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Between Nearly Open Sets and Soft Nearly Open Sets

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Abstract: In this paper, we characterized soft α-open set, soft semi-open set, soft pre-open set, soft β-open set and soft regular-open sets by using the parameterized family of topological spaces induced by the soft topology. Their corresponding operators are also characterized. A few relations between these operators and the soft interior and soft closure operators are also established.

Keywords: Soft set, soft topology, soft α-open, soft semi-open, soft pre-open, soft β-open

1 Introduction

Most of the problems in Engineering, Computer Science, Medical Science, Economics, Environment etc have various uncertainties. The theory of soft set is a vital Mathematical tool used in handling uncertainties about vague concepts. In the year 1999, Molodtsov[17] initiated the concept of soft set theory as a tool for dealing with uncertainties and it is a set associated with parameters and has been applied in several directions. Soft set theory is free from the difficulties where as other existing methods such as Probability theory, Fuzzy set theory[28], Rough set theory[21], Interval Mathematics[8] etc dealing with uncertainties, but they have their own limitations due to the inadequacy of the parameterization tool of these theories as stated in[17]. In the soft set theory we can use any convenient parameterization strategies. The problem of setting the membership function does not arise in soft set theory, which makes soft set theory very convenient and practicable.

Following this, Maji et.al.[16] defined several operations on soft set theory and he described an application of soft set theory to a decision-making problem[15]. In 2009, Ali et.al.[4] presented some new algebraic operations for soft sets. Presently, work on soft set theory is making progress rapidly. Pie et.al.[23] investigated the relationships between soft sets and information systems. They showed that soft sets are a class of special information systems. Zofia Machnicka et.al.[29] have shown that each information system can be considered a soft set and each finite soft set can be considered an information system. Yuksel.et.al [27] applied soft sets to diagnose the prostate cancer risk. Kharal et.al.[13] introduced soft mappings between two soft sets and their application in Medical Expert system.

Many researchers have contributed towards the algebraic structure of soft set theory. The application of soft set theory in algebraic structures was introduced by Aktas et.al[2]. They established the basic notions of soft groups as a generalization of the idea of fuzzy groups. Jun [12] investigated BCK/BCI-algebras and their applications in ideal theory.

Recently Shabir et.al [19] initiated the study of soft topological spaces. Later, soft topological spaces were studied in [9],[10],[22],[24],[26] and [30]. Weak forms of soft open sets were first introduced by Chen[11]. He introduced soft semi-open sets. Arockiya Rani et.al [7]defined soft β-open sets and soft regular open sets. Akdag et.al[1],[3] defined soft α-open sets and soft pre-open sets in soft topological spaces and studied their related properties.

In the present study soft nearly open sets like soft α-open sets, soft semi-open sets, soft pre-open sets, soft β-open sets and soft regular open sets are characterized by using the analogous concepts in the parameterized family of topological spaces induced by the soft topology. Further the properties of soft closure and soft interior operators of these soft nearly open sets are also characterized.

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2 Preliminaries

Throughout this paper X,Y denote the universal sets and E,K are parameter spaces. A pair (F,E) is called a soft set[17] over X where $F : E \to 2^X$ is a mapping. $S(X,E)$ denotes the collection of all soft sets over X with parameter space E. We denote (F,E) by $\widetilde{F}$ in which case we write $\widetilde{F} = \{(e,F(e)) : e \in E\}$. In some occasions, we use $\widetilde{F}(e)$ for $F(e)$. For any two soft sets $\widetilde{F}$ and $\widetilde{G}$ in $S(X,E)$, $\widetilde{F}$ is a soft subset of $\widetilde{G}$ if $F(e) \subseteq G(e)$ for all $e \in E$. If $\widetilde{F}$ is a soft subset of $\widetilde{G}$ then we write $\widetilde{F} \subseteq \widetilde{G}$. $\widetilde{F}$ and $\widetilde{G}$ are equal if and only if $F(e) = G(e)$ for all $e \in E$. That is $\widetilde{F} = \widetilde{G}$ if $\widetilde{F} \subseteq \widetilde{G}$ and $\widetilde{G} \subseteq \widetilde{F}$. $\widetilde{\Phi} = \{(e,\phi) : e \in E\} = \{(e,\Phi(e)) : e \in E\} = (\Phi,E)$ is called the null soft set and $\widetilde{X} = \{(e,X) : e \in E\} = \{(e,X(e)) : e \in E\} = (X,E)$ is the absolute soft set.

