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Influence of the Kerr-Like and the External Classical Field on the Atom-Field Interaction

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Abstract: In this communication we study the interaction between a two-level atom and an electromagnetic field in the presence of the classical field and Kerr-like medium. Under a certain condition the system is transformed to the usual Jaynes-Cummings model. The atomic inversion is investigated where the phenomenon of super-structure is reported for a large value of the classical field coupling parameter. Our results show that the information of variance and entropy squeezing beside the purity can be controlled by both of the coupling parameter of the classical field and the Kerr-like medium parameter.

Keywords: Jaynes-Cummings model, external field, Kerr-like medium

1 Introduction

The Jaynes -Cummings Model (JCM) [1] has been recognized as the simplest and most effective model of the interaction between radiation and matter in quantum optics. It describes a two-level atom interacting with a single mode radiation field in the rotating wave approximation (RWA) . The model has been realized experimentally [2,3]. The success of the JCM prompted many physicists to improve and generalize the model in different ways. They studied multimode and multiphoton instead of a single mode and a single photon interaction [4,5,6]. The effect of Kerr-like medium and Stark Shift [7,8,9] have also been studied. The effect of an external field on the JCM has been studied by authors [10,11,12] The dynamics of the driven JCM has been considered using canonical transformations by transforming it to a series of multiphoton JCM for some special cases, the authors [11] found that there are super revivals related to the long time-scale revivals in the average field in the standard JCM. Also the driven JCM has shown that in the presence of the internal coherent field the phenomenon of collapses and revivals as well as the phenomenon of squeezing can be generated from a thermal photon state [12]. However many efforts have been devoted to study the problem of coupling the external field with the JCM , most of these attempts were limited. The exact solution for the problem of a driven JCM was reported in [13]. Following the same approach we will study the effect of the Kerr-like medium on the JCM under the action of an external classical field.

The aim of this paper is to examine the influence of the Kerr-like medium on the atomic motion by considering some statistical properties. The material in this paper is organized as follows, In Section **2** we present a Hamiltonian to describe the interaction between a two level atom and one cavity mode in the presence of the external field as well as the Kerr-like medium and By using the Heisenberg equation we obtain the wave function in section **3**,and in sections **4**-**6** we calculate some statistical properties for the system, Finally, our conclusions are presented in section **7**.

2 The system Hamiltonian

The interaction between the cavity quantized field and two level atom in the presence of the external field and the Kerr-like medium (through the arbitrary function of the photon number operator $f(\hat{n})$) takes the following forms:

$$\hat{H}(t) = \omega(\hat{a}^{\dagger}\hat{a}+\frac{1}{2})+f(\hat{n})+\frac{\overline{\omega}}{2}\hat{\sigma}_z+i(\overline{\varepsilon}(t)\hat{\sigma}_+-\overline{\varepsilon}^*(t)\hat{\sigma}_-)$$
$$+i\lambda(t)(\hat{\sigma}_+-\hat{\sigma}_-)(\hat{a}+\hat{a}^{\dagger}),(\hbar=1) \quad (1)$$

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where $\hat{\sigma}_z$ and $\hat{\sigma}_\pm$are the usual Pauli operator for the two-level atom, $\lambda(t)$ and $\overline{\varepsilon}(t)$ are the time dependent coupling parameter and the complex amplitude of the external field, respectively, ω is the field frequency and $\overline{\omega}$ is the difference between frequency of the atomic levels while $\hat{a}$ and $\hat{a}^\dagger$are the annihilation and creation operators which satisfy the relation $[\hat{a},\hat{a}^\dagger]=1$. It is well known that the coupling parameter for the quantized field with atom, depends on the function $\cos kz$, where k is the wave number and z is the direction of propagation. For the moving atom $z=(vt)$ is assumed where v is the velocity of the atom, the time dependent coupling parameter $\lambda(t)$ may be also regarded as a modulation to the amplitude of the irradiation laser field.

Considering $\lambda(t)=2\lambda_1\cos(\beta t)$ where $\beta=vk$ while we take $\overline{\varepsilon}(t)=\varepsilon\exp(i\gamma t)$, then the Hamiltonian (2) can be written in the following form:

$$\hat{H}(t)=\omega(\hat{a}^\dagger\hat{a}+\frac{1}{2})+f(\hat{n})+\frac{\overline{\omega}}{2}\hat{\sigma}_z+i\lambda_1\left(e^{i(\beta t)}+e^{-i(\beta t)}\right)(\hat{\sigma}_+-\hat{\sigma}_-)\left(\hat{a}+\hat{a}^\dagger\right)+i\left(\varepsilon\hat{\sigma}_+e^{(i\gamma t)}-\varepsilon^*\hat{\sigma}_-e^{(-i\gamma t)}\right) \tag{2}$$

