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Comments on "The Exponentiated Inverted Weibull Distribution"

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Abstract: In this note, we comment on the recently published paper "Flaih, A; Elsalloukh, H; Mendi, E; Milanova M. (2012). The Exponentiated Inverted Weibull Distribution, Appl. Math. Inf. Sci. 6, No. 2, 167-171", which was intended to introduce a new generalization of the standard Inverted Weibull distribution, namely, Exponentiated Inverted Weibull distribution.

Keywords: Exponentiated Inverted Weibull Distribution, Inverse Weibull Distribution

1 Comments

A simple generalization of the standard Inverted Weibull distribution was intended to be proposed by Flaih et al. (2012), namely Exponentiated Inverted Weibull distribution (please see their Section 2). However, such distribution has already been introduced in the literature by [6], referred to as the Inverse Weibull distribution.

Let X be a random variable with Exponentiated Inverted Weibull (EIW) distribution, then its cumulative density function (c.d.f) is given by

$$F(x|\theta,\beta)=\left(\exp\left(-x^{-\beta}\right)\right)^{\theta}=\exp\left(-\theta x^{-\beta}\right), \qquad (1)$$

for all $x>0$, $\theta>0$ and $\beta>0$.

Proposed by [6], the random variable X has an inverse Weibull (IW) distribution if its cumulative distribution function is given by

$$\begin{aligned}F(x|\alpha,\beta)&=\exp\left(-\left(\frac{\alpha}{x}\right)^{\beta}\right)=\exp\left(-\left(\frac{\alpha}{x}\right)^{\beta}\right)\\&=\exp\left(-\alpha^{\beta}x^{-\beta}\right),\end{aligned} \qquad (2)$$

for all $x>0$, $\alpha>0$ and $\beta>0$. Note that (1) is the same as (2) considering the reparametrization $\theta=\alpha^{\beta}$.

Indeed, [7] considered the IW distribution based on progressive type-II censored using the parametrization (1). Different non-informative priors for the IW distribution in the presence of nuisance parameters using the same parametrization were devepoled by [5]. However, it is important to point out that some authors has been used the IW as the EIW distribution. Some frequentist and Bayesian estimation procedures for the parameters of the EIW distribution considering Type II censored were presented by [1]. While, [3] consider the estimation of population parameters for the EIW distribution based on grouped data. Moreover, Bayesian estimation procedures for the parameters of the EIW distribution were presented by [4]. Therefore, the results presented are analogous for the inverse Weibull distribution.

In Section 2 of Flaih et al. (2012) and considering the c.d.f (1), the mean should be given by

$$E[X]=\theta^{\frac{1}{\beta}}\Gamma\left(1-\frac{1}{\beta}\right), \quad \beta>1. \qquad (3)$$

In Section 3, for the EIW distribution the elements of the Fisher information matrix are given by

$$I_{11}(\theta,\beta)=\frac{n}{\theta^2}, \qquad (4)$$

$$I_{12}(\theta,\beta)=I_{21}(\theta,\beta)=\frac{n(1-\gamma-\log(\theta))}{\theta\beta}, \qquad (5)$$

$$I_{22}(\theta,\beta)=\frac{n}{\beta^2}\left(\frac{\pi^2}{6}+(1-\gamma-\log(\theta))^2\right) \qquad (6)$$

where $\gamma\approx 0.5772156649$ is known as Euler-Mascheroni Constant. However, the elements of the

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variance-covariance matrix should be given by

$$I_{11}^{-1}(\theta,\beta) = \frac{6\theta^2}{n\pi^2}\left(\frac{\pi^2}{6} + (1-\gamma-\log(\theta))^2\right), \quad (7)$$

$$I_{12}^{-1}(\theta,\beta) = I_{21}(\theta,\beta) = -\frac{6\beta\theta(1-\gamma-\log(\theta))}{n\pi^2}, \quad (8)$$

$$I_{22}^{-1}(\theta,\beta) = \frac{6}{n\pi^2\beta^2}. \quad (9)$$

In Section 4, the authors analyzed a data set related to the tensile strength of 100 observations of carbon fibers. They considered the EIW distribution to describe such data and used the maximum likelihood method to obtain the estimates of θ and β. However, from the Figure 3 (available in Flaih et al. (2012)), we can easily observe that the EIW distribution is not adequate since there is a significant difference between the empirical and the theoretical distributions.

