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Vibrations And Instability Control Of A Slender System By Means Of Piezoceramic Elements

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Abstract: The results of numerical studies on linear and non-linear transverse vibrations control of a two member column subjected to Euler's load by means of piezoceramic elements have been presented in this paper. The investigated column is composed of two members. The external member is a single rod and the internal one consists of two rods (one of them is a made of piezoceramic material). The connection of elements of the internal member has been modeled by means of a rotational spring of stiffness C. The boundary problem has been formulated on the basis of Hamiltons principle. The perturbation method was used in the solution process due to non-linearity of the column. An influence of the prestressing force generated by the piezoceramic element on natural vibration frequencies, maximum loading and amplitude - vibration frequency relationship have been presented in this paper. The length of the piezoceramic element and stiffness of the rotational spring C on investigated parameters were also taken into account.

Keywords: Vibrations, Non-linear vibration, Linear vibration, Instability, Vibration control, Piezoceramic

1 Introduction

In the research and development departments the piezoceramic sensors and actuators are being investigated as the elements responsible for shape control or active/passive control of the systems subjected to static or dynamic excitation. The vibration control of a complex beam systems and bridges was investigated by many scientists and presented by Song et. al [1]. It was concluded that piezoceramic elements despite of some limitations can be easily implemented as the parts of the mechanical systems and successfully meet their requirements. In the investigation performed by Irschik [2] the methods of shape control of the initially deformed system have been presented. In the composite structures the piezoceramic elements are being installed symmetrically relatively to the main element. The modification of the base system is mostly insignificant. The voltage applied to the piezoelement causes axial deformation or bending of such structure. Thompson and Loughlan [3] have investigated the buckling of the cantilever column with symmetrically attached two piezoceramic strips in the central part of the system. The control of the applied voltage removed the transversal displacement caused by external load. In this study the piezoceramic elements were subjected to the electric field of the same potential magnitude but with opposite directions, causing elongation of the one plate and compression of the other. The problem of active dynamic instability control of the systems subjected to static, periodic and random types of load by means of piezoelements was investigated by Mukherjee and Chaudhuri [4]. In this case the initial bending have been reduced on the basis of the change in the deformation gradient measured by sensors and properly selected electrical voltage applied to the actuators. The same authors [5] have generalized their the problem formulation by application of the springs in the areas of transversal displacements concluding that this combined method is the most effective for vibration amplitude reduction. Faria [6] studied an increase of the buckling critical forces of beams with piezoceramic elements. The installed piezoelements have generated axial tensile forces in the system with both fixed ends. Przybylski [7] on the basis of the residual longitudinal forces induced by the random number of the symmetirically attached pairs of piezoelements have discussed the instability control of the column strengthened by a pin and a translational spring. Faria and Almeida [8] have proposed the use of piezoelectirc actuator to prebuckling control of the

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columns. In the cited papers the piezoceramic plates were ideally connected to the basic structure (the glue layer was so thin, that the shear deformation has been neglected). In the investigations performed by Tylikowski [9] an influence of the glue layer on the piezoelectirc plate ring actuator behavior have been investigated. The conditions of interaction between the actuator and the plate have been modeled as massless elastic glue layer. The same author [10] has studied an influence of delamination of piezoceramic elements on dynamic behavior of the laminated beams with piezoelements. It have been concluded [11] that the conventional method of gluing of piezoelements on the surface or between the layers of the basic structure are not the only ones available. Chaudhry i Rogers [11] have proposed the discrete eccentric connection of the piezoelement to the beam with both end pinned. After the comparison of the results of numerical and experimental investigations it have been concluded that higher level of control of transversal displacement have been achieved. This method of connection is only available for the actuators with the bending rigidity similar to the host structure. Assuming that engineering structures are characterized by the shape and assembly imperfection Przybylski and Sok [12] have proposed an installation of a piezoceramic rod in the eccentrically loaded column in order to control the defection of the system. The rationale for such design solution is that currently available piezoelemnets are being produced in the variety of shape and sizes and their mechanical features are similar to the material of the host structure. It can been concluded that after the voltage has been applied to the piezoceramic rod the investigated column have regained the rectilinear form of static equilibrium. The main purposes of this paper is to investigate an influence of the residual force generated by the integrated piezoelement on the instability, vibration and amplitude - frequency relationship of the two member column. The control of maximum loading capacity and natural vibration frequency have been also investigated. Due to loading conditions the columns loses stability by divergence. The considered two member column is a non-linear system what was taken into account during problem formulation.

2 Problem Formulation

The investigated cantilever column has been presented in the figure 1a. Rods (2) and (3) are connected by the pin and the rotational spring of stiffness C. The external axially applied force P with constant line of action is located on the free end of the column (point of connection of rods (1) and (3)). The length of rods is described by l1, l2, l3. The physical model of the investigated system may be composed of two coaxial tubes, tube and rod or be flat frame. The bent axes diagram is presented in the figure 1b. It is assumed that one of column's rods is made of piezoceramic material; but for more general problem

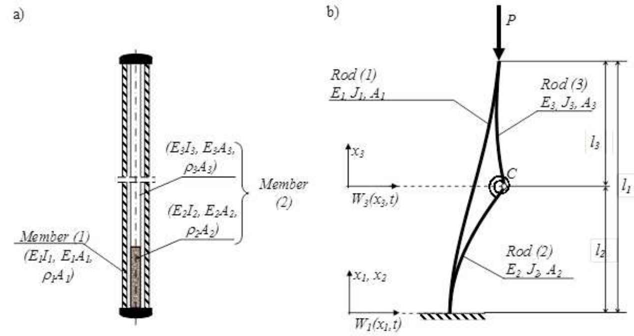

Fig. 1: The investigated system

formulation at this stage there is no need to indicate a specific element of the structure. On the basis of the Hamilton's principle the instability problem has been formulated:

$$(\delta \int_{t_1}^{t_2} (\mathrm{E}^k - \mathrm{E}^p) dt = 0) \tag{1}$$

The kinetic Ek and potential Ep energies are expressed as follows:

$$(\mathrm{E}^k = \frac{1}{2} \sum_{i=1}^{3} \int_0^{l_i} \rho_i A_i \left(\frac{\partial W_i(x_i,t)}{\partial t} \right)^2 dx) \tag{2}$$

$$\mathrm{E}^p = \frac{1}{2} \sum_{i=1}^{3} \left\{ \int_0^{l_i} E_i J_i \left[\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2} \right]^2 dx + \int_0^{l_i} E_i A_i \left[\frac{\partial U_i(x_i,t)}{\partial x_i} + \frac{1}{2} \left(\frac{\partial W_i(x_i,t)}{\partial x_i} \right)^2 \right]^2 dx \right\} + \\ + \frac{1}{2} C \left(\left. \frac{\partial W_3(x_3,t)}{\partial x_3} \right|_{x_3=0} - \left. \frac{\partial W_2(x_2,t)}{\partial x_2} \right|_{x_2=l_2} \right)^2 + P U_1(l_1,t), \tag{3}$$

where

E_i Young modulus, J_i moment of inertia, A_i cross section area, ρ_i material density , C rotational spring stiffness, P external load. Introducing (2) and (3) into equation (1) leads to:

$$\delta \int_{t_1}^{t_2} \left(\frac{1}{2} \sum_{i=1}^{3} \int_0^{l_i} \rho_i A_i \left[\frac{\partial W_i(x_i,t)}{\partial t} \right]^2 dx_i - \frac{1}{2} \left\{ \sum_{i=1}^{3} \int_0^{l_i} E_i J_i \left[\frac{\partial^2 W_i(x_i,t)}{\partial x_i^2} \right]^2 dx_i + \right. \right. \\ \left. + \int_0^{l_i} E_i A_i \left[\frac{\partial U_i(x_i,t)}{\partial x_i} + \frac{1}{2} \left(\frac{\partial W_i(x_i,t)}{\partial x_i} \right)^2 \right]^2 dx_i \right\} + \\ \left. + \frac{1}{2} C \left(\left. \frac{\partial W_3(x_3,t)}{\partial x_3} \right|_{x_3=0} - \left. \frac{\partial W_2(x_2,t)}{\partial x_2} \right|_{x_2=l_2} \right)^2 + P U_1(l_1,t) \right) dt = 0. \tag{4}$$

Completing variation and integration operations on (4) and knowing that virtual displacements $\delta U_i(x,t)$, $\delta W_i(x,t)$ for are arbitrary and independent for $0 < x < \mathrm{l}$ one obtains

(i = 1, 2, 3):
equations of motion in transversal direction:

$$
(E_iJ_i\frac{\partial^4 W_i(x_i,t)}{\partial {x_i}^4}-E_iA_i\frac{\partial}{\partial x_i}\left[\left[\frac{\partial U_i(x_i,t)}{\partial x_i}+\frac{1}{2}\left(\frac{\partial W_i(x_i,t)}{\partial x_i}\right)^2\right]\right.
\left.\frac{\partial W_i(x_i,t)}{\partial x_i}\right]+\rho_iA_i\frac{\partial^2 W_i(x_i,t)}{\partial t^2}=0) \tag{5}
$$

equations of motion in longitudinal direction:

$$
(\frac{\partial}{\partial x_i}\left(\frac{\partial U_i(x_i,t)}{\partial x_i}+\frac{1}{2}\left[\frac{\partial W_i(x_i,t)}{\partial x_i}\right]^2\right)=0) \tag{6}
$$

The axial force in i-th element can be expressed in the form:

$$
(S_i(t)=-E_iA_i\left(\frac{\partial U_i(x_i,t)}{\partial x_i}+\frac{1}{2}\left[\frac{\partial W_i(x_i,t)}{\partial x_i}\right]^2\right)) \tag{7}
$$

The equations of motion of each rod (5) after introducing into it longitudinal force described by equation (7) have the form:

$$
(E_iJ_i\frac{\partial^4 W_i(x_i,t)}{\partial {x_i}^4}-S_i(t)\frac{\partial^2 W_i(x_i,t)}{\partial {x_i}^2}+\rho_iA_i\frac{\partial^2 W_i(x_i,t)}{\partial t^2}=0) \tag{8}
$$

After performing mathematical operations on equation (7), the expression for longitudinal displacement has been obtained:

$$
(U_i(x_i,t)=-\frac{S_i(t)x_i}{E_iA_i}-\frac{1}{2}\int_0^{x_i}\left[\frac{\partial W_i(x_i,t)}{\partial x_i}\right]^2dx_i) \tag{9}
$$

The geometrical boundary conditions can be written in the form:

$$
W_1(0,t)=\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|_{x_1=0}=0 \tag{10a}
$$
$$
W_2(0,t)=\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|_{x_2=0}=0 \tag{10b}
$$
$$
\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|^{x_1=l_1}=\left.\frac{\partial W_3(x_3,t)}{\partial x_3}\right|^{x_3=l_3} \tag{10c}
$$
$$
W_2(l_2,t)=W_3(0,t) \tag{10d}
$$
$$
W_1(l_1,t)=W_3(l_3,t) \tag{10e}
$$
$$
U_1(0,t)=U_2(0,t)=0 \tag{10f}
$$
$$
U_2(l_2,t)=U_3(0,t) \tag{10g}
$$
$$
U_1(l_1,t)=U_3(l_3,t) \tag{10h}
$$