By applying a canonical transformation

$$H_I(t)=U_I^\dagger H(t)U_I-iU_I^\dagger(t)\frac{\partial}{\partial t}U_I(t) \tag{3}$$

where

$$U_I(t)=\exp(i\frac{\gamma\sigma_z}{2}t) \tag{4}$$

to (2) we have

$$\hat{H}_I(t)=\omega(\hat{a}^\dagger\hat{a}+\frac{1}{2})+f(\hat{n})+\frac{\omega_0}{2}\hat{\sigma}_z+i(\varepsilon\hat{\sigma}_+-\varepsilon^*\hat{\sigma}_-)+ i\lambda_1(e^{(i\beta t)}+e^{(-i\beta t)})(\hat{\sigma}_+e^{(-i\gamma t)}-\hat{\sigma}_-e^{(i\gamma t)})\left(\hat{a}+\hat{a}^\dagger\right) \tag{5}$$

where $\omega_0=(\overline{\omega}+\gamma)$. The first two brackets in the last term in the above Hamiltonian lead to four terms, two of them are slowly varying terms which contain the factor $\exp(\pm i(\beta-\gamma)t)$, while the other two terms are the rapidly varying terms which contains the factor $\exp(\pm i(\beta+\gamma)t)$. Therefor, if we discard the rapidly oscillating terms and taking the resonance case in which $\beta=\gamma$ and $\varepsilon=\lambda_2\exp(-\frac{i\pi}{2})$. the Hamiltonian (5) reduced to the following form

$$\hat{H}_I=\omega(\hat{a}^\dagger\hat{a}+\frac{1}{2})+f(\hat{n})+\frac{\omega_0}{2}\hat{\sigma}_z+i\lambda_1(\hat{a}+\hat{a}^\dagger)(\hat{\sigma}_+-\hat{\sigma}_-)+\lambda_2(\hat{\sigma}_++\hat{\sigma}_-) \tag{6}$$

In order to find the wave function $|\psi(t)\rangle$ for the present model, following the approach, [13], let us define the operators $\hat{S}_z$ and $\hat{S}_\pm$ with the new states $|+\rangle$ and $|-\rangle$ (the excited and the ground state for the operators $\hat{S}$) in terms of the states $|e\rangle$ and $|g\rangle$ (the excited and the ground states of the operators $\hat{\sigma}$) as follows:

$$|+\rangle=\cos\eta|e\rangle+\sin\eta|g\rangle,\quad |-\rangle=\cos\eta|g\rangle-\sin\eta|e\rangle. \tag{7}$$

so we can write $\hat{S}$ as

$$\begin{pmatrix}\hat{S}_z\\ \hat{S}_+\\ \hat{S}_-\end{pmatrix}=\begin{pmatrix}\cos 2\eta & \sin 2\eta & \sin 2\eta\\ -\frac{1}{2}\sin 2\eta & \cos^2\eta & -\sin^2\eta\\ -\frac{1}{2}\sin 2\eta & -\sin^2\eta & \cos^2\eta\end{pmatrix}\begin{pmatrix}\hat{\sigma}_z\\ \hat{\sigma}_+\\ \hat{\sigma}_-\end{pmatrix} \tag{8}$$

where $[\hat{S}_+,\hat{S}_-]=\hat{S}_z$, $[\hat{S}_z,\hat{S}_\pm]=\pm2\hat{S}_\pm$, $(\hat{S}_+-\hat{S}_-)$= $(\hat{\sigma}_+-\hat{\sigma}_-)$,$\hat{S}_\pm^2=0$, ,$\hat{S}_z^2=1$ and the determinant of the above matrix is always unity.

From (6,8) after applying the rotating wave approximation for the news operators, we get

$$\hat{H}_I=\omega(\hat{a}^\dagger\hat{a}+\frac{1}{2})+f(\hat{n})+\frac{\Omega_0}{2}\hat{S}_z+i\lambda_1(\hat{a}\hat{S}_+-\hat{a}^\dagger\hat{S}_-) \tag{9}$$

where $\eta=\frac{1}{2}\tan^{-1}\frac{2\lambda_2}{\omega_0}$, and $\Omega_0=\sqrt{\omega_0^2+4\lambda_2^2}$. Note that, the atomic frequency is shifted by the external field coupling parameter λ_2 to $\pm\frac{1}{2}\sqrt{\omega_0^2+4\lambda_2^2}$.

3 Analytical Solution

We devote this section to derive the wave function $|\psi(t)\rangle$ and discuss some statistical properties of the present system. The Heisenberg equation of motion for any operator $\hat{O}(t)$ is given by

$$i\frac{d\hat{O}}{dt}=[\hat{O},\hat{H}] \tag{10}$$

thus, the equation of motion for n̂ and $\hat{S}_z$ are

$$\frac{d\hat{n}}{dt}=-\lambda_1(\hat{a}\hat{S}_++\hat{a}^\dagger\hat{S}_-),\quad \frac{d\hat{S}_z}{dt}=2\lambda_1(\hat{a}\hat{S}_++\hat{a}^\dagger\hat{S}_-) \tag{11}$$

from which we can define the constant of motion as

$$\hat{N}=\hat{n}+\frac{1}{2}\hat{S}_z \tag{12}$$

hence the Hamiltonian (9) can be written in the form

$$\hat{H}_I=\hat{\Lambda}+\hat{C} \tag{13}$$

where

$$\hat{\Lambda}=\omega(\hat{a}^\dagger\hat{a}+\hat{S}_+\hat{S}_-)+\frac{1}{2}\left[f(\hat{N}-\frac{1}{2})+f(\hat{N}+\frac{1}{2})\right] \tag{14}$$

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$$\hat{C} = \frac{\hat{\delta}_1(\hat{n})}{2}\hat{S}_+\hat{S}_- - \frac{\hat{\delta}_2(\hat{n})}{2}\hat{S}_-\hat{S}_+ + i\lambda_1(\hat{a}\hat{S}_+ - \hat{a}^\dagger\hat{S}_-) \quad (15)$$

we note that $[\hat{\Lambda},\hat{C}] = 0$, $[\hat{\Lambda}, \quad \hat{H}] = [\hat{C},\hat{H}] = 0$ i.e. $\hat{\Lambda}$ and $\hat{C}$ are constants of motion

the new detuning parameters $\hat{\delta}_1(\hat{n})$, $\hat{\delta}_2(\hat{n})$ are defined to be

$$\hat{\delta}_1(\hat{n}) = \Delta + f(\hat{n}) - f(\hat{n}+1), \quad \hat{\delta}_2(\hat{n}) = \Delta + f(\hat{n}-1) - f(\hat{n}) \quad (16)$$

where $\Delta = \Omega_0 - \omega$.

