This proves the proposition.

The problem (8), (9) for the system of s particles in the volume V have form:

$$i\frac{\partial}{\partial t}\varphi_s^{\Lambda}(t,x_1,\ldots,x_s;x_1',\ldots,x_s')=$$

$$=\mathscr{H}^{\Lambda}\varphi_s^{\Lambda}(t,x_1,\ldots,x_s;x_1',\ldots,x_s')+$$

$$+\frac{1}{2}\mathscr{W}^{\Lambda}\left(\varphi^{\Lambda},\varphi^{\Lambda}\right)_s(t,x_1,\ldots,x_s;x_1',\ldots,x_s')+$$

$$+\int_{\Lambda}\mathscr{A}^{\Lambda}_{x_{s+1}}e\mathscr{D}^{\Lambda}_{x_{s+1}}\varphi_s^{\Lambda}(t,x_1,\ldots,x_s;x_1',\ldots,x_s')dx_{s+1}+$$

$$+\int_{\Lambda}\left(\mathscr{A}^{\Lambda}_{x_{s+1}}\varphi^{\Lambda}\star\mathscr{D}^{\Lambda}_{x_{s+1}}\varphi^{\Lambda}\right)_s(t,x_1,\ldots,x_s;x_1',\ldots,x_s')dx_{s+1}, \quad (20)$$

$$\varphi_s^{\Lambda}(t,x_1,\ldots,x_s;x_1',\ldots,x_s')|_{t=0}=\varphi_s^{\Lambda}(0,x_1,\ldots,x_s;x_1',\ldots,x_s'). \quad (21)$$

We introduce the quantum operator which is analogy to classical case [6]:

$$U'^{\Lambda}(t)\varphi_s^{\Lambda}(0,x_1,\ldots,x_s;x_1',\ldots,x_s')=$$

$$=\Gamma exp(\Omega^{\Lambda})\Gamma^{-1}[exp(iH^{\Lambda}t)\Gamma(exp(-\Omega^{\Lambda})\Gamma^{-1}\Gamma\times$$

$$\times\varphi_s^{\Lambda}(0,x_1,\ldots,x_s;x_1',\ldots,x_s'))exp(-iH^{\Lambda}t)].$$

Theorem 2.*Let potential $\phi(x_i,x_j)=q^2/|x_i-x_j|$ is Coulomb potential. The operator $U'^{\Lambda}(t)$ generates a strongly continuous semigroup of bounded operators on $\tilde{B}^{\Lambda}$, whose generators coincide with the operator*

$$-i(\mathscr{H}^{\Lambda}+\frac{1}{2}\mathscr{W}^{\Lambda}+\int_{\Lambda}\mathscr{A}^{\Lambda}_{x_{s+1}}\mathscr{D}^{\Lambda}_{x_{s+1}}dx_{s+1}+$$

$$+\int_{\Lambda}\mathscr{A}^{\Lambda}_{x_{s+1}}\star\mathscr{D}^{\Lambda}_{x_{s+1}}dx_{s+1})$$

on $\tilde{B}^{\Lambda}$ everywhere dense in B^{Λ}.

Proof: Using (10) in (4) and $\Gamma^{-1}\Gamma\varphi(t)=\varphi(t)$ we obtain:

$$\rho_s^{\Lambda}(t,x_1,\ldots,x_s;x_1',\ldots,x_s')=\Gamma\varphi_s^{\Lambda}(t,x_1,\ldots,x_s;x_1',\ldots,x_s')=$$

$$=\Gamma exp(\Omega^{\Lambda})\Gamma^{-1}[exp(iH^{\Lambda}t)\Gamma(exp(-\Omega^{\Lambda})\Gamma^{-1}\Gamma\times$$

$$\times\varphi_s^{\Lambda}(0,x_1,\ldots,x_s;x_1',\ldots,x_s'))exp(-iH^{\Lambda}t)]=$$

$$=\Gamma exp(\Omega^{\Lambda})\Gamma^{-1}[exp(iH^{\Lambda}t)\Gamma(exp(-\Omega^{\Lambda})\times$$

$$\times\varphi_s^{\Lambda}(0,x_1,\ldots,x_s;x_1',\ldots,x_s'))exp(-iH^{\Lambda}t)]. \quad (22)$$

