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Concentration Distribution Around the Gas Bubbles in a Bio Tissue with Acceleration Convection under the Effect of Injection Process

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Abstract: The concentration distribution around growing nitrogen gas bubble in the blood and other bio tissues of divers who ascend to surface too quickly is obtained by Mohammadein and Mohamed model (2010)[3] for variant and constant ambient pressure through the decompression process. In this paper, the growing of gas bubbles and concentration distribution under the effect of injection process with convective acceleration are studied as a modification of Mohammadein and Mohamed model (zero injection)[3]. The growth of gas bubble is affected ascent rate, tissue diffusivity, initial concentration difference, surface tension and void fraction. Mohammadein and Mohamed model (2010) is obtained as a special case from the present model. Results showed that the injection process affects the systemic blood circulation and acceleration the growth of gas bubbles the bio tissues. The study warns the divers to take any kind of injection during the dive process to avoid the incidence of decompression sickness(DCS).

Keywords: Gas bubble; concentration distribution; convective growth; method of combined variables.

1 Introduction

Decompression sickness(DCS) is a dangerous and occasionally lethal condition caused by nitrogen gas bubbles that form in the blood and other tissue of divers who surface too quickly or people who flight for long distances from the earth(aviators or astronauts). The decompression process of a diver is the reduction in ambient pressure occurs by the diver during ascent, and also the process of elimination of dissolved inert gases from the diver's body, which occurs both during the actual ascent, during pauses in the ascent known as decompression stops, and after surfacing, unit either the gas concentration reach equilibrium, or another dive is stated.

The same decompression sickness(DCS) can be occurred when aviators or astronauts are exposed to low-pressure environments, in this case $P_\infty = P_{amb} = 101.325kPa$ (the sea level pressure).The normal and critical gas bubbles in a bio tissue throughout a relation of its radius as a function of time, while previous authors presented a numerical or an implicit solution for the problem such as [1,2] and [5,6,7,8,9,10,11,12,13] The growth problem is discussed for unsteady flow in tissue by Mohammadein and Mohamed [5]; which based on the three region model. Moreover, the concentration distribution around a stationary growing gas bubble in a bio tissue is obtained analytically for two main stages.

Strinivasan et al [8] have solved the problem in the case of quasi-static pressure. Mohammadein and Mohamed [5] solved the problem when the effect of changing in concentration with the time takes place. The growth stages can be repeated sequentially , while the diver ascent quickly to a lower-pressure sea level and dives horizontally, and so on until he reaches the sea level pressure (1 atm).

The effort is devoted to study the three-region model(gas bubble, thin boundary layer and well-stirred finite bio tissue). Mohmadein and Mohamed model(zero injection) [5] is extended to observe the effects of injection process in the bio tissues during ascent of divers.

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The growth of gas bubbles and concentration distribution are obtained during the decompression of variable and constant ambient pressure under the effect of injection process for some physical parameters. The extended method of combination variables [8] is used for solving the current problem.

2 Analysis

A single gas bubble as given by Fig.2 is considered to grow inside bio tissue between two finite boundaries R_0 and R_m under the injection process. The growth is affected by some parameters such as pressure difference between bubble pressure $P_g(R(t),t)$ and the ambient pressure $P_{amb}(t)$ surface tension of the mixture σ inside a bio tissue at the bubble boundary, concentration distribution difference between the two phases and other physical parameters.

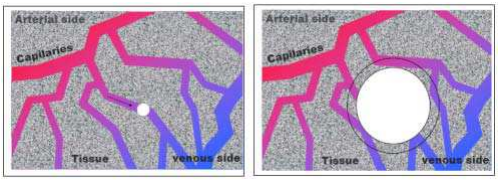


Fig. 1: On the left, in the initial phase of the decompression, an arterial bubble enters a tissue capillary net. It exchanges gas with the surrounding tissues and starts growing. If it reaches a critical radius, it might block the blood supply and cause ischemia. On the right, in the last phase of the decompression, a bubble has grown to a large volume using dissolved gas available in the surrounding tissue. Its mechanical action might cause pain.

