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Computer Construction and Enumeration of All T_0 and All Hyperconnected T_0 Topologies on Finite Sets

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Abstract: There are many axioms on the principal topological spaces. Two of the interesting axioms are the T_0 and hyperconnected topological spaces. There is a well-known and straightforward correspondence (cf. [2]) between the topologies on finite set X_n of n points and reflexive transitive relations (preorders) on those sets. This paper generalizes this result, characterizes the principal hyperconnected T_0-topologies on a nonempty set X and gives their number on a set X_n. It mainly describes algorithms for construction and enumeration of all weaker and strictly weaker T_0 and nT_0-topologieson on X_n. The algorithms are written in fortran 77 and implemented on pentium $II400$ system.

Keywords: Principal topological spaces, preorder relations, $T_0;E$, hyperconnected, hyperconnected T_0-topological spaces

1 Introduction

The ultratopology on a set X is the strictly weaker topology on X than the discrete topology D on X and the infratopology on X is the strictly finer topology on X than the indiscrete topology I on X. In [6] Frohich defined the ultratopology on a set X to be a strictly weaker topology than the discrete topology D on X. The ultratopologies on X are divided into two classes the principal and nonprincipal ultratopologies on X. In [9] Mashhour and Farrag showed that the principal ultratopology on a set X is the topology having the minimal basis $\beta_{yz} = \{\{x\},\{y,z\} : x \in X - \{z\}\}$, where y and z are two distinct points of X and denoted by D_{yz}. In [14] Steiner defined a minimal open set in a topological space $(X,\tau)$to be the open set containing the point x and contained in each open set containing x. The first author defined a principal topology on a set X to be the topology on X having the minimal bases that consist only of open sets minimal at each $x \in X$. It is proved that a topology τ on a set X is principal iff arbitrary intersections of members of τ are members of τ.

In [9]Mashhour et al., defined the S-open set in a topological space (X,τ) to be the open set which cannot be written as a union of distinct open sets which are proper subsets of it. In a principal space (X,τ) the S-open set and the minimal open set are identical and the S-basis i.e., the basis consists only of S-open sets is the minimal basis for τ. In [3], Farrag gave formulas for the numbers of the topologies,hyperconnected and T_0-topologies on a finite set X_n of n points. In [2] Evan et al. established a correspondence between all topologies on a set X_n of n points and all preorders on X_n. In [7] Jason and Stephen gave the number of all preorder relations on a setX_n and so the number of all topologies on X_n. In [4,5], Farrag and Sewisy described algorithms for construction and enumeration of topoloigies on a set X_n. Many authors deal with problem of the number of topologies on X_n as [8,13] and others. Fuzzy topological spaces and algorithms for comparison of Fuzzy sets are described by [12,1].

2 On the T_0 and hyperconnected T_0-topologies

Throughout this paper the topology τ on a nonempty set X is an excluding topology if there is a point $p \in X$ such that $p \notin \cup\{G : G \in \tau - \{X\}\}$: or equivalently if X is minimal at some of its points. Such family of topologies will be denoted by E-topologies. It is a particular topology if there is a point $p \in X$ such that

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$p \in \cap\{G : G \in \tau - \{\emptyset\}\}$ in such topologies X may and may not be minimal at any of its points. Such family of topologies will be denoted by p-topologies. U_x will denote the minimal open set at the point x for each $x \in X$. We denote a hyperconnected topology by h-topology and write nR for non R or not R where R is T_0, E, h, p or Eh and Eh means both E and h. If Q is a family of topologies on a finite set X_n of n points, then $N_n(Q)$ will denote the number of the Q-topologies on X_n. If τ is a topology on X_n,then $N_n(\tau)(Q)$ is the number of all weaker Q-topologies on X_n than τ. **Theorem 2.1.** [5] Let (X,τ) and (X,τ^*) be principal spaces and β be the minimal basis for τ. Then, τ^* is strictly weaker than τ iff there are two distinct points $y,z \in X$ satisfying the conditions.