Introduction of (10a-e) into variational equation allows one to find the set of natural boundary conditions:

$$
E_1J_1\left.\frac{\partial^2 W_1(x_1,t)}{\partial x_1^2}\right|^{x_1=l_1}+E_3J_3\left.\frac{\partial^2 W_3(x_3,t)}{\partial x_3^2}\right|^{x_3=l_3}=0 \tag{11a}
$$

$$
\begin{aligned}
&E_1J_1\left.\frac{\partial^3 W_1(x_1,t)}{\partial x_1^3}\right|^{x_1=l_1}+P\left.\frac{\partial W_1(x_1,t)}{\partial x_1}\right|^{x_1=l_1}\\
&+E_3J_3\left.\frac{\partial^3 W_3(x_3,t)}{\partial x_3^3}\right|^{x_3=l_3}=0
\end{aligned} \tag{11b}
$$

$$
\begin{aligned}
&E_2J_2\left.\frac{\partial^3 W_2(x_2,t)}{\partial x_2^3}\right|^{x_2=l_2}+S_2\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|^{x_2=l_2}+\\
&-E_3J_3\left.\frac{\partial^3 W_3(x_3,t)}{\partial x_3^3}\right|_{x_3=0}-S_3\left.\frac{\partial W_3(x_3,t)}{\partial x_3}\right|_{x_3=0}=0
\end{aligned} \tag{11c}
$$

$$
-E_3J_3\left.\frac{\partial^2 W_3(x_3,t)}{\partial x_3^2}\right|_{x_3=0}+C\left[\begin{array}{l}\left.\frac{\partial W_3(x_3,t)}{\partial x_3}\right|_{x_3=0}\\ -\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|^{x_2=l_2}\end{array}\right]=0 \tag{11d}
$$

$$
E_2J_2\left.\frac{\partial^2 W_2(x_2,t)}{\partial x_2^2}\right|^{x_2=l_2}-C\left[\begin{array}{l}\left.\frac{\partial W_3(x_3,t)}{\partial x_3}\right|_{x_3=0}\\ -\left.\frac{\partial W_2(x_2,t)}{\partial x_2}\right|^{x_2=l_2}\end{array}\right]=0 \tag{11e}
$$

$$
S_2=S_3 \tag{11f}
$$

$$
S_1+S_2=P \tag{11g}
$$

The further investigations have been done in the non-dimensional form, where:

$$
\xi_i=\frac{x_i}{l_1},d_i=\frac{l_i}{l_1},\lambda_i=\frac{A_il^2}{J_i} \tag{12a-c}
$$

$$
w_i(\xi_i,\tau)=\frac{W_i(x_i,t)}{l_1},u_i(\xi_i,\tau)=\frac{U_i(x_i,t)}{l_1} \tag{12d-e}
$$

$$
k_i(\tau)=\frac{S_i(\tau)\,{l_1}^2}{E_iJ_i},p_d=\frac{P{l_1}^2}{E_1J_1+E_2J_2} \tag{12f-g}
$$

$$
\omega_i^2=\Omega_i^2\frac{\rho_iA_i{l_1}^4}{E_iJ_i},\tau=\Omega t \tag{12h-i}
$$

$$
c_b=\frac{Cl_1}{E_1J_1+E_2J_2},r_w=\frac{E_3J_3}{E_2J_2},r_m=\frac{E_2J_2}{E_1J_1} \tag{12j-l}
$$

The solution of the boundary problem has been performed by means of the small parameter method . In this paper vibrations around the rectilinear form of static equilibrium have been presented in [13,14]. The transversal and longitudinal displacements, the longitudinal force and vibration frequency of each rod of the column have been written in the power series with respect to the ε in the form:

$$w_i(\xi,\tau)=\sum_{n=1}^{N}\varepsilon^{2n-1}w_{i2n-1}(\xi,\tau)+O(\varepsilon^{2N+1}) \quad (13)$$

$$u_i(\xi,\tau)=u_{i0}(\xi)+\sum_{n=1}^{N}\varepsilon^{2n}u_{i2n}(\xi,\tau)+O(\varepsilon^{2N+1}) \quad (14)$$

$$k_i(\tau)=k_{i0}+\sum_{n=1}^{N}\varepsilon^{2n}k_{i2n}(\tau)+O(\varepsilon^{2N+1}) \quad (15)$$

$$\omega_i^2={\omega_{i0}}^2+\sum_{n=1}^{N}\varepsilon^{2n}{\omega_{i2n}}^2+O(\varepsilon^{2N+1}) \quad (16)$$

The equations (13-16) are being introduced (6) and (9), and grouped by the terms with the same power of the small parameter ε. The first four equations are listed below:

$$\varepsilon^0: u_{i0}(\xi_i)=-\frac{k_{i0}}{\lambda_i}\xi_i \quad (17)$$

$$\varepsilon^1: w_{i1}^{IV}(\xi_i,\tau)+k_{i0}w_{i1}^{II}(\xi_i,\tau)+\omega_{i0}^2\ddot{w}_{i1}(\xi_i,\tau)=0 \quad (18)$$

$$\varepsilon^2: u_{i2}(\xi_i,\tau)=-\frac{k_{i2}(\tau)}{\lambda_i}\xi_i-\frac{1}{2}\int_0^{l_i}\left[w_{i1}^{I}(\xi,\tau)\right]^2 d\xi_i \quad (19)$$

$$\varepsilon^3: \begin{array}{l} w_{i3}^{IV}(\xi_i,\tau)+k_{i0}w_{i3}^{II}(\xi_i,\tau)+\omega_{i0}^2\ddot{w}_{i3}(\xi_i,\tau)= \\ -k_{i2}(\tau)w_{i1}^{II}(\xi_i,\tau)-\omega_{i2}^2\ddot{w}_{i1}(\xi_i,\tau)\end{array} \quad (20)$$

Equations (17 - 20) are solved sequentially, in order to determine: static longitudinal force, vibration frequency, amplitude of the longitudinal force, second component of vibration frequency.

