Applied Mathematics & Information Sciences
*An International Journal*

http://dx.doi.org/10.18576/amis/100503

# Environmental taxation: Privatization with Different Public Firm's Objective Functions

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Received: 4 Apr. 2016, Revised: 3 Jun. 2016, Accepted: 4 Jun. 2016
Published online: 1 Sep. 2016

**Abstract:** In this paper, we study the effects of environmental taxes and privatization in a mixed market, by considering that the public firm aims to maximize the social welfare. The model has two stages. In the first stage, the government sets the environmental tax. Then, the firms engage in a Cournot competition, choosing output and pollution abatement levels. We also compare the results obtained with the ones got when the public firm aims to maximize the sum of consumer surplus and the firm's profit.

**Keywords:** Industrial Organization; Game Theory, Cournot model, environment, privatization

## 1 Introduction

In recent years, study of mixed markets, where a welfare-maximizing public firm competes with profit-maximizing private firms, are increasingly become popular. Mixed markets are common in many countries. Transportation, energy, steel, telecommunications, oil and hospitals are good examples of mixed oligopolies. DeFraja and Delbono [4] are pioneers in these studies. They showed that in a Cournot competition, privatization of the public firm may improve social welfare. Matsumura and Matsushima [7] proved that, under certain conditions, the partial privatization of the public firm improves social welfare. Some authors (e.g., Ohori [8]) argue that the environmental quality can be affected by the policies of privatization of firms. The imposition of restrictive pollution standards also requires the adoption of costly abatement technology by the firms.

Yin [10] analysed corrective taxes in an oligopoly model with inter-firm externalities and pollution abatement. Bárcena-Ruiz and Garzón [1] studied environmental policies in a duopoly model with incentives. Ohori (2004) examined the interaction among privatization, environmental, and trade policies in international competition with environmental damage. Bárcena-Ruiz and Garzón [2] investigated how the decision on whether to privatize a public firm or not interacts with environmental policy.

Wang and Wang [9] explored whether privatization improves (or deteriorates) the environment in a mixed duopolistic framework with differentiated goods and pollution abatement. They showed that, if the public firm is privatized, less attention is paid to pollution abatement by all the firms coupled with less environment taxes levied by the government, and the environment is more (less) damaged when the goods are less (more) substitutable. Beladi and Chao [3] also showed that privatization can have a negative effect on the environment. Ferreira and Ferreira [6] examined the same questions as in Wang and Wang's paper, by considering a Stackelberg-type sequential-move game with homogeneous goods.

In our paper, we do a similar analysis as done by Wang and Wang [9], but, following Ohori [8], among others, we consider that the public firm aims to maximize the social welfare, instead of the sum of consumer surplus and its profit. The results that we get are different than the ones from Beladi and Chao [3] and Wang and Wang [9]. In fact, in our model, we conclude that the environment is more damaged in the mixed than in the private market. An extended abstract of a previous version of this paper was published in the Proceedings of the $11^{th}$ International Conference on Numerical Analysis and Applied Mathematics (see [5]).

The remained of the paper is organized as follows. In Section II, we present and discuss the mixed model.

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Section III deals with the privatized model. Section IV yields the main results gained by a direct comparison between both the mixed and privatized models. Conclusions are presented in Section V.

## 2 The mixed duopoly

We consider a mixed Cournot duopoly with one public firm $F_1$ and one private firm $F_2$, producing homogeneous goods. We will use a specific model, which is a standard setting in this field. The inverse demand function is given by

$$p = \alpha - Q,$$

where $\alpha > 0$ is the demand parameter, $p$ is the market price and $Q = q_1 + q_2$ is the total output in the market, where $q_1$ and $q_2$ are the outputs of the public firm and the private firm, respectively. We assume that both firms share identical quadratic cost functions: $C(q_i) = F + q_i^2$, where $F = 0$ without loss of generality. The total consumer surplus is

$$CS = \frac{1}{2}(q_1 + q_2)^2.$$

The production of the good in both public and private firms leads to pollution $e_i$. Environmental damage function is given by

$$ED = \frac{(\sum_i e_i)^2}{2}.$$

However, each firm can prevent pollution by undertaking abatement measures. Suppose that if firm $F_i$ chooses pollution abatement level $a_i$, then the corresponding emission level is $e_i = q_i - a_i$. The cost of pollution abatement of firm $F_i$ is $a_i^2/2$.

