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Unified Fixed Point Theorems on $\in -$Chainable Fuzzy Metric Space and Modified Intuitionistic Fuzzy Metric Space

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Abstract: The theory of fuzzy set has been constantly growing by many eminent mathematicians. The core objective of present study is to give a criteria of getting unique fixed point for six mappings on $\in -$ chainable fuzzy metric space. Here, special attention is paid to using the concept of weakly compatible and continuous mapping on complete fuzzy metric spaces. Moreover, we define the concept of $\in -$chain on modified intuitionistic fuzzy metric space (say MIFM-space) . With support of this notion, we ascertain a fixed point result on modified intuitionistic fuzzy metric space. In this paper, we also elongate the main result for finite number of mappings and give an application for integral type contraction on fuzzy metric space and MIFM-space.

Keywords: Common Fixed Point, FM-Space, $\in -$ Chainable FM-Space, Complete Subspace, Weakly Compatible Mappings, MIFM-Space.

1 Introduction and Preliminaries

Fuzzy sets [29] are fully defined by its membership functions. Fuzzy set theory propose a framework where logical as well as trade off operations have their importance. Applications of fuzzy set theory to real problems are abound. FM-space is originated with help of probabilistic metric space given by Menger [19]. The theory of FM- space was initially presented by Kramosil and Michalek [17] after drawn-out the notion of fuzzy set. This concept released a possibility for advance progress of study in such spaces. The significant theory of FM-space has been studied extensively by numerous authors in several means. Georage and Veeramani [9] revised this notion to GV- fuzzy metric space.

Definition 1. [25] An operation $* : [0,1] \times [0,1] \rightarrow [0,1]$ is called continuous triangular norm if $(*, [0,1])$ is a topological abelian monoid and following conditions as
1. $\alpha * 1 = \alpha, \forall \alpha \in [0,1]$
2. $\alpha * \beta \leq \gamma * \eta$ whenever $\alpha \leq \gamma$ and $\beta \leq \eta$, $\forall \alpha, \beta, \gamma, \eta \in [0,1]$.

Definition 2. [9] The triplet $(Y, \mathcal{F}, *)$ is FM-space if Y is an arbitrary set, $*$ is continuous t-norm, $\mathcal{F}$ is fuzzy set in $Y^2 \times (0,\infty)$ satisfying the conditions as for all $\alpha, \beta \in Y, t, s > 0$,
1. $\mathcal{F}(\alpha,\beta,t) > 0$,
2. $\mathcal{F}(\alpha,\beta,t) = 1$, iff $\alpha = \beta$,
3. $\mathcal{F}(\alpha,\beta,t) = \mathcal{F}(\beta,\alpha,t)$,
4. $\mathcal{F}(\alpha,\beta,t) * \mathcal{F}(\beta,\delta,s) \leq \mathcal{F}(\alpha,\delta,t+s)$, $\forall \alpha, \beta, \delta \in Y$,
5. $\mathcal{F}(\alpha,\beta,t) : [0,\infty) \rightarrow [0,1]$ is left continuous.

Jungck and Rhoades [15] named a couple of self maps to be coincidentally commuting if they commute at their coincidence points.

Definition 3. [15] The self-maps P and Q on set Y are known as weakly compatible if $PQx = QPx$ such that $Px = Qx$ for some $x \in Y$

Definition 4. [10] Let $(Y, \mathcal{F}, *)$ be a FM-space and a sequence $\{x_n\}$ in Y is supposed to be convergent to a point $x \in Y$, if for each $\in > 0$ and $t > 0$, there exists δ_0 such that $\mathcal{F}(x_n, x, t) > 1- \in, \forall n \geq \delta_0$.

Definition 5. [9] Let $(Y, \mathcal{F}, *)$ be a FM-space and a sequence $\{x_n\}$ in Y is said to be Cauchy sequence, if for each $\in > 0$ and $t > 0$, there exists δ_0 such that $\mathcal{F}(x_n, x_m, t) > 1- \in, \forall n, m \geq \delta_0$.

Definition 6. [9] A FM-space $(Y, \mathcal{F}, *)$ where every Cauchy sequence is convergent is supposed to be complete.

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Lemma 7. [18] Let $(Y, \mathcal{F}, *)$ be a FM-space. If there occurs $k \in (0,1)$ as $\mathcal{F}(\alpha, \beta, kt) \geq \mathcal{F}(\alpha, \beta, t)$ for all $\alpha, \beta \in Y$, then $\alpha = \beta$.

Cho et al. [5] presented the thought of $\in$-chainable FM-space and achieved common fixed point theorems based on this concept.

Definition 8. [5] Let $(Y, \mathcal{F}, *)$ be a FM-space and $\in > 0$. A finite sequence $x = x_0, x_1, x_2, \cdots, x_n = y$ is called $\in$-chain from x to y if $\mathcal{F}(x_i, x_{i-1}, t) > 1- \in$ for all $t > 0$ and $i = 1, 2, 3, \cdots, n$. Such a FM-space is named as $\in$-chainable FM-space.

Atanassov [2] announced and studied the theory of IF-set as a well-known generality of fuzzy set which has enthused penetrating research bustle around intuitionistic fuzzy set (shorty IF-set). Coker[6] familiarized the notion of the topology of IF-sets. By using the knowledge of IF-sets, Park [20] coined the definition of intuitionistic fuzzy metric spaces with the support of triangular norms, co-norms from the FM-space due to George and Veeramani [9]. Later on, Saadati et al. [24] proposed the very significant idea about modified intuitionistic fuzzy metric spaces (MIFM-spaces).

Lemma 9. [8] Consider the set L^* and operation $\geq_{L^*}$ defined by

$L^* = \left\{(u,v) : (u,v) \in [0,1]^2, u + v \leq 1\right\}$,

$(u,v) \geq_{L^*} (w,z) \Leftrightarrow u \leq w$ and $v \geq z$ for every $(u,v), (w,z) \in L^*$. Then $(L^*, \geq_{L^*})$ is a complete lattice.

Definition 10. [7] A triangular norm on L^* is a mapping $\mathcal{T} : (L^*)^2 \to L^*$ satisfying the following conditions:

1. $\mathcal{T}(x, 1_{L^*}) = x$,
2. $\mathcal{T}(x, y) = \mathcal{T}(y, x)$,
3. $\mathcal{T}(x, \mathcal{T}(y, z)) = \mathcal{T}(\mathcal{T}(x, y), z)$,
4. $x \leq_{L^*} x'$ and $y \leq_{L^*} y' \Rightarrow \mathcal{T}(x, y) \leq_{L^*} \mathcal{T}(x', y')$

for all $x, y, z, x', y' \in L^*$.

Definition 11. [7] A negator on L^* is a decreasing mapping $\mathcal{N} : L^* \to L^*$ satisfying $\mathcal{N}(0_{L^*}) = 1_{L^*}$ and $\mathcal{N}(1_{L^*}) = 0_{L^*}$. A negator on $[0,1]$ is a decreasing mapping $N : [0,1] \to [0,1]$ satisfying $N(0) = 1$ and $N(1) = 0$. In what follows, N_s denotes the standard negator on $[0,1]$ defined as $N_s(x) = 1 - x$ for all $x \in [0,1]$.