The wave function

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle \quad (17)$$

where

$$U(t) = e^{-i\hat{H}_I t} = e^{-i\hat{\Lambda}t}e^{-i\hat{C}t} \quad (18)$$

Assuming that the field is in the coherent state $|\psi\rangle_f$,such that

$$|\psi\rangle_f = \sum_{n=0}^{\infty} q_n|n\rangle, \quad q_n = \exp\left(-\frac{|\alpha|^2}{2}\right)\frac{\alpha^n}{\sqrt{n!}} \quad (19)$$

where α is the coherent complex parameter in general.

and the atom is initially in general atomic state

$$|\psi\rangle_a = \cos\theta|e\rangle + e^{i\phi}\sin\theta|g\rangle \quad (20)$$

where ϕ is a relative phase angle and θ is the atomic coherence angle. In this case the wave function —$\psi(t)\rangle$ at $t = 0$ can be written as

$$|\psi(0)\rangle = |\psi\rangle_a \otimes |\psi\rangle_f \quad (21)$$

In this meantime, from the above equations we can get all information about (JCM) , But we want to examine the effect of the given arbitrary function in the presence of the external field.

from (7,20) can be written in the form

$$|\psi\rangle_a = \left[\cos\eta\cos\theta + e^{i\phi}\sin\eta\sin\theta\right]|+\rangle - \left[\sin\eta\cos\eta - e^{i\phi}\cos\eta\sin\theta\right]|-\rangle \quad (22)$$

In the absence of the external classical field, i.e. $\lambda_2 \to 0$, i.e. $\eta = 0$ it is easy to see $|+\rangle = |e\rangle$ and $|-\rangle = |g\rangle$.

using (17-21) we get

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} q_n\left[\hat{F}(\hat{n},t)|\hat{n},+\rangle - \hat{G}(\hat{n},t)|\hat{n},-\rangle\right] \quad (23)$$

where $\hat{F}(\hat{n},t)$ and $\hat{G}(\hat{n},t)$ are given by

$$\hat{F}(\hat{n},t) = \exp\left(-i\frac{\hat{z}_1(\hat{n})t}{2}\right)\left[c_1\left(\cos\hat{\mu}_1 t - \frac{i\hat{\delta}_1}{2\hat{\mu}_1}\sin\hat{\mu}_1 t\right) - \lambda_1 s_1\frac{\sin\hat{\mu}_1 t}{\hat{\mu}_1}\hat{a}\right],$$

$$\hat{G}(\hat{n},t) = \exp\left(-i\frac{\hat{z}_2(\hat{n})t}{2}\right)\left[s_1\left(\cos\hat{\mu}_2 t + \frac{i\hat{\delta}_2}{2\hat{\mu}_2}\sin\hat{\mu}_2 t\right) + \lambda_1 c_1\frac{\sin\hat{\mu}_2 t}{\hat{\mu}_2}\hat{a}^\dagger\right] \quad (24)$$

with

$$\hat{z}_1(\hat{n}) = 2\omega(\hat{n}+1) + f(\hat{n}) + f(\hat{n}+1),$$
$$\hat{z}_2(\hat{n}) = 2\omega(\hat{n}) + f(\hat{n}-1) + f(\hat{n}) \quad (25)$$

$$\hat{\mu}_1^2 = \frac{\delta_1^2(\hat{n})}{4} + \lambda_1^2\hat{a}\hat{a}^\dagger, \quad \hat{\mu}_2^2 = \frac{\delta_2^2(\hat{n})}{4} + \lambda_1^2\hat{a}^\dagger\hat{a}. \quad (26)$$

$$c_1 = \cos\eta\cos\theta + e^{i\phi}\sin\eta\sin\theta,$$
$$s_1 = \sin\eta\cos\eta - e^{i\phi}\cos\eta\sin\theta. \quad (27)$$