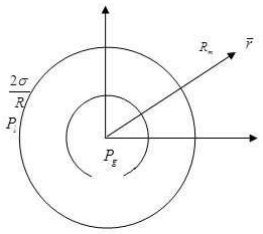


Fig. 2: The problem sketch

The growth of gas bubble as in Fig.2 has been studied on the basis of the following assumptions:
- Gases are considered to be ideal.
- The bubble is assumed to have a spherical geometry.
- Gas density distribution inside the bubble is assumed to be uniform.
- The growth affected by suction process in bio tissue.

The mathematical model describing the current problem consists of four main equations(mass, diffusion, fick's and concentration equation)

2.1 Mass balance equation

The rate of gas uptake by a bio tissue is the amount carried by the blood per unit time less than flux into the gas bubble .assuming equilibration of a bio tissue gas with venous blood gas. thus, the mass equation has the form

$$\alpha_T V_T \frac{dp_T}{dt} = \alpha_b V_T Q(P_a - P_T) - \frac{1}{\Re T}\frac{d}{dt}(P_g V_g) \quad (1)$$

2.2 Pressure balance equation

The relation between pressure inside and outside gas bubble under the effects of surface tension at the gas-liquid interface and neglecting tissue viscoelastic effects, becomes

$$P_g = P_{amb} + \frac{2\sigma}{R} \quad (2)$$

where $P_{amb}(t) = P_0 \dot{\alpha} t$.

2.3 Fick's equation

The molar flux of gas through the bubble surface equals the rate of change of molar concentration of gas in the bubble, then of change of molar concentration of gas in the bubble, then

$$\frac{1}{\Re T}\frac{d}{dt}\left(\frac{4}{3}\pi R^3 P_g\right) = 4\pi R^2 D_T \left(\frac{\partial C}{\partial T}\right)_{r=R} \quad (3)$$

2.4 Diffusion equation

Diffusion Equation with injection process and convection term is described as follow

$$\frac{\partial c}{\partial t} + \frac{\varepsilon R^2 R}{r^2} = D_T\left(\frac{\partial^2 C}{\partial r^2} + \frac{2}{r}\frac{\partial c}{\partial r}\right) + \frac{bD_T}{r}\frac{\partial c}{\partial r}. \quad (4)$$

where

$$b = \begin{cases} 1 \ injection \ process \\ 0 \ zero \ injection \ process \end{cases} \quad (5)$$

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To solve the diffusion equation, we use the combined variables by assuming that

$$C(r,t) = C(s). \tag{6}$$

Where

$$r = \frac{sf(t)}{\beta}. \tag{7}$$

Where $r = R$ then $s = \beta$ and $R = f(t)$.
Based on the above assumptions then equation (5) becomes

$$\left[\frac{-s^2 f}{\beta r} + \frac{\varepsilon f f^2}{r^2}\frac{s}{r}\right]\frac{dC}{ds} = D_T\left[\frac{s^2}{r^2}\frac{d^2C}{dt^2} + \frac{2s}{r^2}\frac{dC}{ds}\right] + \frac{bD_T s}{r^2}\frac{dC}{ds}. \tag{8}$$

By using equation (6) we can get

$$ff' = \frac{D_T\beta^2}{\left(-s + \frac{s\beta^2}{s^2}\right)}\left[\frac{\left(\frac{d^2C}{ds^2}\right)}{\left(\frac{dC}{ds}\right)} + \left(\frac{2+b}{s}\right)\right] = D_T^2\mu. \tag{9}$$

The separation constant in the form D_T^2, where μ is a constant, divides equation (9) into two ordinary differential equation to be solved

$$ff' = D_T^2\mu. \tag{10}$$

And

$$\frac{d}{ds}ln\left(\frac{dC}{ds}\right) = \frac{\mu D_T}{\beta^2}\left[-s + \frac{\beta^3}{s^2}\right] - \frac{(2+b)}{s}. \tag{11}$$