Definition 12. [24] The 3-tuple $(Y, \mathcal{F}_{V,W}, \mathcal{T})$ is said to be an MIFM-space if Y is a non-empty set, $\mathcal{T}$ is a continuous t-representable. Let V and W are fuzzy set such that $V(x,y,t) + W(x,y,t) \leq 1$ for all $x, y, z \in Y$, $t, s > 0$ and $\mathcal{F}_{V,W}$ is a mapping $Y \times Y \times (0, \infty) \to L^*$ satisfying the following conditions:

1. $\mathcal{F}_{V,W}(x, y, t) >_{L^*} 0_{L^*}$,
2. $\mathcal{F}_{V,W}(x, y, t) = 1_{L^*} \Leftrightarrow x = y$,
3. $\mathcal{F}_{V,W}(x, y, t) = \mathcal{F}_{V,W}(y, x, t)$,
4. $\mathcal{F}_{V,W}(x, y, t + s) \geq_{L^*} \mathcal{T}(\mathcal{F}_{V,W}(x, z, t), F_{V,W}(z, y, s))$,
5. $\mathcal{F}_{V,W}(x, y, \cdot) : (0, \infty) \to L^*$ is continuous.

In this instance, $\mathcal{F}_{V,W}$ is called a modified intuitionistic fuzzy metric.

Here, $\mathcal{F}_{V,W}(x, y, t) = (V(x,y,t), W(x,y,t))$.

Lemma 13. [23] Let $(Y, \mathcal{F}_{V,W}, \mathcal{T})$ be a MIFM-space. Then for all $x, y \in Y$, $t > 0$, $\mathcal{F}_{V,W}(x, y, t)$ is non-decreasing with respect to t in $(L^*, \leq_{L^*})$.

Lemma 14. [24] Let $(Y, \mathcal{F}_{V,W}, \mathcal{T})$ be a MIFM-space. Then $\mathcal{F}_{V,W}$ is continuous on $Y^2 \times (0, \infty)$.

Definition 15. [24] The sequence $\{x_n\}$ is said to be convergent to $x \in Y$ in the MIFM-space $(Y, \mathcal{F}_{V,W} \mathcal{T})$ and is generally denoted by $x_n \to x$ if

$\mathcal{F}_{V,W}(x_n, x, t) \to 1_{L^*}$ whenever $n \to \infty$ for every $t > 0$.

Definition 16. [24] A sequence $\{x_n\}$ in a MIFM-space, $(Y, \mathcal{F}_{V,W}, \mathcal{T})$ is called a Cauchy sequence if for each $0 < \epsilon < 1$ and $t > 0$, there exists $n_0 \in N$ such that

$\mathcal{F}_{V,W}(x_n, x_m, t) >_{L^*} (N_s(\epsilon), \epsilon)$

and for each $n, m \geq n_0$, where N_s is the standard negator.

Lemma 17. [24] Let $(Y, \mathcal{F}_{V,W}, \mathcal{T})$ be a modified intuitionistic fuzzy metric spaces. If

$\mathcal{F}_{V,W}(x_n, x_{n+1}, t) \geq_{L^*} \mathcal{F}_{V,W}(x_0, x_1, k^n t)$

for some $k > 1$ and $n \in N$ (set of natural numbers). Then x_n is a Cauchy sequence.

In (2002), Branciari [4] gave an analogue of Banach contraction principle by defining Lebesgue- integrable function and proved a fixed point theorem satisfying contractive condition of integral type.

Definition 18. [4] A function $\hbar(t)$ is called a Lebesgue-integrable function if

1. $\hbar : [0, +\infty) \to [0, +\infty)$ is Lebesgue summable for each compact of R_+,
2. its permitivity $A : [0, +\infty) \to [0, +\infty)$, as $A(t) = \int_0^t \hbar(t)dt$, for all $t > 0$, is well defined, non decreasing and continuous,
3. moreover, if for each $\epsilon > 0, A(\epsilon) > 0$, this permittivity fulfill $A(t) = 0$ iff $t = 0$.

Lemma 19. [4] Let (Y, d) be a complete metric space, $c \in (0,1)$ and let $f : Y \to Y$ be a mapping such that for all $x, y \in Y$,

$\int_0^{d(fx, fy)} \hbar(t)dt \leq c \int_0^{d(x,y)} \hbar(t)dt$,

where $\hbar : R^+ \to R$ is a Lebesgue integrable mapping which is summable on each compact set $[0, \infty)$, non negative and such that $\int_0^\epsilon \hbar(t)dt > 0$ for each $\epsilon > 0$, then f has a unique fixed point $a \in Y$ such that each $x \in Y, \lim_{n\to\infty} f^n = a$.

2 Main Results

Our results use the notion of the continuous, non-decreasing mapping $\phi : [0,1] \to [0,1]$ such as $\phi(r) > r$ for all $r \in (0,1)$and $\phi(1) = 1$.

Theorem 1. Let $(Y, \mathcal{F}, *)$ be a complete $\in$-chainable FM-space with t-norm $a * b = \min\{a, b\}$. Let P, Q, R, S, T and U are self-mappings on Y such that for all $x, y \in Y, t > 0$

$$\mathcal{F}(Tx, Uy, kt) \geq \phi\left\{\begin{array}{c} \mathcal{F}(PQx, RSy, t) * \mathcal{F}(PQx, Tx, t) * \\ \mathcal{F}(RSy, Uy, t) * \frac{1}{2}[\mathcal{F}(PQx, Uy, t) \\ +\mathcal{F}(RSy, Tx, t)] \end{array}\right\}, \tag{1}$$

where $k \in (0,1)$. Also, mappings satisfies following conditions:
2. $T \subset RS(Y)$, $U \subset PQ(Y)$,
3. T and PQ are continuous,
4. the pairs (T, PQ) and (U, RS) are weakly compatible.
Then T, RS, U and PQ have a unique common fixed point in Y.

Proof. Let us choose $x_0 \in Y$, from condition (2), there exist $x_1, x_2 \in Y$ such that
$Tx_0 = RSx_1 = y_0$ and $Ux_1 = PQx_2 = y_1$.
Inductively, we construct sequences $\{x_n\}$ and $\{y_n\}$ in Y such that
$Tx_{2n} = RSx_{2n+1} = y_{2n}$ and
$Ux_{2n+1} = PQx_{2n+2} = y_{2n+1}$,
$\forall\, n = 0, 1, 2 \cdots$.
We will claim that $\{y_n\}$ is a Cauchy sequence in Y.
From (1), we have

$$\mathcal{F}(y_{2n}, y_{2n+1}, kt) = \mathcal{F}(Tx_{2n}, Ux_{2n+1}, kt)$$
$$\geq \phi\left\{\begin{array}{c}\mathcal{F}(PQx_{2n}, RSx_{2n+1}, t)* \\ \mathcal{F}(PQx_{2n}, Tx_{2n}, t)* \\ \mathcal{F}(RSx_{2n+1}, Ux_{2n+1}, t)* \\ \frac{1}{2}[\mathcal{F}(PQx_{2n}, Ux_{2n+1}, t) \\ +\mathcal{F}(RSx_{2n+1}, Tx_{2n}, t)]\end{array}\right\}$$
$$= \phi\left\{\begin{array}{c}\mathcal{F}(y_{2n-1}, y_{2n}, t)* \\ \mathcal{F}(y_{2n-1}, y_{2n}, t)* \\ \mathcal{F}(y_{2n}, y_{2n+1}, t)* \\ \frac{1}{2}[\mathcal{F}(y_{2n-1}, y_{2n}, t) \\ +\mathcal{F}(y_{2n}, y_{2n+1}, t)]\end{array}\right\}$$
$$= \phi\left\{\mathcal{F}(y_{2n-1}, y_{2n}, t) * \mathcal{F}(y_{2n}, y_{2n+1}, t)\right\}.$$