Definition 2.1.[16] The union of two soft sets $\widetilde{F}$ and $\widetilde{G}$ over X is defined as $\widetilde{F} \cup \widetilde{G} = (F \cup G,E)$ where $(F \cup G)(e) = F(e) \cup G(e)$ for all $e \in E$.

Definition 2.2.[4] The intersection of two soft sets $\widetilde{F}$ and $\widetilde{G}$ over X is defined as $\widetilde{F} \cap \widetilde{G} = (F \cap G,E)$ where $(F \cap G)(e) = F(e) \cap G(e)$ for all $e \in E$.

If $\{\widetilde{F_\alpha} : \alpha \in \Delta\}$ is a collection of soft sets in $S(X,E)$ then the arbitrary union and the arbitrary intersection of soft sets are defined below.
$\bigcup\{\widetilde{F_\alpha} : \alpha \in \Delta\} = (\bigcup\{F_\alpha : \alpha \in \Delta\},E)$ and $\bigcap\{\widetilde{F_\alpha} : \alpha \in \Delta\} = (\bigcap\{F_\alpha : \alpha \in \Delta\},E)$ where $(\bigcup\{F_\alpha : \alpha \in \Delta\})(e) = \bigcup\{F_\alpha(e) : \alpha \in \Delta\}$ and $(\bigcap\{F_\alpha : \alpha \in \Delta\})(e) = \bigcap\{F_\alpha(e) : \alpha \in \Delta\}$, for all $e \in E$.

Definition 2.3.[19] The complement of a soft set $\widetilde{F}$ is denoted by $(\widetilde{F})' = (F',E)$ (relative complement in the sense of Ifran Ali.et.al[4]) where $F' : E \to 2^X$ is a mapping given by $F'(e) = X - F(e)$ for all $e \in E$.

It is noteworthy to see that with respect to above complement De Morgan's laws hold for soft sets as stated below.

Lemma 2.4.[30] Let I be an arbitrary index set and $\{\widetilde{F_i} : i \in I\} \subseteq S(X,E)$. Then $(\cup\{\widetilde{F_i} : i \in I\})' = \cap\{(\widetilde{F_i})' : i \in I\}$ and $(\cap\{\widetilde{F_i} : i \in I\})' = \cup\{(\widetilde{F_i})' : i \in I\}$.

Definition 2.5.[19] Let $\widetilde{F}$ be a soft set over X and $x \in X$. We say that $x \in \widetilde{F}$ whenever $x \in F(e)$ for all $e \in E$.

It is easy to see that every soft set in S(X,E) is a soft subset of $\widetilde{X}$

Definition 2.6.[19] Let $\widetilde{\tau}$ be a collection of soft subset of $\widetilde{X}$. Then $\widetilde{\tau}$ is said to be a soft topology on X with parameter space E if
(i) $\widetilde{\Phi},\widetilde{X} \in \widetilde{\tau}$, (ii) $\widetilde{\tau}$ is closed under arbitrary union, and (iii) $\widetilde{\tau}$ is closed under finite intersection.

If $\widetilde{\tau}$ is a soft topology on X with a parameter space E then the triplet $(X,E,\widetilde{\tau})$ is called a soft topological space over X with parameter space E. Identifying (X,E) with $\widetilde{X}$, $(\widetilde{X},\widetilde{\tau})$ is a soft topological space.