Integrating equation (10), and using the initial conditions at $t = t_0, R = R_0$, thus

$$R = \sqrt{2\mu D_T(t-t_0) + R_0^2}. \tag{12}$$

And by integrating equation (11), we obtain

$$\frac{dC}{ds} = \frac{k}{s^{(2+b)}}\exp\left(\frac{-\mu D_T}{\beta^2}\left(\frac{s^2}{2} + \frac{\varepsilon\beta^3}{s}\right)\right). \tag{13}$$

To evaluate the constant k the boundary Condition (6), by using of equation (7) is modified to be

$$\frac{\partial C}{\partial r}|_{r=R} = \frac{R^2\dot{P}_{amb} + 4\sigma R + 3\dot{P}_{amb}R\dot{R}}{3R\Re TD_T}. \tag{14}$$

Where

$$\frac{\partial C}{\partial r} = \frac{s}{r}\frac{dC}{ds}. \tag{15}$$

Using the boundary condition at the bubble wall equation (15) can be rewritten as:

$$\left.\frac{\partial C}{\partial r}\right|_{s=\beta} = \left.\frac{r}{s}\frac{\partial C}{\partial r}\right|_{r=R,s=\beta} = \frac{R^2\dot{P}_{amb} + 4\sigma R + 3\dot{P}_{amb}R\dot{R}}{3R\Re TD_T}. \tag{16}$$

Equations(13) and(16)give

$$k_l = \beta^{(1+b)}\frac{R^2\dot{P}_{amb} + 4\sigma R + 3\dot{P}_{amb}R\dot{R}}{3R\Re TD_T}\exp(\mu D_T(\varepsilon + \frac{1}{2}). \tag{17}$$

Where $\dot{R} = \frac{\mu D_T^2}{R}$.
From equations (13) and(15),we have

$$\frac{\partial C}{\partial r} = \frac{KR^{(1+b)}}{r^{(2+b)}}\exp\left(-\mu D_T\left(\frac{r^2}{2R^2} + \frac{\varepsilon R}{r} - \left(\varepsilon + \frac{1}{2}\right)\right)\right). \tag{18}$$

Where $s = \frac{\beta r}{R}$ and $k = \frac{k_1}{\beta^{(1+b)}}$.
Integrating the previous equation through the interval from any instant t to $t_m t$ which the bubble reaches its maximum radius R_m, at this instant $C(R_m, t_m) = C_\infty$ that is

$$C(r,t) - C_\infty = -kR^{(1+b)}\int_r^{R_m}\frac{1}{r^{(2+b)}} \exp\left(-\mu D_T\left(\frac{r^2}{2R^2} + \frac{\varepsilon R}{r} - \left(\varepsilon + \frac{1}{2}\right)\right)\right)dr. \tag{19}$$

Putting $z = 1 - \frac{R}{r} \Rightarrow dz = \frac{R}{r^2}dr, r = R \Rightarrow z = 0$, and $r - R_m \Rightarrow z = 1 - \frac{R}{R_m}$, we get

$$C - C_\infty = -k\int_0^{1-\frac{R}{R_m}}(1-z)^b \exp\left(-\mu D_T\left(\frac{1}{2(1-z)^2} + \varepsilon(1-z) - \left(\varepsilon + \frac{1}{2}\right)\right)\right)dz. \tag{20}$$

Or in the other form

$$C - C_\infty = -k\int_0^{1-\frac{R}{R_m}}(1-z)^b \exp\left(\frac{-\mu D_T}{2}\left(\frac{1-(1+2\varepsilon z)(1-z)^2}{(1-z)^2}\right)\right)dz. \tag{21}$$

Since $D_T \ll 1$ and $-\leq z < 1 - \frac{R_0}{R_m} < 1$, we can approximate the integral and the previous integral takes the form:

$$C - C_\infty \cong -k\int_0^{1-\frac{R}{R_m}}(1-z)^b(1-\mu D_T(1-\varepsilon)z - \frac{3}{2}\mu D_T z^2 - \frac{4}{2}\mu D_T z^2 - \frac{5}{2}\mu D_T z^4 - \ldots)dz. \tag{22}$$

But we can put $(1-z^b = 1 - bz + 0(z^2)$, the above equation by neglect the variable z of the orders greater

