Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/100437

The Atomic Inversion and the Purity of a Quantum Dot Two-Level Systems

D. A. M. Abo-Kahla[*]

Department of Mathematics, Faculty of Education, Ain Shams University, Roxy, Cairo, Egypt

Received: 19 Mar. 2015, Revised: 11 Dec. 2015, Accepted: 14 Dec. 2015
Published online: 1 Jul. 2016

Abstract: In this paper, we analytically solved the Bloch equations of $In_xG_{a_{1-x}}As$ excitonic two level QD semiconductor system. In addition, we calculated the atomic occupation probabilities ρ_{11}, ρ_{22}, the atomic inversion and the purity for some special values of the total dephasing rate γ_2 and the Rabi frequency Ω_R^0. Hence, we studied the effects of the total dephasing rate γ_2 and the Rabi frequency Ω_R^0 on the atomic occupation probabilities ρ_{11}, ρ_{22}, the atomic inversion, and the purity.

Keywords: Quantum dot, Atomic inversion and the Purity

1 Introduction

Quantum information processing [1] such as quantum communication [2] and quantum computation [3] holds promise for the solution of many intractable problems in information technology. One of the promising candidates for the implementation of the quantum information processing devices is a semiconductor quantum dot (QD) system. Semiconductor quantum dots (QDs) serve as attractive platform for quantum information science and technology. Because of flexibility in controlling number of carriers and spins in atom-like density-of-states and of well-suppressed interaction of quantized electrons and holes with environmental degree of freedom.[4,5,6] Semiconductor quantum dots confine electrons and holes in discrete energy levels a few nanometers in size.[8] These properties have driven speculation that quantum dots may provide physical realization of qubits. Proposed implementations using quantum dots include the presence versus absence of an electron in a certain dot level,[9,10,11] the spin-up versus spin-down state of an electron,[7,12,13,14] or the presence of an electron or a hole in one dot versus another dot.[15,16,17,18] The growth of compositionally uniform alloy crystals is promising for variety of applications because lattic parameters as well as electrical and optical properties can be controlled by composition. Among them, $In_xG_{a_{1-x}}As$ bulk crystals are expected as substrates of laser diodes with emitting wavelength of 1.3 μm.

Here, we focus on quantum computation based on excitonic two-level system in an $In_xG_{a_{1-x}}As$ quantum dot.

This article is organized as follows: in section (2) we solved the Bloch equations analytically, in section (3) we presented the atomic inversion, in section (4) we calculated the purity. Conclusions are summarized at the end of the paper.

2 The optical Bloch equations

We study the $In_xG_{a_{1-x}}As$ excitonic two level QD semiconductor system. The experimental measurements on the photocurrent in $In_xG_{a_{1-x}}As$ excitonic two level systems [19] my be understood within the following optical Bloch equations of the two level system [20]:

$$\frac{d\rho_{11}}{dt} = -i\Omega_R^0\cos(\omega_L t)(\rho_{21}-\rho_{12})+\gamma_1\rho_{22}, \quad (1)$$

$$\frac{d\rho_{22}}{dt} = +i\Omega_R^0\cos(\omega_L t)(\rho_{21}-\rho_{12})-\gamma_1\rho_{22}, \quad (2)$$

$$\frac{d\rho_{12}}{dt} = i\omega_{21}\rho_{12}+i\Omega_R^0\cos(\omega_L t)(\rho_{11}-\rho_{22})-\gamma_2\rho_{12}, \quad (3)$$

$$\frac{d\rho_{21}}{dt} = -i\omega_{21}\rho_{12}-i\Omega_R^0\cos(\omega_L t)(\rho_{11}-\rho_{22})-\gamma_2\rho_{21}, \quad (4)$$

where ρ_{11} and ρ_{22} are the corresponding number occupations of the two levels, ω_L is the laser frequency,

[*] Corresponding author e-mail: doaa_abukahla@ymail.com

Ω_R^0 is the Rabi frequency, defined as

$$\Omega_R = \frac{d \cdot E}{\hbar} = \Omega_R^0 \cos(\omega_L t), \tag{5}$$

where d is the conduction-valence dipole matrix element, γ_1 and γ_2 represent the recombination rate and the total dephasing rate, respectively.