Each firm has to pay an environmental tax per unit of pollutant emitted and, as a result, tax revenues collected by the government are $T = t\sum_i e_i$. The environmental tax is imposed by the government and its objective is to maximize social welfare, which comprises the consumer surplus $CS$, the producer surplus $\pi_1 + \pi_2$, and the tax revenues collected by the government $T$, less the environmental damage $ED$:

$$W = CS + \pi_1 + \pi_2 + T - ED,$$

where the profit of firm $F_i$ is given by

$$\pi_i = p_i q_i - q_i^2 - te_i - \frac{a_i^2}{2}, \quad i = 1, 2.$$

As mentioned above, the public firm's objective function is the social welfare $W$, and the private firm's is to maximize its own profit.

The model consists in the following two-stage game:

–In the first stage, the government chooses the environmental tax rate $t$.

–In the second stage, the firms engage in a Cournot competition, choosing, simultaneously, output and pollution abatement levels.

As usual, the game is solved by backwards induction. In the second stage, both firms choose, simultaneously, output and pollution abatement levels. So, we differentiate the function $W$ with respect to $q_1$ and $a_1$ and the function $\pi_2$ with respect to $q_2$ and $a_2$:

$$\frac{\delta W}{\delta q_1} = \alpha - 4q_1 - 2q_2 + a_1 + a_2 = 0,$$

$$\frac{\delta W}{\delta a_1} = q_1 + q_2 - 2a_1 - a_2 = 0,$$

$$\frac{\delta \pi_2}{\delta q_2} = \alpha - t - q_1 - 4q_2 = 0,$$

$$\frac{\delta \pi_2}{\delta a_2} = t - a_2 = 0.$$

The above first-order conditions yield the following results:

$$q_1 = \frac{5\alpha + 7t}{25}, \quad q_2 = \frac{5\alpha - 8t}{25}. \tag{1}$$

$$a_1 = \frac{5\alpha - 13t}{25}, \quad a_2 = t. \tag{2}$$

From equations (2), we see that the private firm abates pollution to the point where marginal abatement cost equals the tax, but this is not true for the public firm.

Now, putting (1) and (2) into the objective function $W$ of the government, we get

$$W = \frac{25\alpha^2 + 25\alpha t - 119t^2}{125}.$$

Maximizing this function with respect to the variable $t$, and then substituting the result back to $q_i$ and $a_i$, we get the following values at equilibrium:

**Proposition 1.** *In equilibrium, the environmental tax, the quantities and the pollution abatement level in the mixed duopoly are given by*[1]*:*

$$t^M = \frac{25\alpha}{238},$$

$$q_1^M = \frac{39\alpha}{170}, \quad q_2^M = \frac{99\alpha}{595},$$

$$a_1^M = \frac{173\alpha}{1190}, \quad a_2^M = \frac{25\alpha}{238}.$$

From the expressions above, we obtain the profits of each firm, consumer surplus, environmental damage, tax revenues collected by the government and social welfare as shown below.

[1] We use the superscript $M$ to refer to the mixed duopoly.

**Proposition 2.***In equilibrium, the consumer surplus, the environmental damage, the profit of each firm, the tax revenue collected by the government and the social welfare are given by:*

$$\pi_1^M = \frac{26941\alpha^2}{404600}, \qquad \pi_2^M = \frac{172441\alpha^2}{2832200},$$

$$CS^M = \frac{221841\alpha^2}{2832200},$$

$$ED^M = \frac{29929\alpha^2}{2832200},$$

$$T^M = \frac{865\alpha^2}{56644},$$

$$W^M = \frac{501\alpha^2}{2380}.$$

## 3 Case II: private duopoly (post-privatization)

We now proceed to the analysis of post-privatization. As usual, we regard privatization as a change in the objective function of firm $F_1$ form maximizing social welfare to its own profit[2]:

$$\pi_1 = p_1 q_1 - q_1^2 - te_1 - \frac{a_1^2}{2}.$$

Utilizing the same way of calculation as in the previous section, we get the following result.

**Proposition 3.***In equilibrium, the environmental tax, the quantities and the pollution abatement level in the privatized duopoly are given by*[3]*:*

$$t^P = \frac{11\alpha}{101},$$

$$q_i^P = \frac{18\alpha}{101},$$

$$a_i^P = \frac{11\alpha}{101}.$$

From the expressions above, we obtain the following result:

[2] We suppose that the public firm is privatized without cost.

[3] We use the superscript $P$ to refer to the private duopoly.