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than the second order can be rewrite in the form :

$$C(r,t)-C_\infty=-k\int_0^{1-\frac{R}{R_m}}(1-\mu D_T(1-\varepsilon)$$
$$z-\frac{3}{2}\mu D_T z^2-bz+b\mu D_T(1-\varepsilon)z^2\ldots)dz. \quad (23)$$

Therefore,

$$C-C_\infty=-k$$
$$(\frac{2\frac{R(t)}{R_m}(1-\frac{R(t)}{R_m})-\mu D_T(1-\frac{R(t)}{R_m})^2(1-\varepsilon\frac{R(t)}{R_m})-b(1-\frac{R(t)}{R_m})^2}{2\frac{R(t)}{R_m}}). \quad (24)$$

To find the constant μ, using the initial condition, $t=t_0\Rightarrow c=C_0$, $k=k_0$, $\dot{R}=\dot{R}_0$, $P_{amb}=P_{amb0}$ and $\dot{P}_{amb}=\dot{P}_{amb0}$, we get

$$\Delta C_0=h_0$$
$$(\frac{2\varphi_0^{\frac{1}{3}}(1-\varphi_0^{\frac{1}{3}})-\mu D_T(1-\varphi_0^{\frac{1}{3}})^2(1-\varepsilon\varphi_0^{\frac{1}{3}})-2\varphi_0^{\frac{1}{3b}}(1-\varphi_0^{\frac{1}{3}})^2}{2\varphi_0^{\frac{1}{3}}}). \quad (25)$$

Where $\Delta C_0=C_\infty-C(R_0,t_0)$, $\varphi_0=\left(\frac{R_0}{R_m}\right)^2$ and,

$$k_0=\frac{R_0^2\dot{P}_{amb0}+4\sigma\dot{R}_0+3P_{amb0}R_)\dot{R}_0}{3\Re TD_T}. \quad (26)$$

From equation (25), we get the following expression for μ:

$$\mu=\frac{2\varphi_0^{\frac{1}{3}}\left(\left(1-\varphi_0^{\frac{1}{3}}\right)-\frac{\Delta C_0}{k_0}-b\left(1-\varphi_0^{\frac{1}{3}}\right)^2\right)}{D_T\left(1-\varphi_0^{\frac{1}{3}}\right)^2\left(1-\varepsilon\varphi_0^{\frac{1}{3}}\right)}. \quad (27)$$

Substituting for μ into equation (12), we get the relation of the bubble radius as a function of time. That is:

$$R=$$
$$\sqrt{4D_T\frac{\varphi_0^{\frac{1}{3}}\left(\left(1-\varphi_0^{\frac{1}{3}}\right)-\frac{\Delta C_0}{k_0}-b\left(1-\varphi_0^{\frac{1}{3}}\right)^2\right)}{\left(1-\varphi_0^{\frac{1}{3}}\right)^2\left(1-\varepsilon\varphi_0^{\frac{1}{3}}\right)}(t-t_0)+R_0^2}. \quad (28)$$

The constant k_0 has two formulae due to the case of the ambient pressure, for variable ambient pressure at decompression; suppose the ambient pressure linearly decreases with time, i.e. $P_{amb}(t)=P_0-\dot{\alpha}t$, where $\dot{\alpha}$ is the ascent rate [11], k_0 has the form:

$$k_d=\frac{\dot{\alpha}R_0^2+4\sigma\dot{R}_0+3(P_0-\dot{\alpha}t)R_0\dot{R}_0}{3\Re RD_T}. \quad (29)$$

And for constant ambient pressure (after decompression), $\dot{P}_{amb}=0$, i.e. $P_{amb}(t)=const.=P_\infty$, at diving stops or after finishing diving and reaching the sea level it has the formula:

$$k_c=\frac{4\sigma\dot{R}_0+3P_\infty R_0\dot{R}_0}{3\Re TrD_T}. \quad (30)$$

The growth stages (variable or constant ambient pressure) can be repeated sequentially, while the diver ascents quickly to a lower-pressure level till he reaches the sea level pressure (1atm).

The same effect of decompression can be occurred, when aviators or astronauts are exposed to low-pressure environments, in this case $P_0=P_{atm}=101.325N.m^{-2}$ (the sea level pressure).