3 Piezoceramic element as a part of the structure

In the presented formulation, each rod can be treated as made of piezoceramic material. The production of long piezoelements is complicated and expensive, that is why for further investigation the rod (2) has been chosen as a piezoelement. As a part of a member II its length can be much smaller than the length of the whole structure - see fig. 1. The constitutive equations of piezoceramic material polarized in the perpendicular direction to the axis of the rod (2) are as follows:

$$\overset{(2)}{\sigma_x}=E_2\overset{(2)}{\varepsilon_x}-e_{31}E_z \quad (21a)$$

$$D_z=e_{31}\overset{(2)}{\varepsilon_x}+\xi_{33}E_z \quad (21b)$$

where D_z [m] is displacement induced by electrical field E_z [V/m], which is defined as the quotient of the voltage V [V] by the thickness of the piezoelectric h_p [m]. The e_{3_1} is a dielectric constant [C/m2] and ξ_{3_3} effective coefficient of dielectric medium [C/Vm].

Assuming that the column keeps the rectilinear form of the static equilibrium when the electric field is being applied, the strain - displacement relation is as follows:

$$\overset{(i)}{\varepsilon_x}=\frac{dU_i(x_i)}{dx_i} \quad (22)$$

In order to designate the residual force generated by the piezoelement the potential energy has been defined:

$$\mathrm{E}^p=\frac{1}{2}\sum_{i=1}^{3}\int_{\Omega_i}\overset{(i)}{\sigma_x}\overset{(i)}{\varepsilon_x}d\Omega_i-\frac{1}{2}\int_{\Omega_2}D_zE_zd\Omega_2 \quad (23)$$

the normal stress in rods (1) and (3) are formulated on the basis of Hooke's law:

$$\overset{(i)}{\sigma_x}=E_i\overset{(i)}{\varepsilon_x}, i=1,3 \quad (24)$$

Introduction of (21a, 21b, 22, 24) into (23) leads to:

$$\begin{array}{l}\mathrm{E}^p=\frac{1}{2}\sum\limits_{i=1}^{3}\int\limits_0^{l_i}E_iA_i\left[\frac{dU_i(x_i)}{dx_i}\right]^2dx_i+ \\ -\int\limits_0^{l_2}A_2e_{31}E_z\frac{dU_2(x_2)}{dx_2}dx_2-\frac{1}{2}\int\limits_0^{l_2}A_2\zeta_{33}E_z^2dx_2\end{array} \quad (25)$$

where $A_i=b_ih_i$. The variation of the potential energy after performing mathematical operations on (25) has the form:

$$\delta\mathrm{E}^p=\sum_{i=1}^{3}E_iA_i\left[\begin{array}{l}\frac{dU_i(x_i)_i}{dx_i}\delta\,U_i(x_i)|_0^{l_i}+ \\ -\int\limits_0^{l_i}\frac{d^2U_i(x_i)}{dx_i^2}\delta U_i(x_i)dx_i\end{array}\right]-F\delta\,U_2(x_2)|_0^{l_2}=0 \quad (26)$$

The F stands for piezoelectric force, defined as:

$$F=be_{31}V \quad (27)$$

where b is a width of the piezoelement.

On the basis of (26) the three second-order differential equations of longitudinal displacements were obtained.

$$\frac{d^2U_i(x_i)}{dx_i^2}=0, i=1,2,3. \quad (28)$$

The geometrical boundary conditions (29a-c) and natural ones (30a,b) are as follows:

$$U_1(0)=U_2(0)=0, U_2(l_2)=U_3(0), U_1(l_1)=U_3(l_3), \quad (29a\text{-}c)$$

$$|_{x_2=l_2}-E_3A_3\frac{dU_3(x_3)}{dx_3}|_{x_3=0}-F=0 \quad (30a)$$

$$E_1A_1 \frac{dU_1(x_1)}{dx_1}\bigg|_{x_1=l_1} + E_3A_3 \frac{dU_3(x_3)}{dx_3}\bigg|_{x_3=l_3} = 0 \quad \text{(30b)}$$

The solution of (28) with (29a-c) and (30a, b) leads to equations of residual forces in each segment. It can be concluded that the application of electric field with potential V to the piezoceramic element generates the residual force in the first member which is equal to the absolute value of the forces in segments of the second member:

$$|R_1| = |R_2| = |R_3| = |R| = F \frac{l_2}{E_2A_2} \left(\sum_{i=1}^{3} \frac{l_i}{E_iA_i} \right)^{-1} \quad (31)$$

The magnitude of the residual force R depends on piezoelectric force F, relation in the length between rods of the structure and compression stiffness of each rod. The force R causes compression or tension of the system and must be introduced into equations of motion:

$$E_iJ_iW_i^{IV}(x_i,t) + (S_i \pm R)W_i^{II}(x_i,t) + \rho_iA_i\ddot{W}_i(x_i,t) = 0 \quad (32)$$

where dot stands for derivative with respect to time t and roman numeral to space variable.

The effect of those changes depends of direction of electrical field vector. The magnitude of the residual force will have an influence on vibration frequency and maximum loading capacity of the investigated system. The non-dimensional residual force is expressed in the form:

$$f = \frac{Rl^2}{E_1J_1 + E_2J_2} \quad (33)$$

4 Linear problem

The separation of space and time variables have been done according to equation:

$$w_{i1}(\xi_i, \tau) = \overset{(1)}{w_{i_1}}(\xi_i) \cos\tau \quad (34)$$

Introduction of (34) into (32) leads to:

$$\overset{(1)}{w_{i_1}}(\xi_i)^{IV} + k_{i0}\overset{(1)}{w_{i_1}}(\xi_i)^{II} - \omega_{0i}{}^2 \overset{(1)}{w_{i_1}}(\xi_i) = 0 \quad (35)$$