The members of $\widetilde{\tau}$ are called soft open sets in $(X,E,\widetilde{\tau})$. A soft set $\widetilde{F}$ in $S(X,E)$ is soft closed in $(X,E,\widetilde{\tau})$, if its complement $(\widetilde{F})'$ belongs to $\widetilde{\tau}$. $(\widetilde{\tau})'$ denotes the collection of all soft closed sets in $(X,E,\widetilde{\tau})$. The soft closure of $\widetilde{F}$ is the soft set, $\widetilde{s}cl(\widetilde{F}) = \cap\{\widetilde{G} : \widetilde{G}$ is soft closed and $\widetilde{F} \subseteq \widetilde{G}\}$. The soft interior [24] of $\widetilde{F}$ is the soft set, $\widetilde{s}int(\widetilde{F}) = \cup\{\widetilde{O} : \widetilde{O}$ is soft open and $\widetilde{O} \subseteq \widetilde{F}\}$. It is easy to see that $\widetilde{F}$ is soft open if and only if $\widetilde{F} = \widetilde{s}int\widetilde{F}$ and $\widetilde{F}$ is soft closed if and only if $\widetilde{F} = \widetilde{s}cl\widetilde{F}$.

Lemma 2.7.[19] Let $(X,E,\widetilde{\tau})$ be a soft space over X. Then the collection $\widetilde{\tau}_e = \{F(e) : \widetilde{F} \in \widetilde{\tau}\}$ is a topology on X for each $e \in E$.

Thus a soft topology on X gives a parameterized family of topologies on X but the converse is not true.

Definition 2.8. A soft set $\widetilde{F}$ of a soft topological space $(X,E,\widetilde{\tau})$ is said to be

(i)Soft α-open[1] if $\widetilde{F} \subseteq \widetilde{s}int\ (\widetilde{s}cl\ (\widetilde{s}int\ (\widetilde{F})))$.
(ii)Soft pre-open[7] if $\widetilde{F} \subseteq \widetilde{s}int\ (\widetilde{s}cl\ (\widetilde{F}))$.
(iii)Soft semi-open[11] if $\widetilde{F} \subseteq \widetilde{s}cl\ (\widetilde{s}int\ (\widetilde{F}))$.
(iv)Soft β-open[7] if $\widetilde{F} \subseteq \widetilde{s}cl\ (\widetilde{s}int\ (\widetilde{s}cl\ (\widetilde{F})))$.
(v)Soft regular open[7] if $\widetilde{F} = \widetilde{s}int\ (\widetilde{s}cl\ (\widetilde{F}))$.

Definition 2.9. A subset A of a space X in a topological space is called

(i)α-open[20] if $A \subseteq int\ cl\ int\ A$
(ii)Semi-open[14] if $A \subseteq cl\ int\ A$
(iii)Pre-open[18] if $A \subseteq int\ cl\ A$
(iv)β-open[6] if $A \subseteq cl\ int\ cl\ A$
(v)Regular-open[25] if $A = int\ cl\ A$

The complement of an α-open set is α-closed. Semi-closed, pre-closed, β-closed and regular-closed sets are similarly defined. For a subset A of a space X the α-closure (resp.semi-closure, pre-closure and β-closure) of A denoted by αclA(resp.$sclA$, $pclA$ and βclA) is the intersection of all α-closed (resp.Semi-closed, pre-closed and β-closed) subsets of X containing A. Dually the α-interior (resp.semi-interior, pre-interior and β-interior) of A denoted by $\alpha intA$(resp.$sintA$, $pintA$ and $\beta intA$) is the union of all α-open (resp.Semi-open, pre-open and β-open) subsets of X contained in A.

Lemma 2.10.[6] Let A be a subset of a space X. Then the following results hold:

(i)$sclA = A \cup intclA$
(ii)$sintA = A \cap clintA$
(iii)$\alpha clA = A \cup clintclA$
(iv)$\alpha intA = A \cap intclintA$
(v)$pclA = A \cup clintA$
(vi)$pintA = A \cap intclA$
(vii)$\beta clA = A \cup intclintA$
(viii)$\beta intA = A \cap clintclA$

Lemma 2.11.[5] Let A be a subset of a space X. Then the following results hold:

(i)$\alpha cl\alpha intA = clintA$
(ii)$\alpha int\alpha clA = intclA$
(iii)$\alpha int\alpha cl\alpha intA = intclintA$
(iv)$\alpha cl\alpha int\alpha clA = clintclA$

3 Soft Nearly Open Sets

Soft closure and soft interior operators in soft topological spaces can be characterized by the corresponding operators in the parameterized family of topological spaces induced by the soft topology.