We solve the Bloch equations analytically :

Firstly:

We can write the Bloch equations as the form:

$$\frac{d\rho(t)}{dt} = A(t)\rho(t) + B(t), \tag{6}$$

where

$$A(t) = \begin{pmatrix} \frac{-\gamma_1}{2} & i\Omega_R^0 \cos(\omega_L t) & -i\Omega_R^0 \cos(\omega_L t) & \frac{\gamma_1}{2} \\ i\Omega_R^0 \cos(\omega_L t) & i\omega_{21} - \gamma_2 & 0 & -i\Omega_R^0 \cos(\omega_L t) \\ -i\Omega_R^0 \cos(\omega_L t) & 0 & -i\omega_{21} - \gamma_2 & i\Omega_R^0 \cos(\omega_L t) \\ \frac{\gamma_1}{2} & -i\Omega_R^0 \cos(\omega_L t) & i\Omega_R^0 \cos(\omega_L t) & \frac{-\gamma_1}{2} \end{pmatrix},$$

$$B(t) = \begin{pmatrix} \frac{\gamma_1}{2} \\ 0 \\ 0 \\ -\frac{\gamma_1}{2} \end{pmatrix} \text{ and } \rho(t) = \begin{pmatrix} \rho_{11}(t) \\ \rho_{12}(t) \\ \rho_{21}(t) \\ \rho_{22}(t) \end{pmatrix}. \tag{7}$$

Secondly:

We construct the general solution of the homogeneous system

$$\frac{d\rho(t)}{dt} = A(t)\rho(t), \tag{8}$$

Note that the matrix $A(t)$ of the system is symmetric, therefore, the fundamental matrix of the system is given by

$$\begin{aligned} \Phi(t) &= \exp\left(\int_0^t A(\tau)d\tau\right) \\ &= \exp\begin{pmatrix} \frac{-\gamma_1}{2}t & -i\frac{\Omega_R^0}{\omega_L}\sin(\omega_L t) & i\frac{\Omega_R^0}{\omega_L}\sin(\omega_L t) & \frac{\gamma_1}{2}t \\ -i\frac{\Omega_R^0}{\omega_L}\sin(\omega_L t) & (i\omega_{21} - \gamma_2)t & 0 & i\frac{\Omega_R^0}{\omega_L}\sin(\omega_L t) \\ i\frac{\Omega_R^0}{\omega_L}\sin(\omega_L t) & 0 & -(i\omega_{21} + \gamma_2)t & -i\frac{\Omega_R^0}{\omega_L}\sin(\omega_L t) \\ \frac{\gamma_1}{2}t & i\frac{\Omega_R^0}{\omega_L}\sin(\omega_L t) & -i\frac{\Omega_R^0}{\omega_L}\sin(\omega_L t) & \frac{-\gamma_1}{2}t \end{pmatrix} \end{aligned} \tag{9}$$

Now we perform the necessary transformations with the matrix exponential to write the general solution of the homogeneous system.

We find the eigenvalues of the matrix $\left(\int_0^t A(\tau)d\tau\right)$ (λ_i , $i = 1,2,3,4$)

$$\begin{aligned} \lambda_1 &= 0, \\ \lambda_2 &= \frac{1}{3\omega_L^2}\left\{-t\ \omega_L^2(2\gamma_2 - \gamma_1) - \frac{K_1}{K_4} + K_4\right\}, \\ \lambda_3 &= \frac{1}{12\omega_L^2}\left\{\begin{matrix} -4t\ \omega_L^2(2\gamma_2 + \gamma_1) \\ + \frac{2(1+i\sqrt{3})K_1}{K_4} + 2i(i+\sqrt{3})K_4 \end{matrix}\right\}, \\ \lambda_4 &= \bar{\lambda}_3 \text{ (the complex conjugate of } \lambda_3). \end{aligned} \tag{10}$$