There are two cases arise, which are discuses below:
Case 1: If $\mathcal{F}(y_{2n-1}, y_{2n}, t) > \mathcal{F}(y_{2n}, y_{2n+1}, t)$, then by using the property $\phi(t) > t$, we get

$$\mathcal{F}(y_{2n}, y_{2n+1}, kt) \geq \phi\left\{\mathcal{F}(y_{2n}, y_{2n+1}, t)\right\}$$
$$> \mathcal{F}(y_{2n}, y_{2n+1}, t).$$

Thus by using Lemma 7, one can get
$\lim_{n\to\infty} \mathcal{F}(y_{2n}, y_{2n+1}, kt) = 1$.
Case 2: If $\mathcal{F}(y_{2n-1}, y_{2n}, t) < \mathcal{F}(y_{2n}, y_{2n+1}, t)$, then by using the property $\phi(t) > t$, we get

$$\mathcal{F}(y_{2n}, y_{2n+1}, kt) \geq \phi\left\{\mathcal{F}(y_{2n-1}, y_{2n}, t)\right\}$$
$$> \mathcal{F}(y_{2n-1}, y_{2n}, t)$$
$$> \mathcal{F}(y_{2n-2}, y_{2n-1}, t)$$
$$> \mathcal{F}(y_{2n-3}, y_{2n-2}, t).$$

Continue like this for all n

$$\mathcal{F}(y_n, y_{n+1}, kt) \geq \mathcal{F}(y_{n-1}, y_n, t)$$
$$\geq \mathcal{F}(y_{n-2}, y_{n-1}, \tfrac{t}{k})$$
$$\geq \cdots \geq \mathcal{F}(y_1, y_0, \tfrac{t}{k^{n-1}}).$$

This implies $\lim_{n\to\infty} \mathcal{F}(y_n, y_{n+1}, kt) = 1$.
So, we proved $\{y_n\}$ is a Cauchy sequence in Y. Since Y is complete space. So, $y_n \to l$ for some $l \in Y$.
Also, $\{Tx_{2n}\} \to l, \{RSx_{2n+1}\} \to l$,
$\{Ux_{2n+1}\} \to l, \{PQx_{2n+2}\} \to l$.
As Y is $\in -$ chainable, there exists a finite sequence
$x_n = y_1, y_2, y_3, \cdots, y_q = x_{n+1}$ such that
$\mathcal{F}(y_i, y_{i-1}, kt) > 1- \in$
for all $t > 0$ and $i = 1, 2, 3, \cdots, q$.
By using this method, we get $\{x_n\}$ is a Cauchy sequence in Y. From completeness of Y, we have $x_n \to m \in Y$.
So we have $Tx_{2n} \to Tm, PQx_{2n+2} \to PQm$.
Since Y is Hausdorff, then $Tm = l = PQm$.
Now, using weak compatibility condition (4) for the pair $[T, PQ]$, $TPQm = PQTm$ and $Tl = PQl$.
By assuming continuity of T and PQ implies that
$TPQx_{2n} \to Tl = PQl$ and $PQPQ \to PQl$.
By taking $x = PQx_{2n+2}$, $y = x_{2n+1}$ in (1) and letting $n \to \infty$, we have

$$\mathcal{F}(TPQx_{2n+2}, Ux_{2n+1}, kt) \geq$$
$$\phi\left\{\begin{array}{c}\mathcal{F}(PQPQx_{2n+2}, RSx_{2n+1}, t) \\ *\mathcal{F}(PQPQx_{2n+2}, TPQx_{2n+2}, t) \\ *\mathcal{F}(RSx_{2n+1}, Ux_{2n+1}, t)* \\ \frac{1}{2}[\mathcal{F}(PQPQx_{2n+2}, Ux_{2n+1}, t) \\ +\mathcal{F}(RSx_{2n+1}, TPQx_{2n+2}, t)]\end{array}\right\}$$

This implies,

$$\mathcal{F}(PQl, l, kt) \geq \phi\left\{\begin{array}{c}\mathcal{F}(PQl, l, t) * \mathcal{F}(PQl, PQl, t)* \\ \mathcal{F}(l, l, t)* \\ \frac{1}{2}[\mathcal{F}(PQl, l, t) + \mathcal{F}(l, PQl, t)]\end{array}\right\}$$
$$> \phi\left\{\mathcal{F}(PQl, l, t) * \frac{1}{2}[\mathcal{F}(PQl, l, t) + \mathcal{F}(l, PQl, t)]\right\}$$
$$> \mathcal{F}(PQl, l, t).$$

So, we have $PQl = l$ and $Tl = l = PQl$.
Since $T \subset RS(Y)$, therefore there exists $q \in Y$ such that $Tl = RSq$.
From (1), we have

$$\mathcal{F}(Tx_{2n}, Uq, kt) \geq \phi\left\{\begin{array}{c}\mathcal{F}(PQx_{2n}, RSq, t) \\ *\mathcal{F}(PQx_{2n}, Tx_{2n}, t)* \\ \mathcal{F}(RSq, Uq, t)* \\ \frac{1}{2}[\mathcal{F}(PQx_{2n}, Uq, t) \\ +\mathcal{F}(RSq, Tx_{2n}, t)]\end{array}\right\}.$$

Letting $n \to \infty$, we get

$$\mathcal{F}(l, Uq, kt) \geq \phi\left\{\mathcal{F}(l, l, t) * \mathcal{F}(l, l, t) * \mathcal{F}(l, Uq, t) * \frac{1}{2}[\mathcal{F}(l, Uq, t) + \mathcal{F}(l, l, t)]\right\}$$
$$= \phi\left\{\mathcal{F}(l, Uq, t) * \frac{1}{2}[\mathcal{F}(l, Uq, t) + \mathcal{F}(l, l, t)]\right\}$$
$$= \phi\left\{\mathcal{F}(l, Uq, t)\right\} > \mathcal{F}(l, Uq, t).$$

This implies $Uq = l$ and $RSq = l = Uq$.
From condition (3), we have $URSq = RSUq$ and $Ul = RSl$.
We claim that l is the fixed point of T, RS, U and PQ. From (1), we have

$$\mathcal{F}(Tx_{2n}, Ul, kt) \geq \phi\left\{\mathcal{F}(PQx_{2n}, RSl, t) * \mathcal{F}(PQx_{2n}, Tx_{2n}, t) * \mathcal{F}(RSl, Ul, t) * \frac{1}{2}[\mathcal{F}(PQx_{2n}, Ul, t) + \mathcal{F}(RSl, Tx_{2n}, t)]\right\}.$$

As $n \to \infty$, one can get

$$\mathcal{F}(l, Ul, kt) \geq \phi\left\{\mathcal{F}(l, Ul, t)\right\} > \mathcal{F}(l, Ul, t).$$

By considering Lemma 7, we get $Ul = l$.
Hence, $Tl = Ul = l = RSl = PQl$. Thus T, RS, U and PQ have fixed point l in Y.
We show that l is a unique fixed point of T, RS, U and PQ.
Suppose not, therefore there exists $l' \in Y$ such that $Tl' = Ul' = l' = RSl' = PQl'$ and $l' \neq l$.
Now, again from (1)

$$\mathcal{F}(l, l', kt) = \mathcal{F}(Tl, Ul', kt)$$
$$\geq \phi\left\{\mathcal{F}(PQl, RSl', t) * \mathcal{F}(PQl, Tl, t) * \mathcal{F}(RSl', Ul', t) * \frac{1}{2}[\mathcal{F}(PQl, Ul', t) + \mathcal{F}(RSl', Tl, t)]\right\}$$
$$> \mathcal{F}(l, l', t).$$

This implies l is a unique fixed point of T, RS, U and PQ.