The general solution of equation (35) has the form:

$$\begin{aligned}\overset{(1)}{w_i}(\xi_i) &= A_i \sin(g_{1i}\xi_i) + B_i \cos(g_{1i}\xi_i) + \\ &+ C_i \sinh(g_{2i}\xi_i) + D_i \cosh(g_{2i}\xi_i)\end{aligned}, i = 1,2,3, \quad (36)$$

where

$$g_{1i} = \sqrt{\frac{k_{i0}}{2} + \sqrt{\frac{k_{i0}{}^2}{4} + \omega_i^2}} \quad (37)$$

$$g_{2i} = \sqrt{-\frac{k_{i0}}{2} + \sqrt{\frac{k_{i0}{}^2}{4} + \omega_i^2}} \quad (38)$$

$$k_{10} = (1 + r_m)\left(\pm f + p_d \frac{a_m}{a_m + 1}\right) \quad (39)$$

$$k_{20} = \frac{1 + r_m}{r_m}\left(\mp f + p_d \frac{1}{a_m + 1}\right) \quad (40)$$

$$k_{30} = \frac{1}{r_w}\frac{1 + r_m}{r_m}\left(\mp f + p_d \frac{1}{a_m + 1}\right) \quad (41)$$

$$a_m = \frac{E_1A_1}{d_1}\left(\frac{d_2}{E_2A_2} + \frac{d_3}{E_3A_3}\right) \quad \text{(41a)}$$

After substituting equations (36) into boundary conditions, the system of twelve homogenous equations with unknowns A_i, B_i, C_i, D_i ($i = 1,2,3$)) is created. The numerical solution of the determinant gives the relationship between vibration frequency and external load.

5 Non-linear problem

From the equation (19) after introduction of (34) and (15) the dependence between the amplitude and axial force amplitude can be found:

$$\overset{(0)}{k}_{12} = \overset{(2)}{k}_{12} = \begin{pmatrix} -\int_0^1 [\overset{(1)}{w}_{11}{}'(\xi_1)]^2 d\xi_1 + \\ +\int_0^{d_2} [\overset{(1)}{w}_{21}{}'(\xi_2)]^2 d\xi_2 + \int_0^{d_3} [\overset{(1)}{w}_{31}{}'(\xi_3)]^2 d\xi_3 \end{pmatrix} \frac{1}{4\left(\frac{1}{\lambda_1} + \frac{d_2}{r_m\lambda_2} + \frac{d_3}{r_m r_w \lambda_3}\right)} \quad (42)$$

From the equation (20) in the form:

$$\begin{aligned} & w_{i3}{}^{IV}(\xi_i,\tau) + k_{i0}w_{i3}{}^{II}(\xi_i,\tau) + \omega_{0i}{}^2\ddot{w}_{i3}(\xi_i,\tau) = \\ & -k_{i2}(\tau)w_{i1}{}^{II}(\xi_i,\tau) - \omega_{2i}{}^2\ddot{w}_{i1}(\xi_i,\tau) \end{aligned} \quad (43)$$

the second component of natural vibration can be obtained. After performing the mathematical operations on (43) with consideration of (14, 34 and 44).

$$w_{i3}(\xi_i,\tau) = \underset{i3}{\overset{(1)}{w}}(\xi_i)\cos\tau + \underset{i3}{\overset{(3)}{w}}(\xi_i)\cos 3\tau \quad (44)$$

and by means of the orthogonality condition, the second component of natural vibration has the form:

$$\omega_{21}^2 = \frac{\frac{3}{2}\overset{(0)}{k}_{12}\overset{(1)}{w}_{21}(\xi_2)|_{\xi_2=d_2}\frac{\overset{(1)}{w}{}^{II}_{21}(\xi_2)|_{\xi_2=d_2}}{c_b} + 6\overset{(0)}{k}{}_{12}^{2}\left(\frac{1}{\lambda_1}+\frac{1}{r_m\lambda_2}\right)}{\int_0^{d_1}\left[\overset{(1)}{w}_{11}(\xi_1)\right]^2 d\xi_1 + \int_0^{d_2}\left[\overset{(1)}{w}_{21}(\xi_1)\right]^2 d\xi_2 + r_w\int_0^{d_3}\left[\overset{(1)}{w}_{31}(\xi_3)\right]^2 d\xi_3} \tag{45}$$

In the numerator of the equation (45) the component which is dependant on spring stiffness is present. If the spring stiffness tends to infinity (this component is equal to zero. It can be concluded that at great magnitude of this component of equation (45) has no influence on the magnitude of the second component of natural vibration frequency.

6 Results of numerical calculations

The results of numerical calculations (presented in the non-dimensional form on the basis of 12a-l and 33) of an influence of the residual force generated by the piezoelement on natural vibration frequency have been presented in the figures 2 - 5. The critical magnitude of the residual non-dimensional force f is $\pi^2/4$. At the beginning the high stiffness of the rotational spring has been considered (figure 2). After that the reduction of stiffness has been done in order to find the best system configuration in which the control area of the dynamic behavior is the greatest.

In the figure 2 the curves natural vibration frequency vs. external load under the influence of the residual force have been plotted. In the case when the stiffness in the connection of rods (2) and (3) is greater than c_b = 100 regardless to the direction of the electric field vector applied to the piezoceramic element the reduction of natural vibration frequency and critical load have been achieved (dotted line). For the comparison purposes the continuous line is plotted which corresponds to the Eulers column.

In the case when the stiffness of the rotational spring is highly reduced $c_b < 10$ and for random length of the piezoelectric rod, the generation of residual force causes the change in the location of the characteristic curves of natural vibration frequency and maximum load according to the direction of the electric field vector (figures 3 and 4). It can be concluded that the loading capacity of the system can be determined by means of the magnitude and direction of the electric field.