Where

$$\begin{aligned} K_1 &= 12\Omega_R^{0^2}\omega_L^2 \sin[t\ \omega_L]^2 - t^2\omega_L^4(\gamma_1^2 \\ &\quad -2\gamma_1\gamma_2 + \gamma_2^2 - 3\omega_{21}^2), \\ K_2 &= -18\Omega_R^{0^2}\sin[t\ \omega_L]^2 + t^2\omega_L^2(\gamma_1^2 \\ &\quad -2\gamma_1\gamma_2 + \gamma_2^2 + 9\omega_{21}^2), \\ K_3 &= (\gamma_1 - \gamma_2)[18t\ \Omega_R^{0^2}\omega_L^4\sin[t\ \omega_L]^2 \\ &\quad -9t^3\omega_{21}^2\omega_L^6 - 3t^3\omega_L^6\gamma_1^2] + t^3\omega_L^6(\gamma_2^3 - \gamma_1^3), \\ K_4 &= K_3 + \sqrt[3]{t\ \omega_L^8(\gamma_1 - \gamma_2)^2 + K_2^2 + K_1^3}. \end{aligned} \tag{11}$$

For each eigenvalue λ_i , we determine the corresponding eigenvectors.

Hence, we get the transition matrix of the matrix $\left(\int_0^t A(\tau)d\tau\right)$, and denoted by H, the Jordan form J of the matrix $A(\tau)$ is a diagonal matrix with the eigenvalues λ_i on the diagonal:

$$\begin{aligned} J &= H^{-1}\left(\int_0^t A(\tau)d\tau\right)H \\ &= \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{pmatrix}. \end{aligned} \tag{12}$$

Then

$$\Phi(t) = H\exp(J)H^{-1}. \tag{13}$$

Thus, the general solution of the homogeneous system is given by

$$X_0 = \Phi(t)C. \tag{14}$$

Thirdly:

We find a particular solution $X_1(t)$ of the nonhomogeneous system equation(6) such that

$$\begin{aligned} \rho(t) &= X_0 + X_1 \\ &= \Phi(t)C + X_1. \end{aligned} \tag{15}$$

In accordance with the method of variation of parameters (Lagrange method), we replace the constant vector C with the vector function $C(t)$.

$$\rho(t) = \Phi(t)C(t) \tag{16}$$

Substituting this equation(16) into the nonhomogeneous system equation(6), we find the unknown vector $C(t)$:

$$B(t) = \Phi(t)\acute{C}(t). \tag{17}$$

The derivative of this function is given by the relation

$$\acute{C}(t) = \Phi^{-1}(t)B(t) \tag{18}$$

So

$$C(t) = \int_0^t \Phi^{-1}(\tau)B(\tau)d\tau \tag{19}$$

Hence, we get the general solution of the nonhomogeneous system equation(6)

3 The atomic inversion

The atomic population inversion, $\langle\sigma_z\rangle = \rho_z = \rho_{11} - \rho_{22}$, can be considered as one of the simplest important quantities, it is defined as "the difference between the probabilities of finding the atom in its exited state and in its ground state".

4 The purity

-The evolution of the purity $P_S(t)$ is given by

$$\begin{aligned} P_S(t) &= 1 - Tr_S(\rho_S^2(t)) \\ &= \rho_{11}^2 + 2\left|\rho_{12}\right|^2 + \rho_{22}^2 \end{aligned} \tag{20}$$

where $\rho_S(t)$ is the reduced density matrix of the system which is defined by $\rho_S(t) = Tr_F\ \rho(t)$.

Present calculations are performed by using the optical Bloch equations with γ_1 corresponding [19] to a lifetime of $1ns$.

In the numerical results we consider ($\omega_{21} = 1, \Omega_R^0 = 1$). In Fig. 1, we investigate the effect of the total dephasing rate γ_2, on the atomic occupation probabilities ρ_{11}, ρ_{22}, the atomic inversion and the purity. We show that when the total dephasing rate $\gamma_2 = 1$, the occupation probability of the lower level starts from maximum, $\rho_{11} = 1$, then it decreases until it reaches its minimum value. Once the dephasing rate has decreased, $\gamma_2 = 0.5$, the minimum value of ρ_{11} decreases, and when $\gamma_2 = 0.1$ the minimum value of ρ_{11} decreases even more. However, when the total dephasing rate $\gamma_2 = 1$, the occupation probability of the upper level starts from minimum, $\rho_{22} = 0$, then it increases until it reaches its maximum value. Once the dephasing rate has increased, $\gamma_2 = 0.5$, the maximum value of ρ_{22} increases, and when $\gamma_2 = 0.1$ the maximum value of ρ_{22} increases even more. Hence, the decreasing total dephasing rate γ_2 causes both the minimum value of ρ_{11} to decrease and the maximum value of ρ_{22} to increase relatedly. Also, when the total dephasing rate $\gamma_2 = 1$, the atomic inversion starts from maximum value, $\langle\sigma_z\rangle = 1$, then it decreases until it reaches its minimum value, after that the atomic inversion decreases to $\langle\sigma_z\rangle = 0.2$ with small osculations. When $\gamma_2 = 0.5$, the minimum value of the atomic inversion decreases until it reaches $\langle\sigma_z\rangle = 0$, and when $\gamma_2 = 0.1$ the minimum value of the atomic inversion decreases also