Therefore, the expectation value of any operator $\hat{Q}$ is given by

$$\langle\hat{Q}\rangle = \langle\psi(t)|\hat{Q}|\psi(t)\rangle \quad (28)$$

Hence

$$\begin{aligned}\langle\hat{S}_x(t)\rangle = &\sum_{n=0}^{\infty}\cos(\frac{z_1-z_2}{2})t\Big[q_n^2\sin 2(\eta-\theta)\left(A+\frac{\delta_1\delta_2}{4}B\right)\\ &-\lambda^2 n q_{n-1}q_{n+1}\sin 2(\eta-\theta)B\\ &+2\lambda_1\sqrt{n}q_n q_{n-1}\cos^2(\eta-\theta)C\\ &-2\lambda_1\sqrt{n+1}q_n q_{n+1}\sin^2(\eta-\theta)D\Big]\\ &-\sum_{n=0}^{\infty}\sin(\frac{z_1-z_2}{2})t\Big[q_n^2\sin 2(\eta-\theta)(\frac{\delta_1 D}{2}+\frac{\delta_2 C}{2})\\ &+\lambda_1\delta_1\sqrt{n}q_n q_{n-1}\cos^2(\eta-\theta)B\\ &-\lambda_1\delta_2\sqrt{n+1}q_n q_{n+1}\sin^2(\eta-\theta)B\Big]\end{aligned} \quad (29)$$

$$\begin{aligned}\langle\hat{S}_y(t)\rangle = &\sum_{n=0}^{\infty}\cos(\frac{z_1-z_2}{2})t\Big[q_n^2\sin 2(\eta-\theta)(\frac{\delta_1 D}{2}+\frac{\delta_2 C}{2})\\ &+\lambda_1\delta_1\sqrt{n}q_n q_{n-1}\cos^2(\eta-\theta)B\\ &-\lambda_1\delta_2\sqrt{n+1}q_n q_{n+1}\sin^2(\eta-\theta)B\Big]\\ &+\sum_{n=0}^{\infty}\sin(\frac{z_1-z_2}{2})t\Big[q_n^2\sin 2(\eta-\theta)(A+\frac{\delta_1\delta_2}{4}B)\\ &-\lambda^2 n q_{n-1}q_{n+1}\sin 2(\eta-\theta)B\\ &+2\lambda_1\sqrt{n}q_n q_{n-1}\cos^2(\eta\theta)C\\ &-2\lambda_1\sqrt{n+1}q_n q_{n+1}\sin^2(\eta-\theta)D\Big].\end{aligned} \quad (30)$$

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$$\langle \hat{S}_z(t)\rangle = \cos 2(\theta-\eta) - 2\lambda_1^2 \sum_{n=0}^{\infty} q_n^2 \left[(n+1)c^2 \frac{\sin^2 \mu_1 t}{\mu_1^2} - n\, s^2 \frac{\sin^2 \mu_2 t}{\mu_2^2}\right] - \lambda_1 \sin(2(\theta-\eta)) \sum_{n=0}^{\infty} q_n q_{n+1}\sqrt{n+1}\frac{\sin 2\mu_1 t}{\mu_1} \quad (31)$$

where

$$A = \cos\mu_1 t \cos\mu_2 t, \quad B = \frac{(\sin\mu_1 t \sin\mu_2 t)}{\mu_2\mu_1},$$

$$C = \frac{(\cos\mu_1 t \sin\mu_2 t)}{\mu_2}, \quad D = \frac{(\cos\mu_2 t \sin\mu_1 t)}{\mu_1}. \quad (32)$$

$$\mu_1^2 = \frac{\delta_1^2(n)}{4} + \lambda_1^2(n+1), \quad \mu_2 = \frac{\delta_2^2(n)}{4} + \lambda_1^2 n. \quad (33)$$

Now the atomic inversion $\langle\hat{\sigma}_z(t)\rangle$ and the expectation value of $\hat{\sigma}_x(t)$ can be obtained from the expressions,

$$\langle\hat{\sigma}_z(t)\rangle = \langle\hat{S}_z(t)\rangle \cos 2\eta - \langle\hat{S}_x(t)\rangle \sin 2\eta,$$

$$\langle\hat{\sigma}_x(t)\rangle = \langle\hat{S}_x(t)\rangle \cos 2\eta + \langle\hat{S}_z(t)\rangle \sin 2\eta. \quad (34)$$

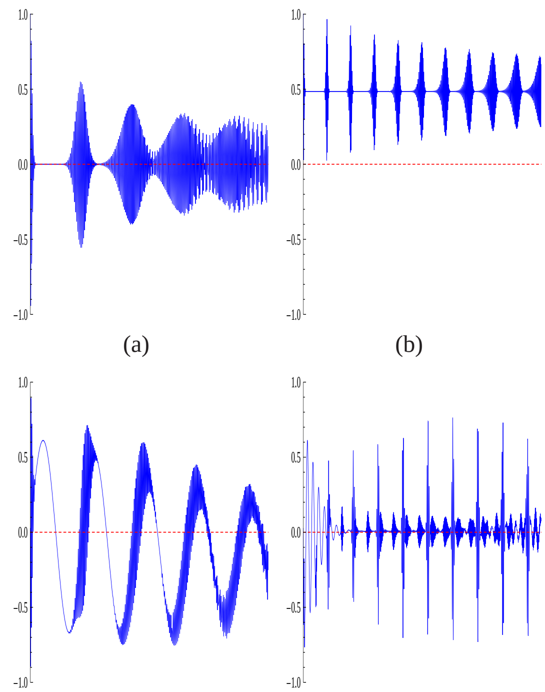
at this stage we are going to discuss some statistical properties for the present system.