Proposition 3.1. Let $(X,E,\widetilde{\tau})$ be a soft topological space over X. Let $\widetilde{F} \in S(X,E)$ and $e \in E$. If $\widetilde{F}$ is soft open in $(X,E,\widetilde{\tau})$ then $\widetilde{F}(e)$ is open in $(X,\widetilde{\tau}_e)$. Conversely if G is open in $(X,\widetilde{\tau}_e)$ then $G=\widetilde{F}(e)$ is soft open in $(X,E,\widetilde{\tau})$ for some soft open set $\widetilde{F}$ in $(X,E,\widetilde{\tau})$.

Proof. Suppose $\widetilde{F} \in \widetilde{\tau}$. Then by using the Lemma 2.7, $\widetilde{F}(e) \in \widetilde{\tau}_e$.

Conversely suppose $G \in \widetilde{\tau}_e$. Again by using the same Lemma 2.7, $G=\widetilde{F}(e)$ for some $\widetilde{F} \in \widetilde{\tau}$.

Proposition 3.2. Let $(X,E,\widetilde{\tau})$ be a soft topological space over X. Let $\widetilde{F} \in S(X,E)$ and $e \in E$. If $\widetilde{F}$ is soft closed in $(X,E,\widetilde{\tau})$ then $\widetilde{F}(e)$ is closed in $(X,\widetilde{\tau}_e)$. Conversely if G is closed in $(X,\widetilde{\tau}_e)$ then $G=\widetilde{F}(e)$ is soft closed in $(X,E,\widetilde{\tau})$ for some soft closed set $\widetilde{F}$ in $(X,E,\widetilde{\tau})$

Proof. Suppose $\widetilde{F}$ is soft closed. Then $\widetilde{F} \in (\widetilde{\tau})'$. Then by using the Lemma 2.7, $\widetilde{F}(e) \in (\widetilde{\tau}')_e$.

Conversely suppose G is closed in $(X,\widetilde{\tau}_e)$. Again by using the same Lemma 2.7, $G=\widetilde{F}(e)$ for some $\widetilde{F} \in (\widetilde{\tau})'$.

Proposition 3.3. Let $(X,E,\widetilde{\tau})$ be a soft topological space. Let $\widetilde{F} \in S(X,E)$. Then $(\widetilde{s}cl\widetilde{F})(e) = cl(\widetilde{F}(e))$ in $(X,\widetilde{\tau}_e)$ for every $e \in E$.

Proof.

$$\begin{aligned}(\widetilde{s}cl\widetilde{F})(e) &= \bigcap\{\widetilde{H}(e) : \widetilde{H} \supseteq \widetilde{F}, \widetilde{H} \text{ is soft closed}\}\\ &= \bigcap\{\widetilde{H}(e) : \widetilde{H}(e) \supseteq \widetilde{F}(e), \widetilde{H} \text{ is soft closed}\}\\ &= \bigcap\{\widetilde{H}(e) : \widetilde{H}(e) \supseteq \widetilde{F}(e), \widetilde{H}(e) \text{ is closed in } \widetilde{\tau}_e\}\\ &= cl(\widetilde{F}(e)).\end{aligned}$$

This shows that $(\widetilde{s}cl\widetilde{F})(e) = cl(\widetilde{F}(e))$ in $(X,\widetilde{\tau}_e)$ for every $e \in E$.

Proposition 3.4. Let $(X,E,\widetilde{\tau})$ be a soft topological space. Let $\widetilde{F} \in S(X,E)$. Then $(\widetilde{s}int\widetilde{F})(e) = int(\widetilde{F}(e))$ in $(X,\widetilde{\tau}_e)$ for every $e \in E$.

Proof.

$$\begin{aligned}(\widetilde{s}int\widetilde{F})(e) &= \bigcup\{\widetilde{G}(e) : \widetilde{G} \subseteq \widetilde{F}, \widetilde{G} \text{ is soft open}\}\\ &= \bigcup\{\widetilde{G}(e) : \widetilde{G}(e) \subseteq \widetilde{F}(e), \widetilde{G} \text{ is soft open}\}\\ &= \bigcup\{\widetilde{G}(e) : \widetilde{G}(e) \subseteq \widetilde{F}(e), \widetilde{G}(e) \text{ is open in } \widetilde{\tau}_e\}\\ &= int(\widetilde{F}(e)).\end{aligned}$$

Therefore $(\widetilde{s}int\widetilde{F})(e) = int(\widetilde{F}(e))$ in $(X,\widetilde{\tau}_e)$ for every $e \in E$.