We reanalysed the data set related to the tensile strength of 100 observations of carbon fibers. For sake of comparison we compared the results with the Weibull distribution. Firstly, to verify the behaviour of the empirical hazard function it will be considered the TTT-plot (total time on test) proposed by [2]. If the curve is concave (convex), the hazard function is increasing (decreasing). When it starts convex and then concave (concave and then convex) the hazard function have a bathtub (unimodal) shape. We also considered the Kolmogorov-Smirnov (KS) test to check the goodness of fit. In the Figure 1, we have the TTT-plot and in the Figure 2 the survival function adjusted by the EIW and the Weibull distributions and the empirical survival function.

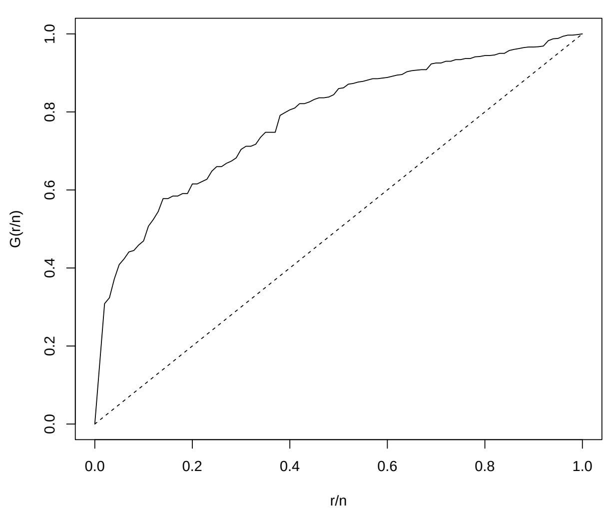

Fig. 1: TTT-plot for the dataset related to the tensile strength of 100 observations of carbon fibres.

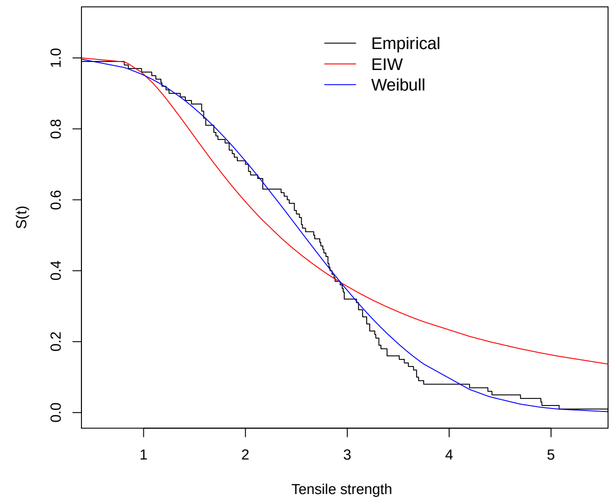

Fig. 2: Survival function adjusted by the EIW, Weibull and a non-parametric method considering the data set related to the tensile strength of 100 observations of carbon fibres

Based on the TTT-plot we observed that the empirical hazard function has an increasing shape. It is worth noting that the EIW distribution only has that hazard function with unimodal shape and therefore should not be used to describe data with increasing failure rate. Comparing the empirical survival function with the adjusted distributions it can be observed a better fit for the Weibull distribution. Considering the KS test we obtain, for the EIW distribution, a KS statistics $D = 0.17614$ with a p-value$= 0.004038$, and, for the Weibull distribution, a KS statistics $D = 0.06320$ with a p-value$= 0.8194$. Therefore, considering a significance level of 5% and since the p-value is smaller than 0.05 for the EIW distribution, we reject the hypothesis that the data comes from a EIW distribution. Moreover, the proposed data can be described by the Weibull distribution, since the p-value returned from the KS test is greater than 0.05.

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