Theorem 2. Let $(Y, \mathcal{F}, *)$ be a complete $\in$-chainable FM-space with t-norm $a * b = \min\{a, b\}$. Let P, Q, R, S, T and U are self-mappings on Y such that for all $x, y \in Y, t > 0$

$$\mathcal{F}(Tx, Uy, kt) \geq \left\{\mathcal{F}(PQx, RSy, t) * \mathcal{F}(PQx, Tx, t) * \mathcal{F}(RSy, Uy, t) * \frac{1}{2}[\mathcal{F}(PQx, Uy, t) + \mathcal{F}(RSy, Tx, t)]\right\},$$

where $k \in (0,1)$. Also, mappings satisfies following conditions:
1. $T \subset RS(Y)$, $U \subset PQ(Y)$,
2. T and PQ are continuous,
3. the pairs (T, PQ) and (U, RS) are weakly compatible.

Then T, RS, U and PQ have a unique common fixed point in Y.

Proof. By using $\phi(t) > t, \phi(1) = 1$, the proof is similar as given in Theorem 1.

Corollary 3. Let $(Y, \mathcal{F}, *)$ be a complete $\in$-chainable FM-space with t-norm $a * b = \min\{a, b\}$. Let P, Q, R, S, T and U are self-mappings on Y such that for all $x, y \in Y, t > 0$

$$\mathcal{F}(Tx, Uy, kt) \geq \left\{\mathcal{F}(PQx, RSy, t) * \mathcal{F}(PQx, Tx, t) * \mathcal{F}(PQx, Uy, 2t) * \mathcal{F}(RSy, Uy, t) * \frac{1}{2}[\mathcal{F}(PQx, Uy, t) + \mathcal{F}(RSy, Tx, t)]\right\},$$

where $k \in (0,1)$ and $t > 0$. Also, mappings satisfies following conditions:
1. $T \subset RS(Y)$, $U \subset PQ(Y)$,
2. T and PQ are continuous,
3. the pairs (T, PQ) and (U, RS) are weakly compatible.

Then T, RS, U and PQ have a unique common fixed point in Y.

Proof. From the definition of FM-space, we have

$$\mathcal{F}(Tx, Uy, kt) \geq \left\{\mathcal{F}(PQx, RSy, t) * \mathcal{F}(PQx, Tx, t) * \mathcal{F}(PQx, Uy, 2t) * \mathcal{F}(RSy, Uy, t) * \frac{1}{2}[\mathcal{F}(PQx, Uy, t) + \mathcal{F}(RSy, Tx, t)]\right\}$$
$$\geq \left\{\mathcal{F}(PQx, RSy, t) * \mathcal{F}(PQx, Tx, t) \mathcal{F}(RSy, Uy, t) * \frac{1}{2}[\mathcal{F}(PQx, Uy, t) + \mathcal{F}(RSy, Tx, t)]\right\}.$$

Then the proof is evidentially follows from Theorem 2.

Corollary 4. Let $(Y, \mathcal{F}, *)$ be a complete $\in$-chainable FM-space with t-norm $a * b = \min\{a, b\}$. Let P, R, T and U are self-mappings on Y such that for all $x, y \in Y$

$$\mathcal{F}(Tx, Uy, kt) \geq \left\{\mathcal{F}(Px, Ry, t) * \mathcal{F}(Px, Tx, t) * \mathcal{F}(Ry, Uy, t) * \frac{1}{2}[\mathcal{F}(Px, Uy, t) + \mathcal{F}(Ry, Tx, t)]\right\},$$

where $k \in (0,1)$ and $t > 0$. Also, mappings satisfies following conditions:
1. $T \subset R(Y)$, $U \subset P(Y)$,
2. T and P are continuous,
3. the pairs (T, P) and (U, R) are weakly compatible.

Then T, R, U and P have a unique common fixed point in Y.

Proof. By assuming $Q = S = I$ in Theorem 2, we obtained the result immediately.

Corollary 5. Let $(Y, \mathcal{F}, *)$ be a complete $\in$-chainable FM-space with t-norm $a * b = \min\{a, b\}$. Let P, R, T and U are self-mappings on Y which satisfies following conditions:
1. $T \subset R(Y)$, $U \subset P(Y)$,
2. T and P are continuous,
3. the pairs (T, P) and (U, R) are weakly compatible,
4. $\mathcal{F}(Tx, Uy, kt) \geq \mathcal{F}(Px, Ry, t)$

for all $x, y \in Y$, $t > 0$ and $k \in (0,1)$. Then T, R, U and P have a unique fixed point in Y.

Proof. Here, $\mathcal{F}(Px, Ry, t) = \mathcal{F}(Px, Ry, t) * 1$
$= \mathcal{F}(Px, Ry, t) * \mathcal{F}(Tx, Tx, 4t)$.

From the definition of fuzzy metric space, we get

$$\mathcal{F}(Px,Ry,t)*\mathcal{F}(Tx,Tx,4t)\geq\left\{\begin{array}{c}\mathcal{F}(Px,Ry,t)\\ *\mathcal{F}(Tx,Uy,t)\\ *\mathcal{F}(Uy,Ry,t)\\ *\mathcal{F}(Ry,Px,t)\\ *\mathcal{F}(Px,Tx,t)\end{array}\right\}.$$

By using contractive condition of Corollary 4, we have

$$\begin{aligned}\mathcal{F}(Px,Ry,t)*\mathcal{F}(Tx,Tx,4t)&\geq\left\{\begin{array}{c}\mathcal{F}(Px,Ry,t)\\ *\mathcal{F}(Tx,Uy,t)\\ *V(Uy,Ry,t)\\ *\mathcal{F}(Ry,Px,t)\\ *\mathcal{F}(Px,Tx,t)\end{array}\right\}\\ &\geq\left\{\begin{array}{c}\mathcal{F}(Px,Ry,t)\\ *\mathcal{F}(Px,Ry,t)\\ *\mathcal{F}(Px,Tx,t)\\ *\mathcal{F}(Ry,Uy,t)\\ *\frac{1}{2}[\mathcal{F}(Px,Uy,t)\\ +\mathcal{F}(Ry,Tx,t)]\\ *\mathcal{F}(Uy,Ry,t)\\ *\mathcal{F}(Ry,Px,t)\\ *\mathcal{F}(Px,Tx,t)\end{array}\right\}\\ &\geq\left\{\begin{array}{c}\mathcal{F}(Px,Ry,t)\\ *\mathcal{F}(Px,Tx,t)\\ *\mathcal{F}(Ry,Uy,t)\\ *\frac{1}{2}[\mathcal{F}(Px,Uy,t)\\ +\mathcal{F}(Ry,Tx,t)]\end{array}\right\}.\end{aligned}$$

From Corollary 4, we get T,R,U and P have a unique fixed point in Y.