The time for the bubble to reach its maximum radius, can be calculated by applying the final conditions on Eq.(12) to get:

$$t_m=\frac{R_m^2-R_0^2}{2\mu D_T}+t_0. \quad (31)$$

From Eq.(28), we can get

$$\dot{R}=$$
$$2D_T\frac{\varphi_0^{\frac{1}{3}}\left(\left(1-\varphi_0^{\frac{1}{3}}\right)-\frac{\Delta C_0}{k_0}-b\left(1-\varphi_0^{\frac{1}{3}}\right)^2\right)}{\left(1-\varphi_0^{\frac{1}{3}}\right)^2\left(1-\varepsilon\varphi_0^{\frac{1}{3}}\right)}\cdot$$
$$\sqrt{4D_T\frac{\varphi_0^{\frac{1}{3}}\left(\left(1-\varphi_0^{\frac{1}{3}}\right)-\frac{\Delta C_0}{k_0}-b\left(1-\varphi_0^{\frac{1}{3}}\right)^2\right)}{\left(1-\varphi_0^{\frac{1}{3}}\right)^2\left(1-\varepsilon\varphi_0^{\frac{1}{3}}\right)}(t-t_0)+R_0^2}. \quad (32)$$

And

$$\dot{R}_0=\frac{2D_T\varphi_0^{\frac{1}{3}}\left(-\frac{\Delta C_0}{k_0}-b\left(1-\varphi_0^{\frac{1}{3}}\right)\right)}{R_0\left(1-\varphi_0^{\frac{1}{3}}\right)\left(1-\varepsilon\varphi_0^{\frac{1}{3}}\right)}. \quad (33)$$

2.5 Concentration Distribution Around a Growing Gas Bubble in a bio Tissue

Concentration Distribution Around a Growing Gas Bubble in a bio Tissue Equation (20) can written in the form

$$C(r,t)=C_\infty-k(t)\int_{(1-\frac{R}{r})}^{(1-\frac{R}{R_m})}(1-z)b$$
$$\exp\left(-\frac{\mu D_T}{2}\left(\frac{1-(1+2\varepsilon z)(1-z)^2}{(1-z)^2}\right)\right)dz. \quad (34)$$

The integral can be approximated as before to give

$$C(r,t)=C_\infty-k(t)$$
$$\left[(\frac{R}{r}-\frac{R}{R_m})+\frac{\mu D_T}{2R}\left((1-\frac{R}{r})^2(r-\varepsilon R)-(1-\frac{R}{R_m})^2(R_m-\varepsilon R)\right)\right.$$
$$\left.+b\left((1-\frac{R}{r})^2-(1-\frac{R}{R_m})\right)\right] \quad (35)$$

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3 Implementation

Suppose a diver at depth at which the ambient pressure $P_0 = 200kp \approx 2atm$ under this pressure more amount of the nitrogen gas is dissolved in some body tissues, if the diver ascents quickly, with ascent rate $\dot{\alpha} = 3066.67N/m^2s$, to the sea level at which the ambient pressure is the atmospheric pressure, then decompression process will take place and the growth of nitrogen bubbles occurs throughout the two stages, the first one takes place throughout the decompression process, for nearly $65.1488s$ till the diver reaches the sea level pressure, the second one takes at constant ambient pressure, sea level, $P_\infty = P_{atm} = 101.325kpa$..

The following table shows the data which used to simulate the problem for decompression stage as given by authors [2,8,11] (see table 1).

By using Mathematica program (version), the following graphs that demonstrate the effect of the physical parameters on the growth of the gas bubble and concentration distribution are obtained.

Table 1: The data which is used to get the graphs needed to show the effect of the physical parameters on the growth of the gas bubble.