Acting to (22) by Γ^{-1} we receive:

$$\varphi_s^{\Lambda}(t,x_1,\ldots,x_s;x_1',\ldots,x_s')=$$

$$=U'^{\Lambda}(t)\varphi_s^{\Lambda}(0,x_1,\ldots,x_s;x_1',\ldots,x_s')=$$

$$exp(\Omega^{\Lambda})\Gamma^{-1}[exp(iH^{\Lambda}t)\Gamma(exp(-\Omega^{\Lambda})\times$$

$$\times\varphi_s^{\Lambda}(0,x_1,\ldots,x_s;x_1',\ldots,x_s'))exp(-iH^{\Lambda}t)]. \quad (23)$$

The generator of the semigroup $U'^{\Lambda}(t)$ coincides with

$$-i(\mathscr{H}^{\Lambda}+\frac{1}{2}\mathscr{W}^{\Lambda}+\int_{\Lambda}\mathscr{A}^{\Lambda}_{x_{s+1}}\mathscr{D}^{\Lambda}_{x_{s+1}}dx_{s+1}+$$

$$\int_{\Lambda}\mathscr{A}^{\Lambda}_{x_{s+1}}\star\mathscr{D}^{\Lambda}_{x_{s+1}}dx_{s+1}),$$

on the set $D(H_s^{\Lambda})$.

So, (23) on $D(-\sum_{1\leq i\leq s}\triangle_i)$ is the unique solution of the Cauchy hierarchy of kinetics equations for correlation matrices with Coulomb potential (20)-(21).

This proves the Theorem 2.

5 Examples

Consider first two equations of the hierarchy of quantum kinetic equations for correlation matrices (8), which are using in plasma physics to describe evolution of plasma. For case s=1 equation (8) has form:

$$i\frac{\partial}{\partial t}\varphi(t,x_1;x_1')=-\frac{1}{2}(\triangle_{x_1}-\triangle_{x_1'})\varphi(t,x_1;x_1')+$$

$$+\frac{N}{V}(1-\frac{1}{N})\int_{\Lambda}(\phi(x_1-x)-\phi(x_1'-x))\varphi(t,x_1,x;x_1',x)dx+$$

$$\frac{N}{V}(1-\frac{1}{N})\int_{\Lambda}(\phi(x_1-x)-\phi(x_1'-x))\varphi(t,x_1;x_1')$$

$$\varphi(t,x;x)dx.$$

For case s=2 equation has form:

$$i\frac{\partial}{\partial t}\varphi(t,x_1,x_2;x_1',x_2)=$$

$$= -\frac{1}{2}\sum_{i}^{2}(\triangle_{x_i} - \triangle_{x'_i})\varphi(t,x_1,x_2;x'_1,x_2)+$$

$$+(\phi(x_1-x_2)-\phi(x'_1-x'_2))\varphi(t,x_1,x_2;x'_1,x'_2)+$$

$$+\frac{1}{2}(\phi(x_1-x_2)-\phi(x'_1-x'_2))\varphi(t,x_1;x'_1)\varphi(t,x_2;x'_2)+$$

$$+\frac{N}{V}(1-\frac{2}{N})\int_{\Lambda}\sum_{i}^{2}(\phi(x_i-x)-\phi(x'_i-x))\times$$

$$\times\varphi(t,x_1,x_2,x;x'_1,x'_2,x)dx+\frac{N}{V}(1-\frac{2}{N})$$

$$\int_{\Lambda}(\phi(x_1-x)-\phi(x'_1-x))\varphi(t,x_1;x'_1)\varphi(t,x_2,x;x'_2,x)dx+$$

$$\frac{N}{V}(1-\frac{2}{N})\int_{\Lambda}(\phi(x_2-x)-$$

$$-\phi(x'_2-x))\varphi(t,x_2;x'_2)\varphi(t,x_1,x;x'_1,x)dx,$$

where $\phi(x_i,x_j)=\frac{q^2}{|x_i-x_j|}$ is Coulomb potential.

Acknowledgement

N.N. Bogolubov JR, is Corr. member of Russian Academy of Science, State Prize of Russia(USSR).,member of Presidium International Academy of Endeavour Moscow.

N.N.(Jr.) Bogoliubov's research has been done in V.A.Steklov Institute of Mathematics.

N.N.(Jr.)Bogoliubov formulated the problem and proposed to apply chain BBGCI to study dynamics of the particle system. He, as well, discussed systematically the obtained results.

M.Yu.Rasulova's research has been done through the CCDST of the Republic of Uzbekistan (No F2-FA-F116) in the Institute of Nuclear Physics of Academy of Sciences of Uzbekistan.

The authors are grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

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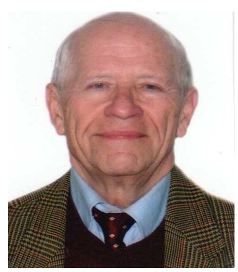

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