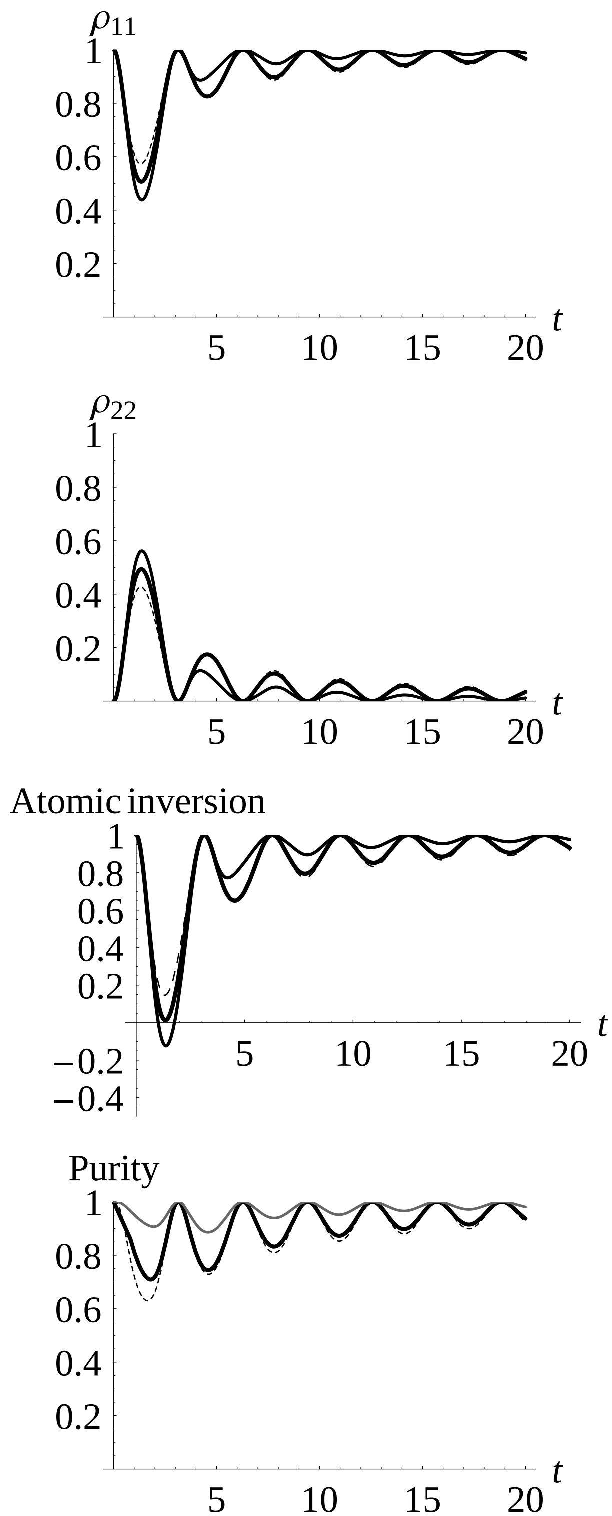

Fig. 1: Figures of the case in which $\omega_{21} = 1, \Omega_R^0 = 1$, where dot, bold solid and grey curves correspond, respectively, to the total dephasing rate γ_2 $(1, 0.5$ and $0.1t \equiv \omega_L t$