Corollary 6. Let $(Y,\mathcal{F},*)$ be a complete $\in$-chainable FM-space. with t-norm $a*b=\min\{a,b\}$. Let T and U are self-mappings on Y satisfies condition there exist $k\in(0,1)$ such that $\mathcal{F}(Tx,Uy,kt)\geq\mathcal{F}(x,y,t)$ for all $x,y\in Y$ $t>0$. Then T and U have a unique fixed point in Y.

Proof. When we take $R=P=I$ in Corollary 5 , then we get important result known as fuzzy Banach contraction theorem.

Before the next result, first we give the concept of $\in$-chain on MIFM-space. Then we prove a fixed point result on it.

Definition 7. Let $(Y,\mathcal{F}_{V,W},\mathcal{T})$ be a modified intuitionistic fuzzy metric space (MIFM-space) . A finite sequence $x=x_0,x_1,\cdots,x_n$ is called $\in-$ chain from x to y if $\mathcal{F}_{V,W}(x_i,x_{i-1},t)\geq_{L^*}(\mathcal{N}_s(\epsilon),\epsilon)$ for all $t>0$, $\epsilon>0$ and $i=1,2,3,\cdots,n$. A modified intuitionistic fuzzy metric space is called $\in-$ chainable if there exists a $\in-$ chain from x to y for all $x,y\in Y$.

Theorem 8. Let $(Y,\mathcal{F}_{V,W},\mathcal{T})$ be a complete modified $\in$-chainable intuitionistic fuzzy metric space. Let P,Q,R,S,T and U are self-mappings on Y such that for all $x,y\in Y,t>0$

$$\mathcal{F}_{V,W}(Tx,Uy,t)\geq_{L^*}\gamma\left\{\min\left(\begin{array}{c}\mathcal{F}_{V,W}(PQx,RSy,kt)\\ ,\mathcal{F}_{V,W}(PQx,Tx,kt),\\ \mathcal{F}_{V,W}(RSy,Uy,kt),\\ \frac{1}{2}[\mathcal{F}_{V,W}(PQx,Uy,kt)\\ +\mathcal{F}_{V,W}(RSy,Tx,kt)]\end{array}\right)\right\}, \quad (1)$$

where $k>1$ and $\gamma:L^*\to L^*$ is a function such that $\gamma(a)\geq a$ for each $a\in L^*$.. Also, mappings satisfy following conditions:

2. $T\subset RS(Y)$, $U\subset PQ(Y)$,
3. T and PQ are continuous,
4. the pairs (T,PQ) and (U,RS) are weakly compatible.

Then T,RS,U and PQ have a unique common fixed point in Y.

Proof. Let us choose $x_0\in Y$, from condition (2), there exist $x_1,x_2\in Y$ such that

$$Tx_0=RSx_1=y_0 \text{ and } Ux_1=PQx_2=y_1.$$

Inductively, we construct sequences $\{x_n\}$ and $\{y_n\}$ in Y such that

$$\begin{aligned}&Tx_n=RSx_{n+1}=y_n \text{ and}\\ &Ux_{n+1}=PQx_{n+2}=y_{n+1},\\ &\qquad\forall\, n=0,1,2\cdots.\end{aligned}$$

Claim that $\{y_n\}$ is a Cauchy sequence in Y.
For this, we have
$\mathcal{F}_{V,W}(y_n,y_{n+1},t)=\mathcal{F}_{V,W}(Tx_n,Ux_{n+1},t)$
$\geq_{L^*}$
$\gamma\{\min(\mathcal{F}_{V,W}(y_{n-1},y_n,kt),\mathcal{F}_{V,W}(y_n,y_{n+1},kt))\}$.
There are two cases arise, which are discuses below:
Case I. If $\mathcal{F}_{V,W}(y_{n-1},y_n,kt)>\mathcal{F}_{V,W}(y_n,y_{n+1},kt)$, then by using the property $\gamma(t)>_{L^*}t$, we get

$$\begin{aligned}\mathcal{F}_{V,W}(y_n,y_{n+1},t)&\geq_{L^*}\gamma\left\{\mathcal{F}_{V,W}(y_n,y_{n+1},kt)\right\}\\ &>_{L^*}\mathcal{F}_{V,W}(y_n,y_{n+1},kt).\end{aligned}$$

which is a contradiction.
Case II. If $\mathcal{F}_{V,W}(y_{n-1},y_n,kt)<\mathcal{F}_{V,W}(y_n,y_{n+1},kt)$, then by using the property $\gamma(t)>t$, we get

$$\begin{aligned}\mathcal{F}_{V,W}(y_n,y_{n+1},t)&\geq_{L^*}\mathcal{F}_{V,W}(y_{n-1},y_n,kt)\\ &\geq_{L^*}\mathcal{F}_{V,W}(y_{n-2},y_{n-1},k^2t)\\ &\geq_{L^*}\cdots\geq_{L^*}\\ &\mathcal{F}_{V,W}(y_1,y_0,k^nt).\end{aligned}$$

From Lemma 17,we proved $\{y_n\}$ is a Cauchy sequence in Y. Since Y is complete space. So, $y_n\to l$ for some $l\in Y$. Also,

$$\begin{aligned}&\{Tx_n\}\to l,\{RSx_{n+1}\}\to l,\{Ux_{n+1}\}\to l,\\ &\{PQx_{n+2}\}\to l.\end{aligned}$$

As Y is $\in-$ chainable, there exists a finite sequence $x_n=y_1,y_2,y_3,\cdots,y_q=x_{n+1}$ such that

$\mathcal{F}_{V,W}(y_i, y_{i-1}, t) \geq_{L^*} (\mathcal{N}_s(\epsilon), \epsilon)$
for all $t > 0$ and $i = 1, 2, 3, \cdots, q$.
Consider,
$\mathcal{F}_{V,W}(x_n, x_{n+1}, t) \geq_{L^*}$
$\mathcal{T}(\mathcal{F}_{V,W}(y_1, y_2, \frac{t}{q}), \mathcal{F}_{V,W}(y_2, y_3, \frac{t}{q}), \cdots,$
$\mathcal{F}_{V,W}(y_{q-1}, y_q, \frac{t}{q}))$
$\geq_{L^*} \mathcal{T}((\mathcal{N}_s(\epsilon), \epsilon), (\mathcal{N}_s(\epsilon), \epsilon), \cdots, (\mathcal{N}_s(\epsilon), \epsilon))$
$>_{L^*} (\mathcal{N}_s(\epsilon), \epsilon)$.
Therefore for all n, m where $n < m$, we have $\mathcal{F}_{V,W}(x_n, x_m, t) >_{L^*} (\mathcal{N}_s(\epsilon), \epsilon)$.
This implies that $\{x_n\}$ is a Cauchy sequence. From completeness of Y, we have $x_n \to m \in Y$ and $Tx_n \to Tm, PQx_{n+2} \to PQm$.
Since Y is Hausdorff, then $Tm = l = PQm$.
Now, using weak compatibility condition for pair $[T, PQ]$. $TPQm = PQTm$ and $Tl = PQl$.
By assuming continuity of T and PQ implies that $TPQx_n \to Tl = PQl$ and $PQPQ \to PQl$.
By taking $x = PQx_{n+2}$, $y = x_{n+1}$ in (1) and letting $n \to \infty$, we have

$$\mathcal{F}_{V,W}(PQl, l, t) \geq_{L^*} \gamma \left\{ \min \begin{pmatrix} \mathcal{F}_{V,W}(PQl, l, kt) \\ , \mathcal{F}_{V,W}(PQl, PQl, kt), \\ \mathcal{F}_{V,W}(l, l, kt), \\ \frac{1}{2}[\mathcal{F}_{V,W}(PQl, l, kt) \\ + \mathcal{F}_{V,W}(l, PQl, kt)] \end{pmatrix} \right\}$$
$$>_{L^*} \mathcal{F}_{V,W}(PQl, l, kt).$$