4 Atomic inversion

The atomic population inversion is defined as the difference between the probabilities of finding the atom in the excited state and in the ground state. This is in fact would give information about the behavior of the atom field interaction through the collapse and revival phenomenon. In order to study the effect of Kerr-like we take (as a special case) $f(\hat{n}) = \chi(\hat{n}^2 - \hat{n})$. In this case we note that

$$\hat{\delta}_1(\hat{n}) = \Delta - 2\chi\hat{n}, \quad \delta_2(\hat{n}) = \Delta - 2\chi(\hat{n}-1). \quad (35)$$

We plot the function $\langle\hat{\sigma}_z(t)\rangle$ against the scaled time $\lambda_1 t$ in the case of the mean photon number $|\alpha|^2 = \bar{n} = 25$ for which the phenomenon of collapes and revivals become pronounced. We have plotted the atomic inversion to display its behaviour for different values of the Kerr-like parameter χ and the coupling parameter ratio λ_2/λ_1. Keeping in mind that the atom is in the excited state. In Fig. (1a) we take $\chi/\lambda_1 = 0$, $\lambda_2/\lambda_1 = 0$ the function shows behaviour similar to that of the usual JCM as would be expected, for $\chi/\lambda_1 = 0.2$ and the coupling parameter ratio λ_2/λ_1 still equal zero it shifts upward and fluctuates around ˜0.5 where the atomic inversion never reaches to zero value at any period of time



(a) (b) (c) (d)

Fig. 1: the atomic inversion against the scaled time $\lambda_1 t$ for the atom initially in the excited state and the field in the coherent state $\alpha = 5$ and for different values of the kerr-like parameter and the coupling parameter ratio λ_2/λ_1. (a) $\chi/\lambda_1 = 0$, $\lambda_2/\lambda_1 = 0$, (b) $\chi/\lambda_1 = 0.2$, $\lambda_2/\lambda_1 = 0$, (c) $\chi/\lambda_1 = 0$, $\lambda_2/\lambda_1 = 2$, (d) $\chi/\lambda_1 = 0.2$, $\lambda_2/\lambda_1 = 2$.

as displaced in Fig.(1b). Different behaviour is observed when the coupling parameter ratio$\lambda_2/\lambda \neq 0$,as a special case $\lambda_2/\lambda_1 = 2$, but the Kerr-like $\chi/\lambda_1 = 0$ we observe a decrease in its amplitude,the oscillation around (0) but not symmetric as in the case of (1a,b), and the mean of the envelop as a whole is slightly shifted down below the value of zero as it goes away from zero Fig.(1c). But in Fig.(1d) since the Kerr-like and the coupling ratio takes values differ from zero namely $\chi/\lambda_1 = 0.2$, and $\lambda_2/\lambda_1 = 2$, the function fluctuates around zero with amplitude extreme around $\sim \pm 0.5$ and symmetric. From the above discussion we show that the Kerr-like shifts the atomic energy only in the upward direction and show that the higher of the Kerr-like the smaller of the revival period but the external field shifts the atomic energy levels and mixes the atomic operator where the effect of $\langle\hat{S}_x(t)\rangle$ in the expression of the atomic inversion apparent.

5 The phenomenon of squeezing

Squeezing phenomenon is one of the most interesting phenomenon in the field of quantum optics. It reflects the non classical behavior for quantum systems.the atomic

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variable squeezing has been a subject of interesting studies. This is due to its relation to quantum entanglement [14,15,16] a basic ingredient of quantum information. Atomic variable squeezing is a useful parameter to quantify entanglement because it is a physically measurable quantity defined by simple spin-$\frac{1}{2}$operators.This in fact would give us an advantage to see the variation in the usual JCM as a result of the external driving field. Therefore we devote the next two subsections to study the information of the variance as well as the entropy squeezing.

5.1 Variance squeezing

It is well known that the entropy and variance squeezing are built up on the concept of the uncertainty relations To continue our progress we devote this subsection to discuss the variance squeezing and to see the effect of the Kerr-like parameter and the driving field on the quadrature variances.This has been discussed dy the authors of Ref.[17]. In quantum mechanical system one can write the Heisenberg uncertainty relation in the form

$$\left\langle (\Delta\hat{A})^2\right\rangle \left\langle (\Delta\hat{B})^2\right\rangle \geq \frac{1}{4}|\left\langle C\right\rangle|^2 \qquad (36)$$

where Â and B̂ are two physical observable Hermitian operators satisfying the commutation relation $\left[\hat{A},\hat{B}\right] = i\hat{C}$, while $\left\langle (\Delta\hat{A})^2\right\rangle$ and $\left\langle (\Delta\hat{B})^2\right\rangle$ are the quadrature variances. Also we can see the uncertainty relation for a two level atom characterized by Pauli operator $\hat{\sigma}_x, \hat{\sigma}_y$ and $\hat{\sigma}_z$, satisfying the commutation relation $[\hat{\sigma}_x, \hat{\sigma}_y] = 2i\hat{\sigma}_z$. can also write as $\Delta\hat{\sigma}_x\Delta\hat{\sigma}_y \geq |\langle\hat{\sigma}_z\rangle|$. Fluctuations in the component $\hat{\sigma}_\alpha$ of the atomic dipole said to be squeezed if $\Delta\hat{\sigma}_\alpha$ satisfies the condition

$$V_\alpha(t) = \left(\Delta\hat{\sigma}_\alpha(t) - \sqrt{|\langle\hat{\sigma}_z(t)\rangle|}\right) < 0, \alpha = xory. \qquad (37)$$