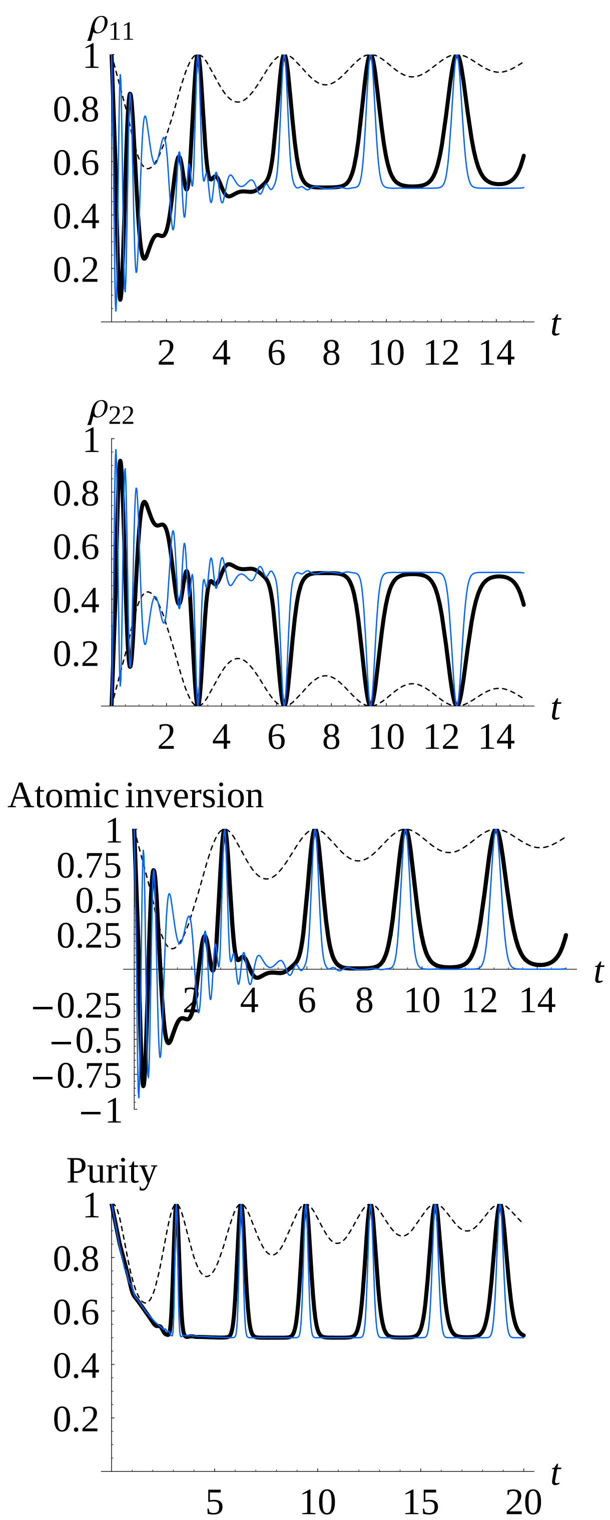

Fig. 2: Figures of the case in which $\omega_{21} = 1, \gamma_2 = 1$ where dot, bold solid and blue curves correspond, respectively, to the Rabi frequency Ω_R^0 (1, 5 and 10$t \equiv \omega_L t$

until it reaches $\langle\sigma_z\rangle = -0.2$. We find that the increase of the purity is remarkably related with the increase of the time. The purity increases also remarkably, when the total dephasing rate γ_2 decreases. This means that the effect of the field is very weak. This clearly shows the effect of the total dephasing rate γ_2.

In Fig. 2, we investigate the effect of the Rabi frequency Ω_R^0, on the atomic occupation probabilities ρ_{11}, ρ_{22}, the atomic inversion and the purity. When $\Omega_R^0 = 1$, the atomic occupation probabilities ρ_{11}, ρ_{22}, and the atomic inversion have regular oscillations. When the Rabi frequency Ω_R^0 increases, it is noticed that, in the interval $0 \le \omega_L t \le 6$, they have irregular oscillations. After that, in the interval $6 \le \omega_L t \le \infty$, they have regular and periodic oscillations with a larger phase. However, when the Rabi frequency Ω_R^0 increases, the purity decreases with small osculations. This clearly shows the effect of the Rabi frequency Ω_R^0.