So, we have $PQl = l$ and $Tl = l = PQl$.
Since $T \subset RS(Y)$, therefore there exists $q \in Y$ such that $Tl = RSq$.
From (1), we have
$\mathcal{F}_{V,W}(Tx_n, Uq, kt)$

$$\geq_{L^*} \gamma \left\{ \min \begin{pmatrix} \mathcal{F}_{V,W}(PQx_n, RSq, t) \\ , \mathcal{F}_{V,W}(PQx_n, Tx_n, t), \\ \mathcal{F}_{V,W}(RSq, Uq, t), \\ \frac{1}{2}[\mathcal{F}_{V,W}(PQx_n, Uq, t) \\ + \mathcal{F}_{V,W}(RSq, Tx_n, t)] \end{pmatrix} \right\}.$$

Letting $n \to \infty$ and from equation (1) , we get

$$\mathcal{F}_{V,W}(l, Uq, t) \geq_{L^*} \gamma \left\{ \min \begin{pmatrix} \mathcal{F}_{V,W}(l, l, kt), \\ \mathcal{F}_{V,W}(l, l, kt), \\ \mathcal{F}_{V,W}(l, Uq, kt), \\ \frac{1}{2}[\mathcal{F}_{V,W}(l, Uq, kt) \\ + \mathcal{F}_{V,W}(l, l, kt)] \end{pmatrix} \right\}$$

$$>_{L^*} \gamma \Big\{ \mathcal{F}_{V,W}(l, Uq, kt) \Big\} \quad >_{L^*} \mathcal{F}_{V,W}(l, Uq, kt).$$

This implies $Uq = l$ and $RSq = l = Uq$. Since (U, RS) are weakly compatible, so
$URSq = RSUq$ and $Ul = RSl$.
We claim that l is the fixed point of T, RS, U and PQ.
From (1), we have
$\mathcal{F}_{V,W}(Tx_n, Ul, t) \geq_{L^*}$

$$\gamma \left\{ \min \begin{pmatrix} \mathcal{F}_{V,W}(PQx_n, RSl, kt) \\ , \mathcal{F}_{V,W}(PQx_n, Tx_n, kt), \\ \mathcal{F}_{V,W}(RSl, Ul, kt), \\ \frac{1}{2}[\mathcal{F}_{V,W}(PQx_n, Ul, kt) \\ + \mathcal{F}_{V,W}(RSl, Tx_n, kt)] \end{pmatrix} \right\}.$$

As $n \to \infty$,

$$\mathcal{F}_{V,W}(l, Ul, t) \geq_{L^*} \gamma \Big\{ \mathcal{F}_{V,W}(l, Ul, kt) \Big\}$$
$$>_{L^*} \mathcal{F}_{V,W}(l, Ul, kt).$$

This implies $Ul = l$ and we have
$Tl = Ul = l = RSl = PQl$.
Thus T, RS, U and PQ have fixed point l in Y.
Show that l is a unique fixed point of T, RS, U and PQ.
Suppose not, therefore there exists $l' \in Y$ such that $Tl' = Ul' = l' = RSl' = PQl'$ and $l' \neq l$.
Now, again from (1)

$$\mathcal{F}_{V,W}(l, l', t) = \mathcal{F}_{V,W}(Tl, Ul', t)$$
$$\geq_{L^*} \gamma \left\{ \min \begin{pmatrix} \mathcal{F}_{V,W}(PQl, RSl', kt) \\ , \mathcal{F}_{V,W}(PQl, Tl, kt), \\ \mathcal{F}_{V,W}(RSl', Ul', kt), \\ \frac{1}{2}[\mathcal{F}_{V,W}(PQl, Ul', kt) \\ + \mathcal{F}_{V,W}(RSl', Tl, kt)] \end{pmatrix} \right\}$$
$$>_{L^*} \mathcal{F}_{V,W}(l, l', kt)$$

This shows that l is a unique fixed point of T, RS, U and PQ.

3 Applications

Theorem 9. Let $(Y, \mathcal{F}, *)$ be a complete $\in$-chainable FM-space with t-norm $a * b = min\{a, b\}$. Let P, Q, R, S, T and U are self-mapping on Y such that for all $x, y \in Y, t > 0$,

$$\int_0^{\mathcal{F}(Tx, Uy, kt)} \hbar(t)dt \geq \phi\left(\int_0^u \hbar(t)dt\right) \qquad (1)$$

where $\hbar(t)$ is Lebesgue- integrable function, $k \in (0, 1)$ and

$$u = \mathcal{F}(PQx, RSy, t) * \mathcal{F}(PQx, Tx, t) * \mathcal{F}(RSy, Uy, t)$$
$$* \frac{1}{2}[\mathcal{F}(PQx, Uy, t) + \mathcal{F}(RSy, Tx, t)]$$

Also, mappings satisfies following conditions:
2. $T \subset RS(Y)$, $U \subset PQ(Y)$,
3. T and PQ are continuous,
4. the pairs (T, PQ) and (U, RS) are weakly compatible.
Then T, RS, U and PQ have a unique common fixed point in Y.
Proof. Same as in Theorem 1, define x_n, y_n in Y as
$Tx_{2n} = RSx_{2n+1} = y_{2n}$
and $Ux_{2n+1} = PQx_{2n+2} = y_{2n+1}$,

$\forall\ n = 0, 1, 2 \cdots$.
From (1), we have

$$\int_0^{\mathcal{F}(y_{2n},y_{2n+1},kt)} \hbar(t)dt = \int_0^{\mathcal{F}(Tx_{2n},Ux_{2n+1},kt)} \hbar(t)dt \geq \phi\left(\int_0^u \hbar(t)dt\right),$$

where

$$u = \mathcal{F}(PQx_{2n}, RSx_{2n+1}, t) * \mathcal{F}(PQx_{2n}, Tx_{2n}, t) * \mathcal{F}(RSx_{2n+1}, Ux_{2n+1}, t) * \frac{1}{2}[\mathcal{F}(PQx_{2n}, Ux_{2n+1}, t) + \mathcal{F}(RSx_{2n+1}, Tx_{2n}, t)].$$

By considering two cases discussed in Theorem 1 and Y is complete $\in$-chainable FM-spaces, we get x_n, y_n are Cauchy sequences in Y.
This implies $x_n \to m \in Y, y_n \to l \in Y$.
The weak compatibility and continuity of $[T, PQ]$ shows that

$$TPQx_{2n} \to Tl = PQl \text{ and } PQPQ \to PQl.$$

By using (1) and $n \to \infty$, we get

$$\int_0^{\mathcal{F}(TPQx_{2n+2},Ux_{2n+1},kt)} \hbar(t)dt \geq \phi\left(\int_0^u \hbar(t)dt\right),$$

where

$$u = \mathcal{F}(PQx_{2n+2}, RSx_{2n+1}, t) * \mathcal{F}(PQx_{2n+2}, Tx_{2n+2}, t) * \mathcal{F}(RSx_{2n+1}, Ux_{2n+1}, t) * \frac{1}{2}[\mathcal{F}(PQx_{2n+2}, Ux_{2n+1}, t) + \mathcal{F}(RSx_{2n+1}, Tx_{2n+2}, t)].$$