To discuss the variance squeezing for the present system we have to calculate either $\Delta\hat{\sigma}_x(t)$ or $\Delta\hat{\sigma}_y(t)$ which can be obtained from Eqs.(29-34). To get some insight about the quadrature variances, we have plotted Fig.(2) to display the behavior of both quadratures $V_x(t)$ and $V_y(t)$ against the scaled time $\lambda_1 t$ for different values of the Kerr-like parameter χ/λ_1 and the coupling parameter ratio λ_2/λ_1. As before we restrict our examination for the case in which $\bar{n} = 25$ This means that for $\chi/\lambda_1 = 0$, $\lambda_2/\lambda_1 = 0$ the system turns to the usual JCM at exact resonance. In this case no squeezing can be reported in both quadrature variances as should be expected (not displayed here). and when we take $\lambda_2/\lambda_1 = 1$ and take different values for χ we observe squeezing occurs nearly one time in the half of the meantime in the first quadrature at $\chi/\lambda_1 = 0.3$ (Fig. 2(a) but in the second quadrature the phenomenon is absent over all time and for all values of the kerr like parameter

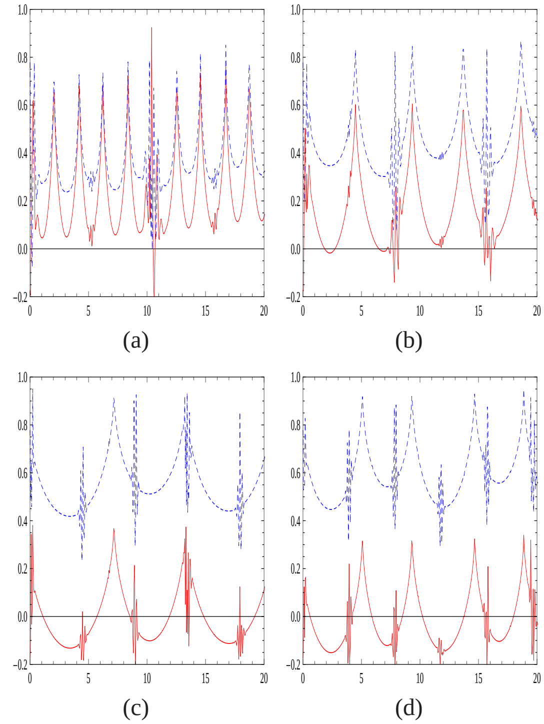

Fig. 2: The Variances squeezing $V_x(t)$ (solid line) and $V_y(t)$ (dashed line) against the scaled time $\lambda_1 t$ for the atom initially in the excited state and the field in the coherent state $\alpha = 5$ and for constant coupling parameter ratio $\lambda_2/\lambda_1 = 1$,and for different values of the Kerr-like parameterχ (a)$\chi/\lambda_1 = 0.3$, (b)$\chi/\lambda_1 = 0.4$, (c)$\chi/\lambda_1 = 0.7$, (d)$\chi/\lambda_1 = 0.8$.

see Fig. (2). When $\chi/\lambda_1 = 0.4$ the squeezing occurs nearly two times in the first quadrature Fig. (2b), but when $\chi/\lambda_1 = 0.7$ and $\chi/\lambda_1 = 0.8$ we observe the squeezing occur four times and nearly five times (respectively) however, we realize that the maximum value of the squeezing is fixed just above -0.2 as in Fig.(2c,d) Finally, we report that an increase in the value of the Kerr-like parameter would lead to increase of squeezing in the quadrature $v_x(t)$ but still no squeezing in the second quadrature $v_y(t)$, see Ref. [18].

5.2 Entropy squeezing

Since the entropy squeezing is a key concept of quantum information theory, our consideration of the entropy squeezing would give us more information about the JCM in the presence of the external classical field. As is well known in an even N-dimensional Hilbert space, the optimal entropic uncertainty relation for sets of $N+1$

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complementary observable with non degenerate eigenvalues can be described by the inequality[19, 20]

$$\sum_{r=0}^{N+1} H(\hat{\sigma}_\beta) \geq \left[\left(\frac{N}{2}\right) \ln \left(\frac{N}{2}\right) + \left(1+\frac{N}{2}\right) \ln \left(1+\frac{N}{2}\right) \right] \quad (38)$$

where H($\hat{\sigma}_\beta$) represents the Shannon information entropy of the variable $\hat{\sigma}_\beta$. The corresponding Shannon information entropies are defined as

$$H(\hat{\sigma}_\beta) = -\sum_{r=1}^{N} p_j(\hat{\sigma}_\beta) \ln p_j(\hat{\sigma}_\beta) \beta = x, y, z. \quad (39)$$

where $p_j(\hat{\sigma}_\beta)$ are the probability distributions for N possible outcomes of measurements of the operators $\hat{\sigma}_\beta$. To obtain the information entropies of the atomic operators $\hat{\sigma}_\beta$ for a two-level atom, with $N = 2$, we use the expression

$$H(\hat{\sigma}_\beta) = -\left[\left(\frac{1}{2} + \langle\hat{\sigma}_\beta\rangle\right) \ln \left(\frac{1}{2} + \langle\hat{\sigma}_\beta\rangle\right) + \left(\frac{1}{2} - \langle\hat{\sigma}_\beta\rangle\right) \ln \left(\frac{1}{2} - \langle\hat{\sigma}_\beta\rangle\right) \right] \quad (40)$$

where $\beta = x, y, z$, together with Eqs. (29-34) It should be noted that the uncertainty relation of the entropy for the present system can be used as a general criterion for the squeezing of atomic variables. More precisely we study squeezing in terms of the information entropy for a two-level atom in interaction with a quantized electromagnetic field under the influence of Kerr-like medium. For a two-level atom, where $N = 2$, we have$0 \leq H(\hat{\sigma}_\beta) \leq \ln 2$ and hence the information entropies of the operators $\hat{\sigma}_\beta$, $\beta = x, y, z$, satisfy the inequality

$$H(\hat{\sigma}_x) + H(\hat{\sigma}_y) + H(\hat{\sigma}_z) \geq 2 \ln 2. \quad (41)$$