In the figure 5 the natural vibration curves in the function of residual force under different level of external load have been presented. The greater magnitude of the external load the instability of the system occurs at a lower voltage level applied to the piezoceramic rod. As shown the buckling of the column may be obtained by

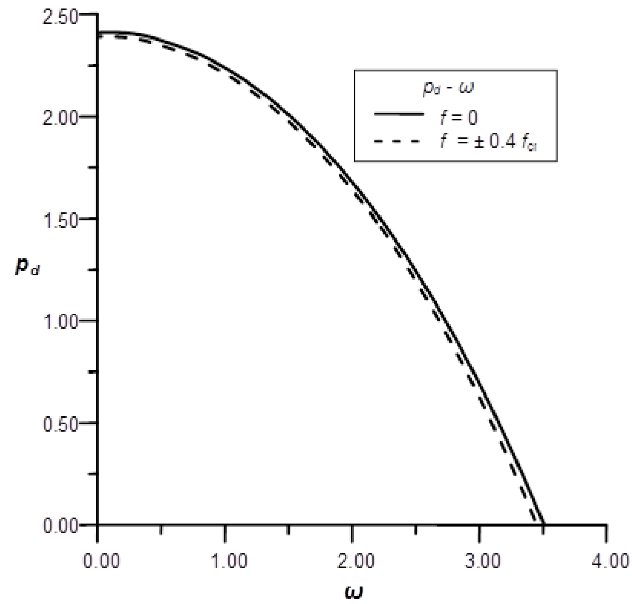

Fig. 2: The influence of the residual force on natural vibration frequency (d_2 = 0.7, c_b = 100, r_m = 1, r_w = 1)

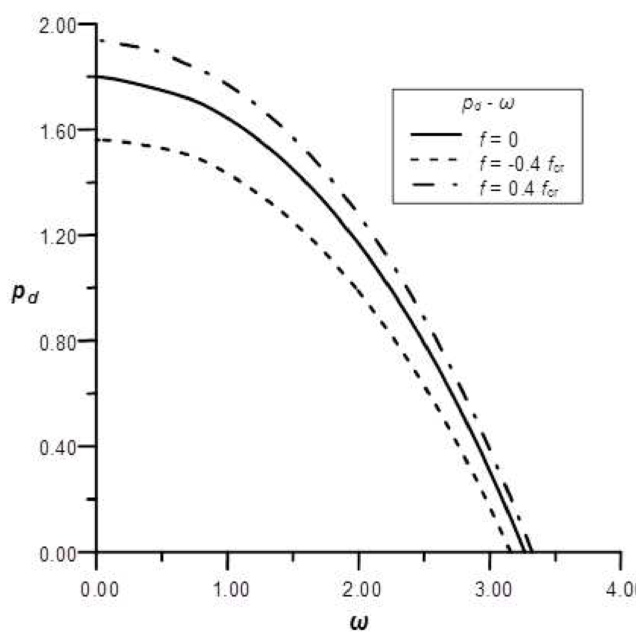

Fig. 3: The influence of the residual force on natural vibration frequency (d_2 = 0.5, c_b = 1, r_m = 1, r_w = 1)

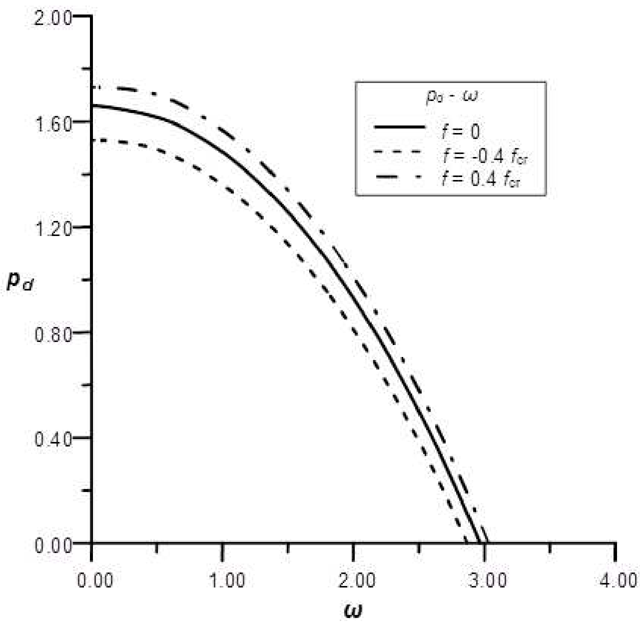

Fig. 4: The influence of the residual force on natural vibration frequency (d_2 = 0.3, c_b = 1, r_m = 1, r_w = 1)

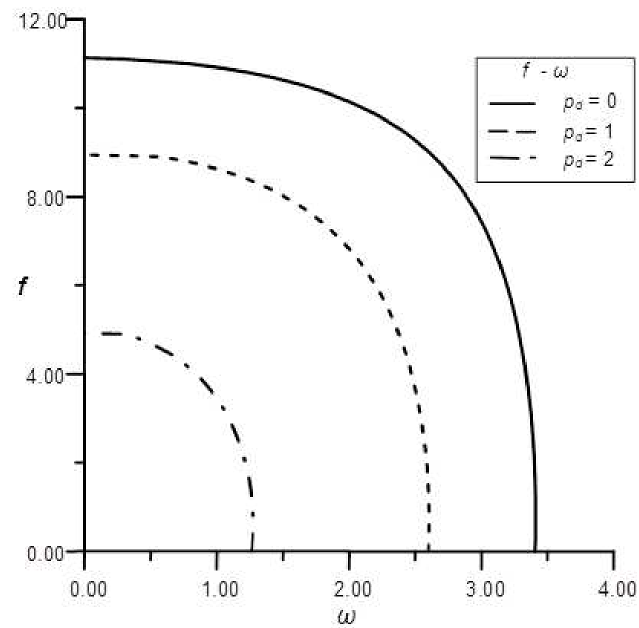

Fig. 5: The change in natural vibration in the function of residual force for different magnitude of external load (d_2 = 0.7, c_b = 10, r_m = 1, r_w = 1)

means of the piezoceramic element regardless to the magnitude of the external load and the geometrical parameters of the investigated system.