This implies $Tl = l = PQl$. Since $T \subset RS(Y)$ therefore there exists $q \in Y$ such that $Tl = RSq$ Since $[U, RS]$ are weakly compatible, it gives $Ul = RSl$.
By taking (1) and $n \to \infty$, we have

$$\int_0^{\mathcal{F}(Tx_{2n},Ul,kt)} \hbar(t)dt \geq \phi\left(\int_0^u \hbar(t)dt\right),$$

where

$$u = \mathcal{F}(PQx_{2n}, RSl, t) * \mathcal{F}(PQx_{2n}, Tx_{2n}, t) * \mathcal{F}(RSl, Ul, t) * \frac{1}{2}[\mathcal{F}(PQx_{2n}, Ul, t) + \mathcal{F}(RSl, Tx_{2n}, t)].$$

We obtain $Tl = Ul = l = RSl = PQl$.
Thus T, RS, U and PQ have fixed point in Y. With help of condition (1), we get the fixed point proved in this result is unique. Thus, we get that the mappings T, RS, U and PQ have a fixed point which is unique.

Theorem 10. Suppose $(Y, \mathcal{F}_{V,W}, \mathcal{T})$ be a complete $\in$-chainable MIFM-space. Let P, Q, R, S, T and U are mappings on Y itself such that $\forall x, y \in Y, t > 0$

$$\int_0^{\mathcal{F}_{V,W}(Tx,Uy,t)} \hbar(t)dt \geq_{L^*} \gamma\left(\int_0^u \hbar(t)dt\right), \qquad (1)$$

where $\hbar\,(t)$ is Lebesgue- integrable function and

$$u = min(\mathcal{F}_{V,W}(PQx, RSy, kt), \mathcal{F}_{V,W}(PQx, Tx, kt), \mathcal{F}_{V,W}(RSy, Uy, kt), \frac{1}{2}[\mathcal{F}_{V,W}(PQx, Uy, kt). + \mathcal{F}_{V,W}(RSy, Tx, kt)])$$

for some $k > 1$ and $\gamma : L^* \to L^*$ is a function such that $\gamma\,(a) \geq a$ for each $a \in L^*$. Also, mappings satisfies following conditions:
2. $T \subset RS(Y)$, $U \subset PQ(Y)$,
3. T and PQ are continuous,
4. the pairs (T, PQ) and (U, RS) are weakly compatible.

Then T, RS, U and PQ have a unique common fixed point in Y.

Proof. Suppose $x_0 \in Y$, then there exist $x_1, x_2 \in Y$ such that $Tx_0 = RSx_1 = y_0$ and $Ux_1 = PQx_2 = y_1$. Define $\{x_n\}$ and $\{y_n\}$ in Y such that
$Tx_n = RSx_{n+1} = y_n$ and
$Ux_{n+1} = PQx_{n+2} = y_{n+1}$,
$\forall\ n = 0, 1, 2 \cdots$.
From (1), we have

$$\int_0^{\mathcal{F}_{V,W}(y_n,y_{n+1},t)} \hbar(t)dt = \int_0^{\mathcal{F}_{V,W}(Tx_n,Ux_{n+1},t)} \hbar(t)dt \geq_{L^*} \gamma\left(\int_0^u \hbar(t)dt\right),$$

where

$$u = min(\mathcal{F}_{V,W}(PQx_n, RSx_{n+1}, kt), \mathcal{F}_{V,W}(PQx_n, Tx_n, kt), \mathcal{F}_{V,W}(RSx_{n+1}, Ux_{n+1}, kt), \frac{1}{2}[\mathcal{F}_{V,W}(PQx_n, Ux_{n+1}, kt) + \mathcal{F}_{V,W}(RSx_{n+1}, Tx_n, kt)]).$$

Thus by using steps for proving Cauchy sequence in Theorem 8, we get y_n is Cauchy sequence in Y and Completeness of Y implies $y_n \to \infty$ for some $l \in Y$. Then

$$\{Tx_n\} \to l, \{RSx_{n+1}\} \to l, \{Ux_{n+1}\} \to l, \{PQx_{n+2}\} \to l.$$

As Y is $\in -$ chainable, there exists a finite sequence $x_n = y_1, y_2, y_3, \cdots, y_q = x_{n+1}$ such that

$\mathcal{F}_{V,W}(y_i, y_{i-1}, t) \geq_{L^*} (\mathcal{N}_s(\epsilon), \epsilon)$ for all $t > 0$ and $i = 1, 2, 3, \cdots, q$.
This implies that $\{x_n\}$ is a Cauchy sequence. From completeness of Y, we have $x_n \to m \in Y$.
Since Y is Hausdorff, then $Tm = l = PQm$.
Now, using weak compatibility condition for pair $[T, PQ]$, we have
$TPQm = PQTm$ and $Tl = PQl$. By assuming continuity of T and PQ implies that

$$TPQx_n \to Tl = PQl \text{ and } PQPQ \to PQl.$$

As using(1) , we have

$$\int_0^{\mathcal{F}_{V,W}(TPQx_{n+2}, Ux_{n+1}, t)} \hbar(t)dt \geq_{L^*} \gamma\left(\int_0^u \hbar(t)dt\right),$$

where

$$\begin{aligned} u = min(&\mathcal{F}_{V,W}(PQPQx_{n+2}, RSx_{n+1}, kt) \\ &, \mathcal{F}_{V,W}(PQx_{n+2}Tx_n, kt) \\ &, \mathcal{F}_{V,W}(RSx_{n+1}, Ux_{n+1}, kt) \\ &, \frac{1}{2}[\mathcal{F}_{V,W}(PQPQx_{n+2}, Ux_{n+1}, kt) \\ &+ \mathcal{F}_{V,W}(RSx_{n+1}, TPQx_{n+2}, kt)]). \end{aligned}$$

we get $PQl = l = Tl$. Since $T \subset RS(Y)$ therefore there exists $q \in Y$ such that $Tl = RSq$.
Again from (1) and $n \to \infty$, we get

$$\int_0^{\mathcal{F}_{V,W}(Tx_n, Uq, t)} \hbar(t)dt \geq_{L^*} \gamma\left(\int_0^u \hbar(t)dt\right),$$

where

$$\begin{aligned} u = min(\mathcal{F}_{V,W}(PQx_n, RSq, kt), &\mathcal{F}_{V,W}(PQx_n, Tx_n, kt) \\ &, \mathcal{F}_{V,W}(RSq, Uq, kt) \\ &, \frac{1}{2}[\mathcal{F}_{V,W}(PQx_n, Uq, kt) \\ &+ \mathcal{F}_{V,W}(RSq, Tx_n, kt)]). \end{aligned}$$

The above equations and weak compatibility of $[U, RS]$ gives $Ul = RSl$.
Assuming condition (1) and $n \to \infty$, one can have

$$\int_0^{\mathcal{F}_{V,W}(Tx_n, Ul, t)} \hbar(t)dt \geq_{L^*} \gamma\left(\int_0^u \hbar(t)dt\right),$$

where

$$\begin{aligned} u = min(\mathcal{F}_{V,W}(PQx_n, RSl, kt), &\mathcal{F}_{V,W}(PQx_n, Tx_n, kt) \\ &, \mathcal{F}_{V,W}(RSl, Ul, kt) \\ &, \frac{1}{2}[\mathcal{F}_{V,W}(PQx_n, Ul, kt) \\ &+ \mathcal{F}_{V,W}(RSl, Tx_n, kt)]). \end{aligned}$$

Hence, we have $Tl = Ul = l = RSl = PQl$. Thus T, RS, U and PQ have fixed point in Y. With help of condition (1), we get the fixed point proved in this result is unique. Thus, we get that the mappings T, RS, U and PQ have a fixed point which is unique.