Proposition 3.5. Let $(X,E,\widetilde{\tau})$ be soft topological space. Let $\widetilde{F} \in S(X,E)$. Then

(i)$(\widetilde{s}cl\ \widetilde{s}int\ \widetilde{F})(e) = cl\ int\ (\widetilde{F}(e))$ in $(X,\widetilde{\tau}_e)$ for all $e \in E$.
(ii)$(\widetilde{s}int\ \widetilde{s}cl\ \widetilde{F})(e) = int\ cl\ (\widetilde{F}(e))$ in $(X,\widetilde{\tau}_e)$ for all $e \in E$.
(iii)$(\widetilde{s}cl\ \widetilde{s}int\ \widetilde{s}cl\ \widetilde{F})(e) = cl\ int\ cl\ (\widetilde{F}(e))$ in $(X,\widetilde{\tau}_e)$ for all $e \in E$.
(iv)$(\widetilde{s}int\ \widetilde{s}cl\ \widetilde{s}int\ \widetilde{F})(e) = int\ cl\ int\ (\widetilde{F}(e))$ in $(X,\widetilde{\tau}_e)$ for all $e \in E$.

Proof.

$$\begin{aligned}(\widetilde{s}cl\ \widetilde{s}int\ \widetilde{F})(e) &= \widetilde{s}cl(\widetilde{s}int\ \widetilde{F})(e)\\ &= cl\ ((\widetilde{s}int\widetilde{F})(e))\ in\ (X,\widetilde{\tau}_e)\\ &= cl\ int\ (\widetilde{F}(e))\ in\ (X,\widetilde{\tau}_e).\end{aligned}$$

This proves (i). The proof for (ii),(iii) and (iv) is analogous.

Proposition 3.6. Let $(X,E,\widetilde{\tau})$ be a soft topological space. Let $\widetilde{F} \in S(X,E)$. Then

(i)$\widetilde{F}$ is soft semi-open in $(X,E,\widetilde{\tau})$ if and only if $\widetilde{F}(e)$ is semi-open in $(X,\widetilde{\tau}_e)$ for all $e \in E$.
(ii)$\widetilde{F}$ is soft α-open in $(X,E,\widetilde{\tau})$ if and only if $\widetilde{F}(e)$ is α-open in $(X,\widetilde{\tau}_e)$ for all $e \in E$.
(iii)$\widetilde{F}$ is soft pre-open in $(X,E,\widetilde{\tau})$ if and only if $\widetilde{F}(e)$ is pre-open in $(X,\widetilde{\tau}_e)$ for all $e \in E$.
(iv)$\widetilde{F}$ is soft β-open in $(X,E,\widetilde{\tau})$ if and only if $\widetilde{F}(e)$ is β-open in $(X,\widetilde{\tau}_e)$ for all $e \in E$.
(v)$\widetilde{F}$ is soft regular-open in $(X,E,\widetilde{\tau})$ if and only if $\widetilde{F}(e)$ is regular-open in $(X,\widetilde{\tau}_e)$ for all $e \in E$.

Proof. $\widetilde{F}$ is soft semi-open in $(X,E,\widetilde{\tau})$
$\Leftrightarrow \widetilde{F} \subseteq \widetilde{s}cl\ \widetilde{s}int\ \widetilde{F}$
$\Leftrightarrow \widetilde{F}(e) \subseteq (\widetilde{s}cl\ \widetilde{s}int\ \widetilde{F})(e)$ for all $e \in E$.
By using the proposition 3.5 and by using the definition 2.9 $\widetilde{F}(e)$ is semi-open in $(X,\widetilde{\tau}_e)$
$\Leftrightarrow \widetilde{F}(e) \subseteq cl\ int\ (\widetilde{F}(e))$ for all $e \in E$.

This proves (i). The proof for (ii),(iii),(iv) and (v) is analogous.