1)$y \notin U_z$,
2)$z \in U_x$ and $x \notin U_z$ imply that $y \in U_x$,
3)$x \in U_y$ and $y \notin U_x$ imply that $x \in U_z$,

and $\tau^* = \tau \cap D_{yz} = \tau_{yz}$ having the minimal basis $\beta_{yz} = \{U_x, U_y \cup U_z : U_x \in \beta - \{U_z\}\}$.

If we add the condition $U_x \cup U_z \neq X$ to the conditions of Theorem 2.1, then one can obtain all strictly weaker non E-topologies on X than τ.

Theorem 2.2. [3] The number of all topologies on a finite set X_n of n points is given by:

$$N_n = \sum_{r=0}^{n-1} C_r^n N_r + N_n(nE)$$

Where $N_0 = 1$ and N_r is the number of all topologies on a set $X_r, n > 0$.

A topological space (X,τ) is hyperconnected [11] or irreducible [10] iff the intersection of any two nonempty open sets is nonempty.

Theorem 2.3.[5] A principal topological space (X,τ) is h iff $\cap\{G \in \tau : G \neq \emptyset\} \neq \emptyset$ iff τ is a P-topology on X.

Theorem 2.4. [3] The number of all h-topologies on a set X_n of n points is given by:

$$N_n(h) = \sum_{r=0}^{n-1} C_r^n N_r$$

Where $N_0 = 1$ and N_r is the number of all topologies on $X_r, n > 0$.

Theorem 2.5. A principal topological space (X,τ) is T_0 iff $U_x \neq U_y$ iff $x \in U_y$ implies that $y \notin U_x$ for each two distinct points $x,y \in X$.

Proof. Suppose that $x,y \in X$ are any two distinct points. If $U_x \neq U_y$ then $x \notin U_y$ or $y \notin U_x$ which implies that (X,τ) is T_0.

Conversely; if (X,τ) is T_0,then there is an open set $G \in \tau$ such that $x \in G$ and $y \notin G$ or $y \in G$ and $x \notin G$. If $x \in G$,then $U_x \subseteq G$ and so $y \notin G$ implies that $y \notin U_x$ which implies that $U_x \neq U_y$.

As a consequence of Theorem (2.5), if (X,τ) is a principal T_0-topological space one can reform Theorem (2.1) and introduce two equivalent easy but main and important results as follows:

Theorem 2.6. The topology τ^* on a nonempty set X is a strictly weaker principal topology than a principal T_0-topology τ on X iff there are two distinct points $y,z \in X$ such that:

1)$y \notin U_z$,
2)$z \in U_x$ implies that $y \in U_x$,
3)$x \in U_y$ implies that $x \in U_z$,

such that $\tau^* = \tau_{yz} = \tau \cap D_{yz}$ and its minimal basis is $\beta_{yz} = \{U_x, U_y \cup U_z : x \in X - \{z\}\}$.

And we have the following two corollaries.

Corollary 1. *The topology τ_{yz} is T_0 iff $z \notin U_y$,*

Corollary 2. *The topology τ_{yz} is nT_0 iff $z \in U_y$,*

where $U_y \in \tau$ is the minimal open set at y.

Theorem 2.7. A principal T_0-topological space (X,τ) is h iff there is a point $p \in X$ such that $\cap\{G \in \tau : G \neq \emptyset\} = \{p\}$.

Proof. If $\cap\{G \in \tau : G \neq \emptyset\} = \{p\}$, then (X,τ) is h. Conversely, if (X,τ) is h, then by Theorem (2.3) $\cap\{G \in \tau : G \neq \emptyset\} = U \neq \emptyset$. Since τ is principal,then $U \in \tau$ is the minimal open set at each of its points and τ is T_0 implies that there is a point $p \in X$ such that $U = \{p\}$.

Theorem 2.8. The number of all T_0-topologies on a set X_n of n points is given by:

$$N_n(T_0) = nN_{n-1}(T_0) + N_n(nET_0)$$

where $N_{n-1}(T_0)$ is the number of all T_0-topologies on a set of $n-1$ points.