4 Conclusion

Fuzzy set theory has establish practical use in a number of structures of information science. Information retrieval is one zone where fuzzy set theory is very valuable. The motivation for the use of fuzzy set theory to the strategy of database and information storage and retrieval system lies in the essential to handle vague information. For the withdrawal of information by reflecting and modeling the hesitancy existing in actual life condition, IF-set theory has been playing a vital part. The use of IF-sets in its place of fuzzy sets gives the outline of alternative degree of liberty into set description. In view of above results, we have dealt with the concept of convergence of sequences on complete $\in$-chainable FM-spaces and find the unique fixed point result for six mappings which follows contractive condition. Efforts have also been taken to broaden main result to some finite number of mappings. In this paper, the results also apply to generalize the Fuzzy Banach contraction theorem on FM-spaces. Also, the notion of MIFM-space is used for proving new fixed point results.

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References

[1] Alaca C., Turkoglu D. and Yildiz C., **2006**. Fixed points in intuitionistic fuzzy metric spaces. *Chaos, Solitons and Fractals.*vol.29, pp.1073–1078 .
[2] Atanassov K. T., **1986**. Intuitionistic fuzzy set. *Fuzzy Sets and System.* vol.20, no.1, pp.87–96.
[3] Beg I., Gupta V. and Kanwar A., **2015**. Fixed points on intuitionistic fuzzy metric spaces using the E.A. property. *J. Nonlinear Funct. Anal.* 2015 (2015), Article ID 20.
[4] Branciari A., **2002**. A fixed point theorem for mapping satisfying a general contractive condition of integral type. *International Journal of Mathematics and Mathematical Sciences.* vol.29, pp.531–536.
[5] Cho S. H., Jung J. H., **2006**. On common fixed point theorems in fuzzy metric spaces. *Int. Mathematical Forum.* vol.1, no.29, pp.1441–1451.
[6] Coker D., **1997**. An introduction to intuitionistic fuzzy topological spaces. *Fuzzy Sets and System.* vol.88, no.1, pp.81-99.
[7] Deschrijver G., Cornelis C. and Kerre E.E., **2004**. On the representation of intuitionistic fuzzy t-norms and t-conorms. *IEEE Trans Fuzzy Syst.* vol.12, no.3, pp.45–61.
[8] Deschrijver G., Kerre E.E., **2003**. On the relationship between some extensions of fuzzy set theory, *Fuzzy Sets and System.* vol.133, no.2, pp.227–235.
[9] George A., Veeramani P., **1994**. On some results in fuzzy metric spaces. *Fuzzy Sets and System.* vol.64, no.3, pp.395–399.

[10] Grebiec M., **1988**. Fixed points in fuzzy metric spaces. *Fuzzy Sets and System.* vol.27, no.3, pp.385–389.
[11] Gregori V., Romaguera S. and Veereamani P., **2006**. A note on intuitionistic fuzzy metric spaces *Chaos, Solitons and Fractals.* vol.28, no.4, pp.902–905.
[12] Gupta V., Kanwar A., **2012**. Fixed Point Theorem in Fuzzy Metric Spaces Satisfying E.A Property. *Indian Journal of Science and Technology.* vol.5, no. 12, pp.3767–3769.
[13] Gupta V., Kanwar A., **2016**. V-Fuzzy metric space and related fixed point theorems. *Fixed Point Theory and Applications.* 2016:51 ,pp.1–17, DOI 10.1186/s13663-016-0536-1.
[14] Jungck G., **1986**. Compatible mappings and common fixed points. *Int. J. Math Sci.* vol.9, no.4, pp.771–779.
[15] Jungck G., Rhoades BE., **1998**. Fixed point for set valued functions without continuity. *Indian J. Pure Appl. Math.* vol.29, no.3, pp.227–238.
[16] Kang S. M., Gupta V., Singh B. and Kumar S., **2013**. Some common fixed point theorems using implicit relations in fuzzy metric spaces, *International Journal of Pure and Applied Mathematics.* vol.87, no.2, pp.333–347.
[17] Kramosil I., Michalek J.,**1975**. Fuzzy metric and Statistical metric spaces, *Kybernetica.* vol.11, pp.336–344.
[18] Mishra S. N., Sharma N. and Singh S. L., **1994**. Common fixed points of maps on fuzzy metric spaces. *Int. J. Math. Sci.* vol.17, no.2, pp.253–258.
[19] Menger K., **1942**. Statistical Matrices, *proceedings of National academy of sciences of the United States of America*. vol.28, no.12, pp.535–537.
[20] Park JH. **2004**. Intuitionistic fuzzy metric spaces. *Chaos, Solitons and Fractals.* vol.22, pp.1039–1046.
[21] Saini R.K., Gupta V. and Singh S.B., **2007**. Fuzzy Version of Some Fixed Points Theorems On Expansion Type Maps in Fuzzy Metric Space. *Thai Journal of Mathematics.* vol.5, no.2, pp.245–252.
[22] Saadati R., Razani A. and Adibi H., **2007**. A common fixed point theorem in L-fuzzy metric spaces. *Chaos, Solitons and Fractals.* vol.33, no.1, pp.358–363.
[23] Saadati R., Park J.H., **2006**. On the intuitionistic fuzzy topological spaces. *Chaos, Solitons and Fractals.* vol.27, no.1, pp.331–44.
[24] Saadati R., Sedghi S. and Shobe N., **2008** Modified intuitionistic fuzzy spaces and some fixed point theorems. *Chaos, Solitons and Fractals.* vol.38, no.1, pp.36–47.
[25] Schweizer B., Sklar A., **1960**. Statistical metric spaces. *Pacific J. Math.* vol.10, pp.314–334.
[26] Sharma S., Deshpande B., **2010**. Common fixed point theorems for finite number of mappings without continuity and compatibility on fuzzy metric spaces. *Fuzzy Systems and Math.* vol.24 , no.2, pp.73–83.
[27] Tanveer M., Imdad M., Gopal D. and Patel D. K., **2012**. Common fixed point theorems in modified intuitionistic fuzzy metric spaces with common property (E.A.). *Fixed Point Theory and Applications.* vol.2012, no.1, article 36, 2012.
[28] Turkoglu D., Alaca C., Cho Y. J.and Yildiz C., **2006**, Common fixed points in intuitionistic fuzzy metric spaces. *Journal of Applied Mathematics and Computing.* vol.22, no.1-2, pp. 411–24.
[29] Zadeh L. A., **1965**. Fuzzy Sets. *Inform. And Control*. vol.8, pp.338–353.

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