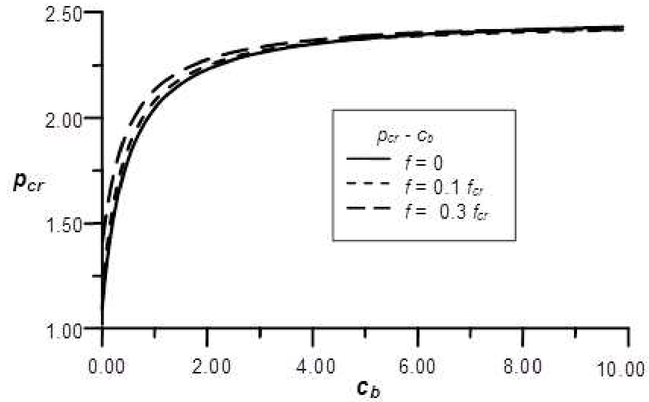

Fig. 6: The influence of the rotational spring stiffness on the critical loading under action of the residual force (d_2 = 0.5, r_m = 1, r_w = 1)

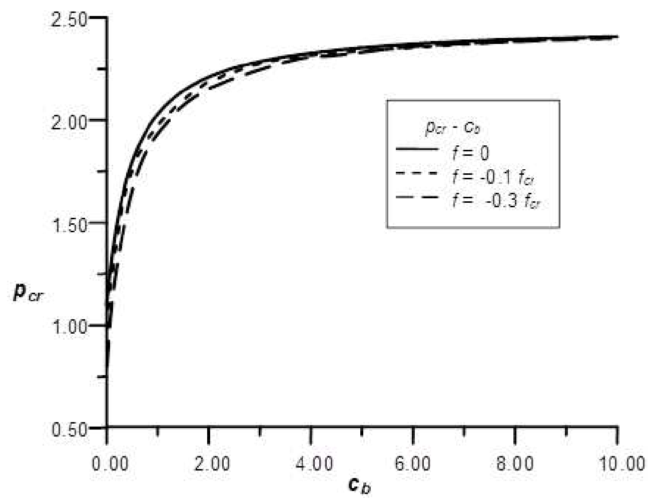

Fig. 7: The influence of the rotational spring stiffness on the critical loading under action of the residual force (d_2 = 0.5, r_m = 1, r_w = 1)

In the figures 6 and 7 an influence of the connection stiffness of the piezorod to the host structure on the loading capacity of the investigated system under action of the residual force have been presented. In both figures the continuous line shows the capacity of the column when no voltage is being applied to the piezorod. In the case when the residual force causes the compression of the first member the increase in maximum loading magnitude has been achieved. This increase is highly dependant on the stiffness in the connection of rods (2) and (3) and the length of rod (2). The decrease of critical loading is caused by extension of the first member (figure

7) or by installation of very stiff rotational spring. It can be concluded that it is better to generate compressive forces in the range of spring stiffness $0 < c_b < c_{b_max}$. Where cbmax is a limiting spring stiffness which depends on its location.

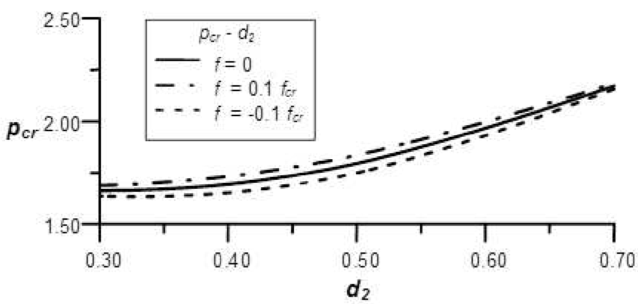

Fig. 8: The influence of the rotational spring location on the critical loading under action of the residual force (c_b = 1, r_m = 1, r_w = 1)

The influence of the rotational spring location on the critical loading under action of the residual force have been shown in the figure 8. When the rotational spring is being moved from the support up to the free end on the column the increase in the magnitude of maximum loading can be observed. The application of the voltage to the piezoceramic element causes the change in maximum loading (the change depends on direction of the electrical field vector). It can be observed that the area of control is being reduced at high spring stiffness or at low spring stiffness and location of the spring near to the free end of the column. Evensen [15] has studied an influence of amplitude on natural frequency of systems with different types of supports. Przybylski [16] has presented the result of investigations on divergence instability of the cantilever column with supporting springs and amplitude - natural frequency relationship. Sok [17] studied the amplitude vibration frequency relationship in the cracked supporting columns. It can be concluded that the type of instability is independent from the spring stiffness. The vibration amplitude change corresponds to vibration frequency change. In the investigated system these changes depend on external load magnitude, rotational spring stiffness, residual force and flexural rigidity factors (all parameters have an influence on shape modes which are related to second vibration frequency component). The non-linear vibration frequency is being computed as dependent on amplitude and the point at which the displacement has the greatest magnitude for give shape mode. The amplitude can expressed by the following formula $A = \varepsilon\sqrt{\lambda}$ (λ- slenderness ratio). The non - linear vibration frequency has been computed as $\omega_n = \sqrt{\omega_0^2 + \varepsilon^2\omega_2^2}$. In the figures 9 - 11 the amplitude - frequency relationship have been presented, for different spring stiffness and location.

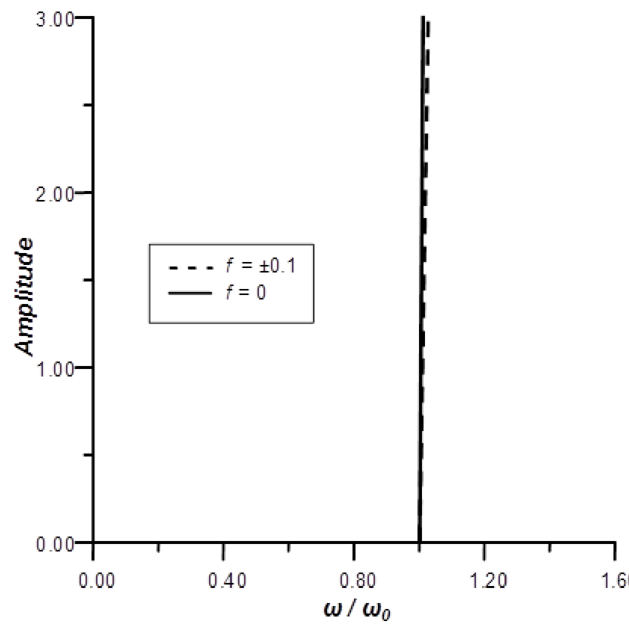

Fig. 9: An influence of the residual force on natural frequency vs. amplitude (d_2 = 0.5, c_b = 100, r_m = 1,r_w = 1)