Proof. Evidently (X_n,τ) is not T_0 if X_n is minimal at more than one of its points. So, if β is the minimal basis for a T_0-topology on $X_{n-1} = X_n - \{p\}$, where $p \in X_n$ is any point. Then, $\beta^* = \{U, X_n : U \in \beta\}$ is the minimal basis for a T_0-topology. τ^* on X_n in which X_n is minimal at the point p. This completes the proof.

Theorem 2.9. The number of all hT_0-topologies on a set X_n of n points is:

$$N_n(hT_0) = nN_{n-1}(T_0)$$

Proof. Suppose that X_n is a set of n points, $p \in X_n$ and $X_{n-1} = X_n - \{p\}$. If β is the minimal basis for a T_0-topology τ on X_{n-1}, then $\beta_p = \{\{p\}, U \cup \{p\} : U \in \beta\}$ is the minimal basis for an hT_0-topology τ_p on X_n and each weaker topology than τ_p is h. This completes the proof.

In [2] Evans et.al. Induced a correspondence between the reflexive transitive relations and the topologies on a set X_n of n points. We generalize this result to the principal topologies on any nonempty set X as follows:

Theorem 2.10. Let X be a nonempty set, then there is a correspondence between the preorders and the principal topologies on X.

Proof. If X is a nonempty set, τ is a principal topology on X and $\beta = \{U_x : x \in X\}$ is the minimal basis for τ,where U_x is the minimal open set at the point x for each $x \in X$. Define the relation $P = \{(x,y) : y \in U_x\}$ as x varies on X.Then, (1) $(x,x) \in P$ because $x \in U_x$ for each $x \in X$. So $P$is reflexive. (2) For any points x,y and z of X if $(x,y) \in P$,then $y \in U_x$ and $(y,z) \in P$ implies that $z \in U_y$. Hence $z \in U_x$ which implies $(x,z) \in P$ and so P is transitive.
Conversely; if P is a preorder relation on X and $U_x = \{y \in X : (x,y) \in P\}$ as x varies on X. Then, (1) $\cup\{U_x; x \in X\} = X$ because P is reflexive (2) If x,y are any two distinct points of X and $z \in U_x \cap U_y$, then $(x,z),(y,z) \in P$. Now $u \in U_z$ implies that $(z,u) \in P$ and since P is transitive,$(x,u),(y,u) \in P$, which implies that $U_z \subset U_x \cap U_y$. Therefore, $\beta = \{U_x : x \in X\}$ is a basis for a topology τ on X. If $x \in X$ and $G \in \tau$ such that $x \in G$,then there is a point $y \in X$ for which $x \in U_y \subset G$ which implies that $(y,x) \in P$. Now $z \in U_x$ implies that $(x,z) \in P$ and since P is transitive, then $(y,z) \in P$, which implies that $z \in U_y$ which implies that $U_x \subseteq G$ which implies that τ is a principal topology on X.

Theorem 2.11. If P is the preorder relation on a nonempty set X corresponding to the principal topology τ on X. Then, the topology τ is:

1)T_0 iff P is partial.
2)h iff there is a point $p \in X$ such that $(x,p) \in P$ for each $x \in X$.
3)E iff there is a point $p \in X$ such that $(p,x) \in P$ for each $x \in X$.
4)Eh i.e both E and h iff there are two points $p,q \in X$ such $(p,x),(x,q) \in P$ for each $x \in X$.

3 Proposed algorithm

We present an algorithm in Fortran 77 for construction and enumeration of all strictly weaker and all weaker $T_0, ET_0, nET_0, hT_0, nhT_0, EhT_0, nEnhT_0, EnhT_0, hnET_0,$ $nEhT_0, nT_0, EnT_0, nEnT_0, hnT_0, nhnT_0, EhnT_0, nEnhnT_0,$ $EnhnT_0, hnEnT_0, nEhnT_0$-topologies on X_n.