This means that, if we define $\delta H(\hat{\sigma}_\beta) = \exp[H(\hat{\sigma}_\beta)]$, the above inequality can be written in the form

$$\delta H(\hat{\sigma}_x) \delta H(\hat{\sigma}_y) \delta H(\hat{\sigma}_z) \geq 4., \quad (42)$$

In this case the atom is in a completely mixed state when $\delta H(\hat{\sigma}_\beta) = 2$. On the other hand the atom is in the pure state when $\delta H(\hat{\sigma}_\beta)$ takes the value 1. Now we define the squeezing of the atom using the entropy uncertainty relation (42). The fluctuations in the components $\hat{\sigma}_\beta$ ($\beta = x$ or y) of the atomic dipole are said to be squeezed in the entropy if the information entropy $H(\hat{\sigma}_\beta)$ satisfies the condition

$$E_\beta(t) = \left(\delta H(\hat{\sigma}_\beta) - \frac{2}{\sqrt{|\delta H(\hat{\sigma}_z)|}} \right) < 0, \beta = x, y. \quad (43)$$

In Fig.(3) we have plotted the entropy squeezing $E_x(t), E_y(t)$ against the scaled time $\lambda_1 t$ for different values of the kerr-like parameter χ/λ_1 and the coupling parameter ratio, for the case in which $\chi/\lambda_1 = 0$, $\lambda_2/\lambda_1 = 0$ the function shows JCM behavior where the entropy squeezing occurs in the first quadrature $E_x(t)$ in the onest of the interaction while the quadrature $E_y(t)$ in the half of the period Fig. (3a). The case of $\lambda_2/\lambda_1 = 2$, and χ/λ_1 takes different values such that $\chi/\lambda_1 = 0.3$ the squeezing in the first quadrature $E_x(t)$ occurs two times and takes maximum value -0.2 while the quadrature $E_y(t)$ occurs in the onest of the interaction only for very short period as in Fig.(3b) and for $\chi/\lambda_1 = 0.4$ the squeezing in the first quadrature $E_x(t)$ occurs three times and takes maximum value -0.3 but no squeezing in the second quadrature $E_y(t)$as in Fig.(3c). Different behavior when $\chi/\lambda_1 = 0.6$ the squeezing in all the first quadrature $E_x(t)$ but no squeezing in the second quadrature $E_y(t)$ see Fig.(3d), then the squeezing occurs all the time in $E_x(t)$ as χ/λ_1increase and decrease in $E_y(t)$ as χ/λ_1 increase.

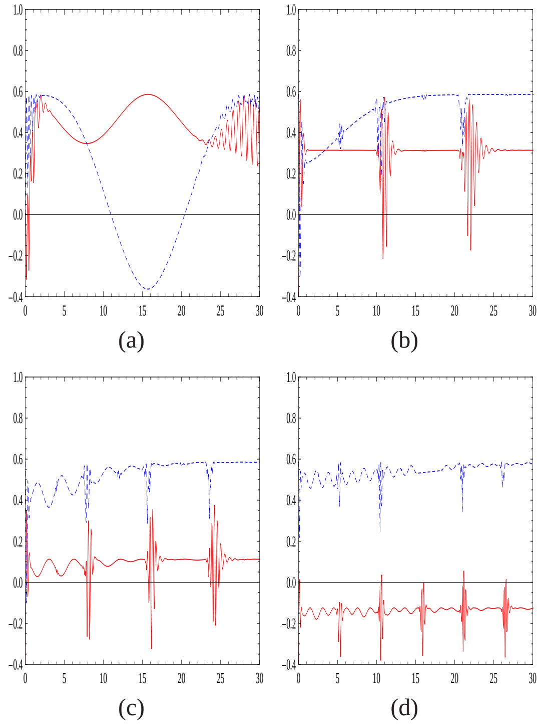

Fig. 3: The entropy squeezing $E_x(t)$ (solid line) and $E_y(t)$ (dashed line) against the scaled time $\lambda_1 t$ for the atom initially in the excited state and the field in the coherent state $\alpha = 5$ and for different values of the Kerr-like parameterχ/λ_1 and the coupling parameter ratioλ_2/λ_1. (a)$\chi/\lambda_1 = 0, \lambda_2/\lambda_1 = 0$, (b)$\chi/\lambda_1 = 0.3, \lambda_2/\lambda_1 = 2$, (c)$\chi/\lambda_1 = 0.4, \lambda_2/\lambda_1 = 2$, (d)$\chi/\lambda_1 = 0.6, \lambda_2/\lambda_1 = 2$.