Proposition 3.7. Let $(X,E,\widetilde{\tau})$ be soft topological space. Let $\widetilde{F} \in S(X,E)$. Then

(i)$\widetilde{s}\alpha cl\widetilde{F} = \widetilde{F} \cup \widetilde{s}cl\ \widetilde{s}int\ \widetilde{s}cl\ \widetilde{F}$ and $\widetilde{s}\alpha int\widetilde{F} = \widetilde{F} \cap \widetilde{s}int\ \widetilde{s}cl\ \widetilde{s}int\ \widetilde{F}$.
(ii)$\widetilde{s}scl\widetilde{F} = \widetilde{F} \cup \widetilde{s}int\ \widetilde{s}cl\ \widetilde{F}$ and $\widetilde{s}sint\widetilde{F} = \widetilde{F} \cap \widetilde{s}cl\ \widetilde{s}int\ \widetilde{F}$.
(iii)$\widetilde{s}pcl\widetilde{F} = \widetilde{F} \cup \widetilde{s}cl\ \widetilde{s}int\ \widetilde{F}$ and $\widetilde{s}pint\widetilde{F} = \widetilde{F} \cap \widetilde{s}int\ \widetilde{s}cl\ \widetilde{F}$.
(iv)$\widetilde{s}\beta cl\widetilde{F} = \widetilde{F} \cup \widetilde{s}int\ \widetilde{s}cl\ \widetilde{s}int\ \widetilde{F}$ and $\widetilde{s}\beta int\widetilde{F} = \widetilde{F} \cap \widetilde{s}cl\ \widetilde{s}int\ \widetilde{s}cl\ \widetilde{F}$.

Proof.

$$\begin{aligned}(\widetilde{s}\alpha cl\widetilde{F})(e) &= \cap\{H(e) : \widetilde{H} \supseteq \widetilde{F} : \widetilde{H}\ is\ soft \alpha - closed\}\\ &= \cap\{H(e) : H(e) \supseteq F(e) : \widetilde{H}\ is\ soft\ \alpha - closed\}\\ &= \cap\{H(e) : H(e) \supseteq F(e) : \widetilde{H}(e)\ is \alpha - closed\\ &\quad in\ (X,\widetilde{\tau}_e)\}\\ &= \alpha cl\ (\widetilde{F}(e)).\end{aligned}$$

Then by applying Lemma 2.10(c),

$$\begin{aligned}\alpha cl\ (\widetilde{F}(e)) &= \widetilde{F}(e) \cup cl\ int\ cl\ \widetilde{F}(e)\\ &= (\widetilde{F} \cup \widetilde{s}cl\ \widetilde{s}int\ \widetilde{s}cl\widetilde{F})(e).\end{aligned}$$

Therefore $\widetilde{s}\alpha cl\ \widetilde{F} = \widetilde{F} \cup \widetilde{s}cl\ \widetilde{s}int\ \widetilde{s}cl\widetilde{F}$

and

$$\begin{aligned}(\widetilde{s}\alpha int\widetilde{F})(e) &= \cup\{G(e) : \widetilde{G} \subseteq \widetilde{F} : \widetilde{G}\ is\ soft \alpha - open\}\\ &= \cup\{G(e) : G(e) \subseteq F(e) : \widetilde{G}\ is\ soft\ \alpha - open\}\\ &= \cup\{G(e) : G(e) \subseteq F(e) : \widetilde{G}(e)\ is \alpha - open\\ in\ (X, \widetilde{\tau}_e)\}\\ &= \alpha int\ (\widetilde{F}(e)).\end{aligned}$$

Then by applying Lemma 2.10(d),

$$\begin{aligned}\alpha int\ (\widetilde{F}(e)) &= \widetilde{F}(e) \cap int\ cl\ int\ \widetilde{F}(e)\\ &= (\widetilde{F} \cap \widetilde{s}int\ \widetilde{s}cl\ \widetilde{s}int\widetilde{F})(e).\end{aligned}$$

Therefore $\widetilde{s}\alpha int\ \widetilde{F} = \widetilde{F} \cap \widetilde{s}int\ \widetilde{s}cl\widetilde{s}int\ \widetilde{F}$.

This proves (i). Proofs for (ii),(iii) and (iv) is analogous.