Suppose that $X_n = \{1,2,3,...,n\}$, in the data of our program for constructing the minimal bases for topologies on X_n,we present the minimal basis β for a given topology τ on X_n by an $n \times n$ matrix $[U(i,j)]$. If $U_i \in \beta$ is the minimal open set at $i \in X_n$ then,

1)The row number i of this matrix $[U(i,j)]$ represents U_i and $U(i,j)$ of this row is such that:

$$U(i,j) = \begin{cases} j & \text{if } j \in U_i \\ 0 & \text{if } j \notin U_i \end{cases}$$

2)If $U_i = U_j$,then the rows of the numbers i and j are coincide.
3)Each column j of the matrix has at least nonzero element that is $U(j,j)$ for each $j \in X_n$.
4)When we say a singleton row (column) if $U(i,j) = 0$ for each $j \in X_n - \{i\}$,$(U(i,j) = 0$ for each $i \in X_n - \{j\})$.
5)The full column j is such that $U(i,j) = j$ for each $i \in X_n$.
6)The full row i is such that $U(i,j) = j$ for each $j \in X_n$ in such case $U_i = X_n$.
7)The output of the program will be matrices of the mentioned properties and each of which represents a basis β^* for a topology τ^* on X_n weaker than τ.

Theorem 3.1. [4] A topology τ on a set X_n is h or $P(E)$ iff the matrix which represents its minimal basis β for τ has a full column (a full row).

Theorem 3.2. A topology τ on a set X_n is $T_0(nT_0)$ iff the matrix which represents its minimal basis β has no coincided rows (at least two coincided rows).

*Remark.*The minimal bases for the nE-topologies on X_n represented by the matrices which has no full rows and the minimal bases for nh-topologies represented by the matrices which has no full columns. In both cases for any two distinct points $i,j \in X_n$ there is a point $k \in X_n$ such that $U(i,k) \neq U(j,k)$.

*Remark.*The minimal bases for the $hnET_0$ and $EnhT_0$-topologies on X_n can be easily obtained by obtaining the matrices representing the minimal bases for all T_0-topologies each of which has a full column and no full rows and the matrices each of which has a full row and no full columns.

*Remark.*By using Theorem (3.2) one can easily obtain the minimal bases for all hT_0-topologies on X_n. This by obtaining the matrices that represent the minimal bases for all T_0-topologies on X_n each of which has a full column.

*Remark.*One can easily obtain the minimal bases for all ET_0-topologies on X_n by obtaining the matrices which represent the minimal bases for all T_0-topologies on X_n each of which has a full row.

*Remark.*One can obtain the minimal bases for all nT_0-topologies (T_0-topologies) by obtaining the matrices,which has two distinct coincided (no coincide) rows using Theorem (3.2).

*Remark.*It should be noted that $N_n(ET_0) = N_n(hT_0)$,$N_n(nET_0) = N_n(nhT_0)$, $N_n(EnhT_0) = N_n(hnET_0)$, $N_n = N_n(T_0) + N_n(nT_0)$, $N_n = N_n(ET_0) + N_n(nET_0) = N_n(hT_0) + N_n(nhT_0) = N_n(EhT_0) + N_n(nEhT_0)$ and $N_n(nEhT_0) = N_n(EnhT_0) + N_n(hnET_0) + N_n(nEnhT_0)$ where N_n are the number of all topologies on X_n. We have similar equations for the same classes of the nT_0-topologies.

3.1 Algorithm (X_n, β, N)

*Remark.*This algorithm constructs and enumerates the minimal bases for all strictly weaker T_0 and nT_0 topologies than a given T_0-topology τ on X_n.

This algorithm is the algorithm in [4] and we may or may not check the conditions of Theorem (2.6) in stead of the conditions of the proposition in [4] which used in this algorithm. This is together with the checking of:

1)The condition of the Corollary (2.7) to obtaining the minimal bases for all strictly weaker T_0-topologies on X_n than τ.
2)The condition of the Corollary (2.8) to obtaining the minimal bases for all strictly weaker nT_0-topologies on X_n than τ.