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6 Linear entropy and entanglement

Now we turn our attention to consider the linear entropy as a tool to discuss the entanglement for the present system.In fact the entanglement is a basic ingredient of many applications of quantum-information technology [21]-[26]. To do so we start with the definition of the linear entropy which is given by

$$\Omega(t) = \frac{1}{2}(1 - \zeta(t)), \quad (44)$$

where $\zeta(t)$ is the well known Bloch sphere radius defined as[27],[28]

$$\zeta(t) = \sqrt{\langle \hat{\sigma}_x(t) \rangle^2 + \langle \hat{\sigma}_y(t) \rangle^2 + \langle \hat{\sigma}_z(t) \rangle^2} \quad (45)$$

The Bloch sphere has been used as a tool in the field of

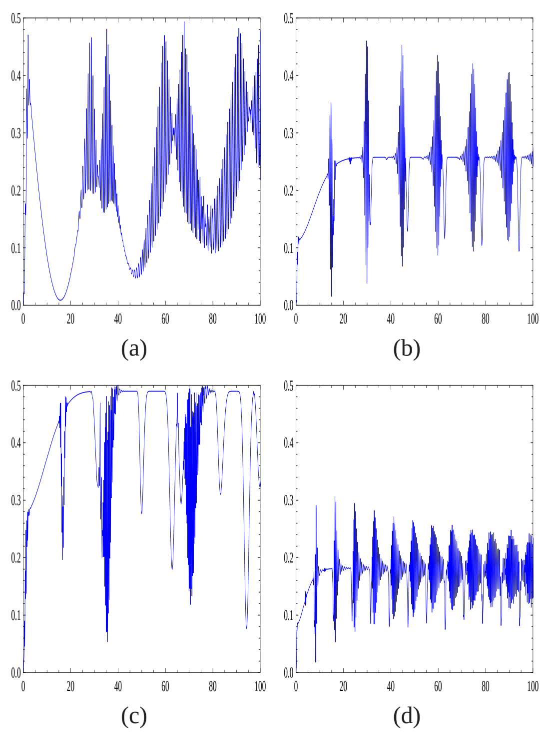

Fig. 4: Linear entropy against the time λ_1t for different values of the kerr-like parameter and the coupling parameter ratio λ_2/λ_1.a)$\chi = 0$, $\lambda_2/\lambda_1 = 0$, b)$\chi = 0.2$, $\lambda_2/\lambda_1 = 0$, c)$\chi = 0.1$, $\lambda_2/\lambda_1 = 3$, d)$\chi = 0.4$, $\lambda_2/\lambda_1 = 3$.

quantum optics. In fact the entanglement is a basic ingredient of many applications of quantum-information technology. In the meantime it forms the pillars of experiments in the realm of quantum information. On the other hand the disentanglement of the two quantum systems suggests interesting applications, e.g. in preparation of atomic states through interacting quantum systems to detect cavity fields. However, for the systems consisting of two subsystems and being prepared in a pure state, a linear entropy of the reduced atomic (or field) density matrix can serve for the degree of entanglement. Therefore analytical conclusions about the system state vector dynamics and atom-field entanglement can be drawn through linear entropy.

In Fig.(4) we plotted the function $\Omega(t)$ against the scaled time $\lambda_1 t$ we consider the case in which the atom in the excited state and $\alpha = 5$ for $\omega = \omega_0 = 0.1\lambda_1$for different values of the kerr-like parameter χ/λ_1 and the coupling parameter ratio λ_2/λ_1 we observe that if we take $\chi/\lambda_1 = 0$, $\lambda_2/\lambda_1 = 0$ the function shows the JCM behavior for which the most entanglement can be seen as in Fig.(4a), and for $\chi/\lambda_1 = 0.1$, $\lambda_2/\lambda_1 = 0$ we can see that a decrease in the maximum value (~ 0.1) of the entanglement after the onest of the interaction and then it starts to increase its value again as in Fig.(4b) in the case of $\chi/\lambda_1 = 0.1$, $\lambda_2/\lambda_1 = 3$ the function shows a decrease in the maximum value (˜0.3) of the entanglement after the onest of the interaction in a short period of time and then the most entanglement (0.5) are observed as in Fig.(4c) ,but for $\chi/\lambda_1 = 0.4$, $\lambda_2/\lambda_1 = 3$ the function shows a decrease of the entanglement (0.25) as in Fig.(4d) we observe that the Kerr decreased the value of the entanglement.

7 Conclusion

In this paper,we have explained the effect of the Kerr-like medium in the system of Hamiltonian with external classical field. Using a certain canonical transformation to obtain a system as for as JCM with kerr-like medium and we derived the wave function and then we discussed the atomic inversion where we show the difference between the system and JCM for different values of Kerr-like and external field parameters, also we have mentioned the variance and entropy squeezing as well as the linear entropy where the system shows squeezing and partial entanglement for a large values of χ.

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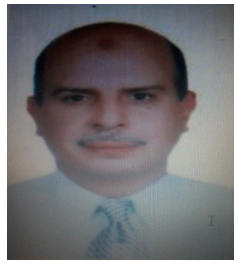

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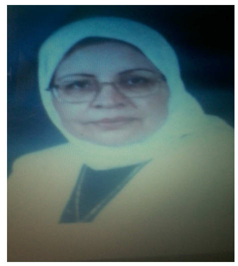

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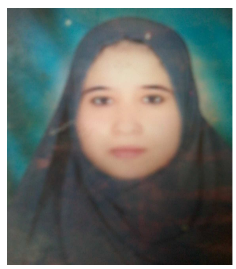

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