**Proposition 4.***In equilibrium, the consumer surplus, the environmental damage, the profit of each firm, the tax revenue collected by the government and the social welfare in the privatized duopoly are given by:*

$$CS^P = \frac{648\alpha^2}{10201},$$

$$ED^P = \frac{98\alpha^2}{10201},$$

$$\pi_i^P = \frac{648\alpha^2}{10201},$$

$$T^P = \frac{154\alpha^2}{10201},$$

$$W^P = \frac{21\alpha^2}{101}.$$

## 4 Effects of privatization

In this section, we compare the mixed and privatized duopoly equilibria. The following theorem summarizes our results.

**Theorem 1.***In equilibrium,*

$$q_2^M < q_i^P < q_1^M, \qquad Q^P < Q^M,$$
$$a_2^M < a_i^P < a_1^M,$$
$$t^M < t^P,$$
$$\pi_2^M < \pi_i^P < \pi_1^M,$$
$$ED^P < ED^M, \qquad CS^P < CS^M,$$
$$T^P < T^M, \qquad W^P < W^M.$$

We observe that in the private competition, the market is more competitive, and thus the private firm produces more than the mixed competition ($q_i^P > q_2^M$). On the other hand, the privatized public firm reduces its production, since its competitor acts more aggressively ($q_i^P < q_1^M$). The overall effect when the public firm is privatized is a decreasing in the aggregate quantity in the market ($Q^P < Q^M$). For the private firm, the increase in the output level increases its profit ($\pi_2^M < \pi_2^P$). For the privatized public firm, the decrease in the output level is not fully compensated by the exchange of its objective from social welfare maximization to profit maximization, and, so, decreases its profit ($\pi_1^M > \pi_1^P$). Furthermore, higher (resp., lower) output levels in the mixed competition than in the private competition induces higher (resp., lower) corresponding abatement levels also in the mixed than in the private competition ($q_1^M > q_1^P \Rightarrow a_1^M > a_1^P$; $q_2^M < q_2^P \Rightarrow a_2^M < a_2^P$). The overall effect is that the environmental tax is lower in the mixed than in the private

competition ($t^M < t^P$). Another consequence is that the environment is more damaged in the mixed than in the private market ($ED^M > ED^P$). Moreover, the consumer surplus and the social welfare are also higher in the mixed than in the private competition.

## 5 Comparison with different public firm's objective functions

Up to now, we have used the social welfare as the public firm's objective function. An alternative of that public firm's objective function is the sum of consumer surplus and its own profit:

$$G = CS + \pi_1,$$

as it is used by Wang and Wang [9]. They got the following result:

**Proposition 5.** *[9] In equilibrium*[4],

$$q_1^{M,G} > q_i^{P,G} > q_2^{M,G},$$
$$Q^{M,G} > Q^{P,G},$$
$$\pi_i^{P,G} > \pi_2^{M,G} > \pi_1^{M,G},$$
$$a_1^{M,G} = a_2^{M,G} = t^{M,G} > t^{P,G} = a_i^{P,G},$$
$$CS^{M,G} > CS^{P,G},$$
$$ED^{P,G} > ED^{M,G},$$
$$W^{P,G} > W^{M,G}.$$

From the previous results, we can conclude that the definition of the public firm's objective function plays an important role in the effects of privatization. In fact, in our model, privatization decreases the environmental damage and the welfare, results that are in contrast with the ones got through the formalization of Wang and Wang [9]: In their case, privatization increases environmental damage and also the welfare.

## 6 Conclusions

In this paper, we analysed the effects of privatization of a public firm in the environment, by considering a mixed Cournot duopoly assuming that each firm can prevent pollution by undertaking abatement measures.

We concluded that, in the mixed market the industry output is higher than in the private market. Furthermore the environmental tax rate in the mixed duopoly is lower than that in the privatized duopoly, and the environment is more damaged in the mixed than in the private market. The overall effect on the social welfare is that it will becomes higher in the mixed than in the private market.

We have also compared the results of our model with the ones got for a different public firm's objective function: the sum of consumer surplus and its own profit. We concluded that the definition of the public firm's objective function plays an important role in the effects of privatization. In fact, in our model, privatization decreases the environmental damage and the social welfare, which is not the case when the public firm aims to maximize the sum of consumer surplus and its own profit.

[4] We use the superscript $M,G$ (resp., $P,G$) to refer to the mixed (resp., private) duopoly, when the public firm's objective function is $G$.

## Acknowledgement

We thank ESEIG and Polytechnic Institute of Porto for their financial support.

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