3.2 Algorithm

$(X_n, \beta(T_0), \beta_n(T_0), N_n(T_0), \beta_n(ET_0), N_n(ET_0), \beta_n(nET_0),$
$\beta_n(hT_0), N_n(hT_0), N_n(nhT_0), \beta_n(EhT_0), N_n(EhT_0),$
$\beta_n(nEnhT_0), N_n(nEnhT_0), \beta_n(EnhT_0), N_n(EnhT_0),$
$\beta_n(hnET_0), N_n(hnET_0), \beta_n(nEhT_0), N_n(nEhT_0),$
$\beta_n(nT_0), N_n(nT_0), \beta_n(EnT_0), N_n(EnT_0),$
$\beta_n(nEnT_0), N_n(nEnT_0), \beta_n(hnT_0), N_n(hnT_0),$
$\beta_n(nhnT_0), N_n(nhnT_0), \beta_n(EhnT_0), N_n(EhnT_0),$
$\beta_n(nEnhnT_0), N_n(nEnhnT_0), \beta_n(EnhnT_0), N_n(EnhnT_0),$
$\beta_n(hnEnT_0), N_n(hnEnT_0), \beta_n(nEhnT_0), N_n(nEhnT_0)).$

*Remark.*This algorithm constructs and enumerates the minimal bases for all weaker nET_0-topologies and all T_0-topologies on X_n than a given T_0-topology τ on X_n. If τ is the discrete topology on X_n then we obtain the minimal bases for all T_0 and all nET_0-topologies on X_n.

This algorithm is algorithm of [5] together with the checking of the condition of the Corollary (2.7). This leads to obtaining the minimal bases for all weaker T_0 and nET_0-topologies on X_n than a given topology τ on X_n.

The minimal bases for the nT_0-topologies on X_n can not be obtained directly by using the algorithm of [5] and the Corollary (2.8) because the ultratopologies on X_n are all T_0. So, the algorithm of [5] to obtaining a store of the matrices,which represents the minimal bases for all weaker topologies on X_n than the given T_0-topology τ on X_n. Then, use the condition of Theorem (3.2) to select from the store all weaker nT_0-topologies on X_n than τ. Of course by using Theorem (3.2) one can also selected all weaker T_0-topologies on X_n than τ.

4 Computer experiments

In this section the proposed algorithms are demonstrated by applying them to different finite sets and bases.

*Example 1.***Input**: $X_{10} = \{1,2,3,4,5,6,7,8,9,10\}$ and $\beta = \{\{1,5,8\},\{2,6\},\{3,9,10\},\{4\},\{5\},$ $\{6\},\{6,7\},\{5,8\},\{9,10\},\{10\},\}$. Where β is the minimal basis for a T_0-topology τon X_n.
Output:
(a) Using algorithm (3.1) with the Corollary (2.7) to obtain all minimal bases for the strictly weaker T_0-topology on X_{10} than the topology τ which are 17 bases. We write the first and the end of them:
$\beta(10,7) = \{\{1,5,8\},\{2,6\},\{3,9,10\},\{4\},\{5\},\{6\},\{6,7,10\},\{5,8\},\{9,10\},\{10\}\}$.
(b) Using algorithm (3.9) with the Corollary (2.8) to obtain all minimal bases for the strictly weaker nT_0-topology than the topology τ on X_{10} which are 4 bases. We write the end of them:
$\beta(9,10) = \{\{1,5,8\},\{2,6\},\{3,9,10\},\{4\},\{5\},\{6\},\{6,7\},\{5,8\},\{9,10\}\}$.
(c) Using algorithm of [4] to obtain the minimal bases for all strictly weaker topologies on X_{10} than the topology τ on X_{10} which are 21 bases. These are just the union of the T_0 and non T_0-topologies obtained in (a) and(b).