It can be concluded that regardless of spring location for great stiffness of connection between rods (2) and (3);($c_b = 10$, 100 the amplitude - frequency relationship is equal to unity if the magnitude of the applied voltage is equal to zero. While the voltage has non zero magnitude; the curves of amplitude natural vibration frequency relationship deviates from the vertical position only in one direction regardless from direction of the electric field vector. For the smaller magnitude of stiffness of connection between rods (2) and (3) the deviation of amplitude - frequency relationship curves from the vertical position takes place. Induction of residual forces allows one to obtain amplitude - frequency control. For the positive magnitudes of the electric field vector the investigated curves regain the vertical position and for the negative ones the curves deflection is increasing.

It can be concluded that if the difference in displacements of the rods of the column are small the deflection of the amplitude - frequency relationship curves from the vertical position and the area of control are also small.

7 Concluding remarks

The investigations on instability of the cantilever column have been performed by means of dynamic instability

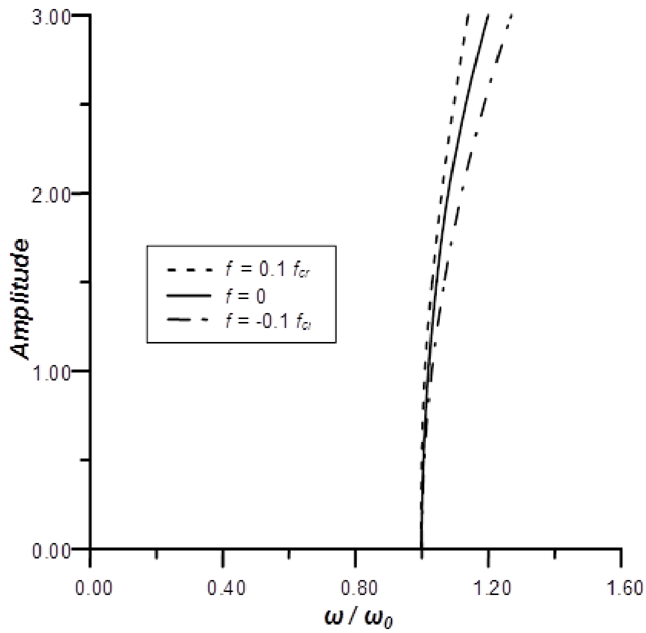

Fig. 10: An influence of the residual force on natural frequency vs. amplitude (d_2 = 0.5, c_b = 1, r_m = 1, r_w = 1)

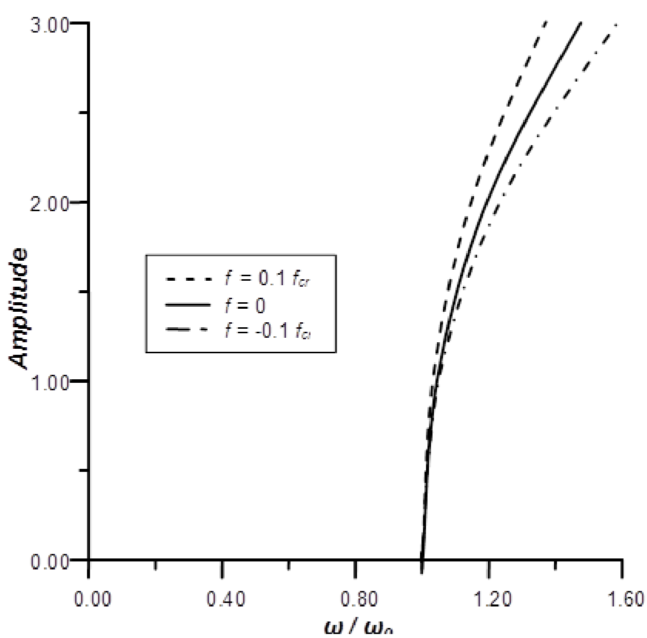

Fig. 11: An influence of the residual force on natural frequency vs. amplitude (d_2 = 0.3, c_b = 2, r_m = 1, r_w = 1)

criteria which allows one to observe the change of vibration frequency in the function of external load. Because of this, after the energetic formulation of the problem the small parameter method has been used in order to perform the solution of the boundary problem. The non-linear component of natural vibration frequency have been obtained on the basis of the orthogonality condition. The general conclusion is that passive control of vibration and instability of the two member column with the piezoceramic rod can be achieved by the proper selection of physical and geometrical features as well as by the induction of prestressing by means of electric field. On the basis of the analysis of the obtained results of numerical simulations it can be concluded that:
- the induction of the additional compressive or tensile forces after the electrical field is being applied to the piezoceramic rod allows one to control vibration frequency what is very important if the system is located in the area of a wide range of excitation frequencies,
- the generation of prestressing of the system by means of residual force, changes the magnitude of maximum load.
the localization of the rotational spring has a great influence of loading capacity of the investigated system. The change in location of the spring form the fixed end up to the free one causes the stabilization of critical forces. Along with this the vibration frequency change,
-the residual force generated by the piezoceramic rod allows to control the amplitude - vibration frequency despite of geometrical features of the column.

The piezoceramic element discretely connected to the host structure can be used to generate residual forces in the mechanical system. That force depends not only on the applied electrical field but also on the geometrical and physical features of the system. Residual forces can be used to correct natural vibration frequency and amplitude - vibration frequency as well as maximum loading capacity.

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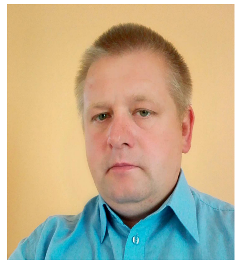

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