Proposition 3.8. Let $(X, E, \widetilde{\tau})$ be soft topological spaces. Let $\widetilde{F} \in S(X, E)$. Then

(i)$(\widetilde{s}\alpha cl\widetilde{F})(e) = \alpha cl(\widetilde{F}(e))$
(ii)$(\widetilde{s}\alpha int\widetilde{F})(e) = \alpha int(\widetilde{F}(e))$
(iii)$(\widetilde{s}scl\widetilde{F})(e) = scl(\widetilde{F}(e))$
(iv)$(\widetilde{s}sint\widetilde{F})(e) = sint(\widetilde{F}(e))$
(v)$(\widetilde{s}pcl\widetilde{F})(e) = pcl(\widetilde{F}(e))$
(vi)$(\widetilde{s}pint\widetilde{F})(e) = pint(\widetilde{F}(e))$
(vii)$(\widetilde{s}\beta cl\widetilde{F})(e) = \beta cl(\widetilde{F}(e))$
(viii)$(\widetilde{s}\beta int\widetilde{F})(e) = \beta int(\widetilde{F}(e))$
(ix)$(\widetilde{s}\alpha cl\ \widetilde{s}\alpha int\ \widetilde{F})(e) = \alpha cl\ \alpha int\ (\widetilde{F}(e))$
(x)$(\widetilde{s}\alpha int\ \widetilde{s}\alpha cl\ \widetilde{F})(e) = \alpha int\ \alpha cl\ (\widetilde{F}(e))$
(xi)$(\widetilde{s}\alpha cl\ \widetilde{s}\alpha int\ \widetilde{s}\alpha cl\ \widetilde{F})(e) = \alpha cl\ \alpha int\ \alpha cl\ (\widetilde{F}(e))$
(xii)$(\widetilde{s}\alpha int\ \widetilde{s}\alpha cl\ \widetilde{s}\alpha int\ \widetilde{F})(e) = \alpha int\ \alpha cl\ \alpha int\ (\widetilde{F}(e))$

Proof. Follows from Proposition 3.7.

Proposition 3.9. Let $(X, E, \widetilde{\tau})$ be soft topological spaces. Let $\widetilde{F} \in S(X, E)$. Then

(i)$\widetilde{s}\alpha cl\ \widetilde{s}\alpha int\ \widetilde{F} = \widetilde{s}cl\ \widetilde{s}int\ \widetilde{F}$
(ii)$\widetilde{s}\alpha int\ \widetilde{s}\alpha cl\ \widetilde{F} = \widetilde{s}int\ \widetilde{s}cl\ \widetilde{F}$
(iii)$\widetilde{s}\alpha cl\ \widetilde{s}\alpha int\ \widetilde{s}\alpha cl\ \widetilde{F} = \widetilde{s}cl\ \widetilde{s}int\ \widetilde{s}cl\ \widetilde{F}$
(iv)$\widetilde{s}\alpha int\ \widetilde{s}\alpha cl\ \widetilde{s}\alpha int\ \widetilde{F} = \widetilde{s}int\ \widetilde{s}cl\ \widetilde{s}int\ \widetilde{F}$

Proof. Follows from Proposition 3.8(ix),(x),(xi) and (xii) and from Lemma 2.11.

4 Conclusion

Soft topological space is based on soft set which is a collection of information granules is the Mathematical formulation of appropriate reasoning about information systems. In this paper, we have characterized the concepts of soft α-open sets, soft semi-open sets, soft pre-open sets, soft β-open sets and soft regular open sets by using the analogous concepts in the parameterized family of topological spaces induced by the soft topology. Further the interior and closure operators of these soft nearly open sets are also characterized. We hope that the findings in the paper are just the beginning of a new structure in the soft topology and not only will form the theoretical basis but also will lead to the development of Information system and various fields in engineering. There is a compact connection between soft sets and information systems. It can be seen that a classical soft set indeed is a simple information system in which the attributes only take two values 0 and 1. Moreover, the concepts in this this paper can be extended to Rough set theory and thus get more affirmative solution in decision making problems in real life situations. And it is useful in many areas of health sciences particularly in diagnosis of diseases.

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