P	Value	Unit	P	Value	Unit
T	$310(36C^0)$	K	R_0	1.0×10^{-6}	m
P_0	200.000	$N.m^{-2}.$	P_∞	101.325	$N.M^{-2}$
ΔC_0	0.7	$Mol.m^3$	$\Re$	8.314472	$m/mol.K$
σ	0.03.	$N.m.$	$\dot{\alpha}$	3066.67	$N/m^2.s$
D_T	$2.2\times10^{-12}.$	$m^2.s$	φ_0	]0,1[.	—

4 Discussion of the results

The diffusion equation (4), for a convective growing gas bubble in tissue with acceleration convective under the effect of injection in ambient when the pressure is constant and variant is solved by the method of combined variables. The solution of the problem equation (28), gives the growth of gas bubble radius as a function of time combined with the physical parameters that affect on the growth process, such as the initial void fraction φ_0, the diffusivity of the tissue D_T, the ascent rate $\dot{\alpha}$, surface tension σ, the initial radial velocity $\dot{R}_0$, and the initial difference in concentration ΔC_0.

The growth of gas bubbles in terms of time for two different values of parameter ”b” is shown in Fig.[3]. It is observed that, the growth under the effect of injection process performs values greater than Mohammadein and Mohamed (zero injection) [3] when ambient pressure is constant or variant. The growth of gas bubble in terms of time for two different values of gas diffusion coefficient in a bio tissue when ambient pressure is variant is shown in Fig.[4].

It observed that the growth of gas bubbles is directly proportional with gas diffusion coefficient for all values of parameter ”b”.

The growth of gas bubble in terms of time for two different values of void fraction in a bio tissue when ambient pressure is variant is shown in Fig.[5].

The concentration distribution surrounded a growing gas bubble in a bio tissue under injection and zero-injection processes is shown by Figs.[6],[7] respectively.

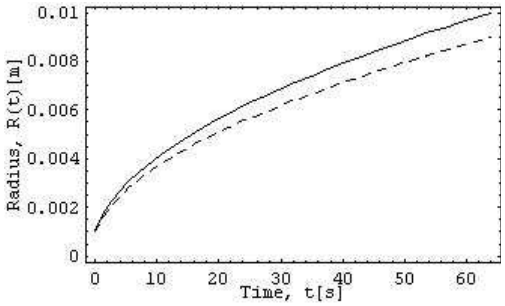


Fig. 3: The effect of injection and zero injection process on the growth of gas bubble in a bio tissue for variant ambient pressure(⋯ b=0, —b=1.0).

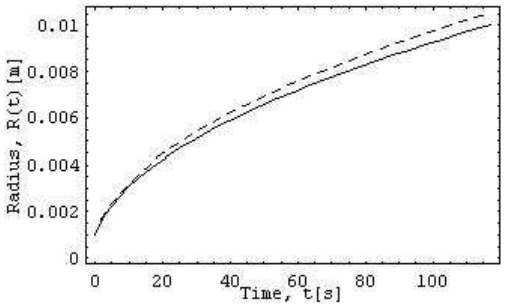


Fig. 4: The increasing of growing gas bubble in a bio tissue for gas diffusion coefficient when ambient pressure is variant($\cdots .D_T = 2.2\times10^{-12}, - - - D_T = 4.4\times10^{-12}$).

The concentration distribution around a growing gas bubble in a bio tissue is presented for the two main stages as obtained by equations [29],[30] and [35].

5 Conclusion

The gas diffusion problem is discussed for unsteady flow in a bio tissue, based on the three-region model [8]. The concentration distribution around a growing gas bubble in

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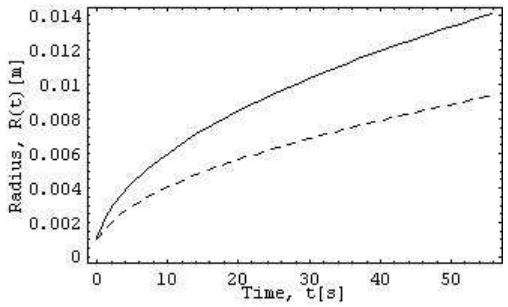


Fig. 5: The growth of gas bubble in a bio tissue when ambient pressure is variant for two different values of void fraction($- -$ $-\varphi_0 = 0.01$ and — $\varphi_0 = 0.02$).