5 Conclusion

In this paper, we calculated the atomic occupation probabilities ρ_{11}, ρ_{22}, the atomic inversion and the purity for some special values of the total dephasing rate γ_2 and the Rabi frequency Ω_R^0. It is obviouse that when the total dephasing rate γ_2 decreases, both of the minimum value of ρ_{11} and the atomic inversion decrease, while both of the maximum value of ρ_{22} and the purity increase remarkably. When the Rabi frequency Ω_R^0 increases, the oscillations of ρ_{11}, ρ_{22} and the atomic inversion become irregular at first. Then they gradually become more regular in shape and larger in phase. The purity, on the other hand, decreases with small osculations. From the above-mentioned we concluded that the changes of the total dephasing rate γ_2 and the Rabi frequency Ω_R^0 have remarkable effects on the atomic occupation probabilities ρ_{11}, ρ_{22}, the atomic inversion and the purity.

References

[1] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, (Cambridge Univ. Press, Cambridge, 2000).
[2] D. P. DiVicenzo, "Quantum computation," Science 270, 255-261 (2000).
[3] N. Gisin, and R. Thew, "Quantum communication," Nature Photon. 1, 165-171 (2007).
[4] D. P. DiVicenzo: quant-phy/0002077 (2000).
[5] H. Kamada et al., Phys. Rev. B 58, 16243 (1998).
[6] D. Deutsch, A. Brenco, and A. Eckert: Proc. Roy. Soc. London Ser. A 474, 969 (1995).
[7] D. Loss and D. P. DiVincenzo, Phys. Rev. A 57, 120 (1998).
[8] D. Bimberg, M. Grundman, and N. N. Ledentsov, Quantum Dots Heterostructures (Wiley, New York, 1999).
[9] P. I. Tamborenea and H. Metiu, Europhys. Lett. 53, 776 (2001).

[10] G. W. Bryant and W. Jaskolski, Physica E (Amsterdam) 13, 293 (2002).
[11] P. Zhang, X.-G. Li, and X.-G. Zhao, Phys. Lett. A 294, 108 (2002).
[12] A. Imamoglu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. 83, 4204 (1999).
[13] D. P. DiVincenzo, D. Bacon, J. Kempe, G. Burkard, and K. B. Whaley, Nature (London) 408, 339 (2000).
[14] D. Loss and E. V. Sukhorukov, Phys. Rev. Lett. 84, 1035 (2000).
[15] M. Bayer, P. Hawrylak, K. Hinzer, S. Fafard, M. Korkusinski, Z. R. Wasilewski, O. Stern, and A. Forchel1, Science 291, 451 (2001).
[16] K. Hinzer, M. Bayer, J. P. McCaffrey, P. Hawrylak, M. Korkusin-ski, O. Stern, Z. R. Wasilewski, S. Fafard, and A. Forchel, Phys. Status Solidi B 224, 385 (2001).
[17] M. Bayer, G. Ortner, A. Larionov, V. Timofeev, A. Forchel, P. Hawrylak, K. Hinzer, M. Korkusinski, S. Fafard, and Z. Wasilewski, Physica E (Amsterdam) 12, 900 (2002).
[18] M. Korkusinski, P. Hawrylak, M. Bayer, G. Ortner, A. Forchel, S. Fafard, and Z. Wasilewski, Physica E (Amsterdam) 13, 610 (2002).
[19] A. Zrenner, E. Beham, S. Stufler, F. Findels, M. Bichler, G. Abstreiter, Nature 418, 612 (2002).
[20] H.S. Brandi, A. Latge´, Z. Barticevic, L.E. Oliveira, Solid State Communications 135, 386–389 (2005).

Doaa Abo-Kahla received her Ms and PhD degrees in quantum optics, from Ain Shams University, Roxy, Cairo, Egypt, in 2007 and 2011 respectively. She is currently full time lecturer of quantum optics at the department of mathematics, faculty of education, Ain Shams University. Her main research interests are quantum information and quantum computation. She is a co-author of 9 papers in Optics Communications, Physica A, Nonlinear Optics and Quantum Optics, Physica E, Quantum Information Review, Canadian Journal Physics, Optik and Laser Physics. She is the main developer of scientific researches data base of faculty of education, Ain Shams University. She is also an ETMS member.