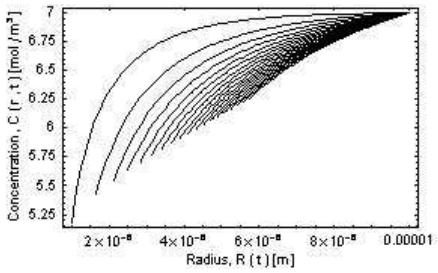


Fig. 6: The cosentration distrebution around a growing nitrogen bubble in a bio-tissue for diver under the effect of zero injection procecess (b=0).

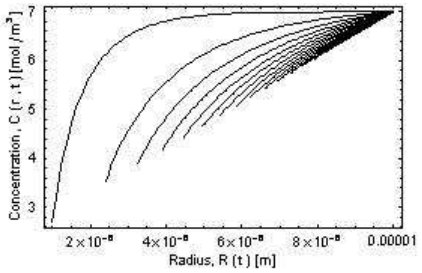


Fig. 7: The cosentration distrebution around a growing nitrogen bubble in a bio-tissue under the effect of injection procecess (b=1).

a bio tissue is obtained analytically for two main stages as given by Eqs. (29), (30) and (35) respectively. The discussion of results and figures concluded the following remarks:

1. The growth of gas bubble radius is directly proportional to the ascent rate $\dot{\alpha}$, the initial difference of concentration ΔC_0, the diffusivity of the tissue D_T, the initial void fraction φ_0 and inversely proportional to the surface tension σ of the bio tissue.
2. The growth of gas bubble radius is directly proportional to all values of parameter "b".
3. The effect of injection process (b=1) on the growth and concentration distribution performs values greater than that obtained by Mohammadein and Mohamed model [5].
4. When b=0 Mohammadein and Mohamed model [5] is obtained as a special case from the present model
5. The effect of injection process is very dangerous on the bio tissues increases pressure difference in blood circulation of divers when ascending quickly to the surface of sea and causes the a schema.
6. The injection process affect on the systemic blood circulation and delays the growth of gas bubbles; which increasing the incidence of decompression sickness (DCS).
7. The study warns the divers to take any kind of injection during the dive process to avoid the incidence of decompression sickness(DCS).

Nomenclature

C : Concentration of dissolved gas $[mol.m-3]$.
C_∞: Concentration of dissolved gas in the tissue far from the bubble $[mol.m-3]$.
$\Delta C_0 := C_\infty - C_0$. The concentration difference $[mol.m-3]$.
k : Time-dependent, concentration variable; defined by equation (18a) $[mol.m-3]$.
P_a: Gas partial pressure in arterial blood $[N.m^{(-2)}]$, equation (10).
P_{amb} : Ambient pressure $[N.m^{(-2)}]$.
P_{atm}: Atmospheric pressure $[N.m^{(-2)}]$.
P_g: Pressure of the bubble wall $[N.m^{(-2)}]$.
$\dot{Q}$: Blood flow per unit tissue volume $[s^{(-1)}]$.
$\Re$: general gas constant $[N.m/mol.K]$.
D_T : Gs diffusion coefficient in tissue $[mol.m-1]$.
r : The distance from the origin of the bubble.
R : Instantaneous bubble wall radius $[m]$.
R_0 : Initial bubble wall radius $[m]$.
$\dot{R}$: Instantaneous bubble wall radius velocity $[m.s^{(-1)}]$.
t : Time elapsed $[s]$.
T : Temperature of the gas inside the bubble $[K]$.

Greek symbols

$\dot{\alpha}$: Ascent rate $[N.m^{(-2)}.s^{(-1)}]$.
α_b : Gas solubility in blood $[s^2.m^{(-2)}]$.
α_t : Gas solubility in tissue $[s^2.m^{(-2)}]$.
μ : Constant given by equation (27) $[s.m^{(-2)}]$.
σ : The surface tension of liquid surrounding the bubble $[N.m^{(-1)}]$.

Subscripts

0 : Initial value quantities.
c : After the decompression process (constant ambient pressure).
d : Through the decompression process (variable ambient pressure).
g : Constant and variables corresponding to the gas bubble.
m : Final or maximum value.
T : Constant and variables corresponding to the tissue.

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