*Example 2.***Input**: $X_6 = \{1,2,3,4,5,6\}$ and $\beta = \{\{1\},\{2\},\{3\},\{4\},\{5\},\{6\}\}$.
Output:
(a) Using algorithm (3.10) with the condition of the Corollary (2.7) to obtain the minimal bases for all nondiscrete T_0-topologies on X_6. These T_0-topologies will also be divided into nine classes of $E, nE, h, nh, Eh, nEnh, Enh, hnE$ and nEh-topologies. We write the number of each class and some of each of which.

i)The number of all T_0-topologies on X_6 is $N_6(T_0) = 130023$ and: $\beta(130022) = \{X_6, X_6 - \{1\},\{3,4,5,6\},\{4,5,6\},\{5,6\},\{6\}\}$.
ii)The number of all ET_0-topologies on X_6 is $N_6(ET_0) = 25386$ and: $\beta(25386) = \{X_6, X_6 - \{1\},\{3,4,5,6\},\{4,5,6\},\{5,6\},\{6\}\}$.
iii)The number of all nondiscrete nET_0-topologies on X_6 is $N_6(nET_0) = 104637$ and: $\beta(104636) = \{X_6 - \{2\}, X_6 - \{1\},\{3,4,5,6\},\{4,5,6\},\{5,6\},\{6\}\}$.
iv)The number of all T_0-topologies on X_6 is $N_6(hT_0) = 25386$ and: $\beta(25386) = \{X_6 - \{2\}, X_6 - \{1\},\{3,4,5,6\},\{4,5,6\},\{5,6\},\{6\}\}$.
v)The number of all nhT_0-topologies on X_6 is $N_6(nhT_0) = 104637$ and: $\beta(104636) = \{X_6, X_6 - \{1\},\{3,4,5,6\},\{4,5,6\},\{5\},\{6\}\}$.
vi)The number of all $hnET_0$-topologies on X_6 is $N_6(hnET_0) = 18816$ and: $\beta(18816) = \{X_6 - \{2\}, X_6 - \{1\},\{3,4,5,6\},\{4,5,6\},\{5,6\},\{6\}\}$.
vii)The number of all $EnhT_0$-topologies on X_6 is $N_6(EnhT_0) = 18816$ and:

$\beta(18816) = \{X_6, X_6-\{1\},\{3,4,5,6\},\{4,5,6\},\{5\},\{6\}\}$.

viii)The number of all $nEnhT_0$-topologies on X_6 is $N_6(nEnhT_0) = 85821$ and:
$\beta(85820) = \{X_6-\{2\}, X_6-\{1\},\{3,4,5,6\},\{4,5,6\},\{5\},\{6\}\}$.

ix)The number of all EhT_0-topologies on X_6 is $N_6(EhT_0) = 6750$ and:
$\beta(6570) = \{X_6, X_6-\{1\},\{3,4,5,6\},\{4,5,6\},\{5,6\},\{6\}\}$.

x)The number of all $nEhT_0$-topologies on X_6 is $N_6(nEhT_0) = 123452$ and:
$\beta(123452) = \{X_6, X_6-\{1\},\{3,4,5,6\},\{4,5,6\},\{5,6\},\{6\}\}$.

(b) Using algorithm (3.10) and the condition of Theorem (3.2) to obtain the minimal bases for all nT_0-toologies on X_7. These nT_0-topologies will also divided into nine classes of E, nE, h, nh, Eh, $nEnh, Enh, hnE$ and nEh-topologies. We write the number of each class and the end of each of which.

i)The number of all nT_0-topologies on X_6 is $N_6(nT_0) = 79504$ and:
$\beta(79503) = \{\{1,3,4,5\}, X_6, \{3,4,5\},\{4,5\},\{5\}\}$.

ii)The number of all EnT_0-topologies on X_6 is $N_6(EnT_0) = 22238$ and:
$\beta(22237) = \{\{1,3,4,5\}, X_6, \{3,4,5\},\{4,5\},\{5\}\}$.

iii)The number of all $nEnT_0$-topologies on X_6 is $N_6(nEnT_0) = 57266$ and:
$\beta(57266) = \{\{X_6-\{2\}, X_6-\{1\},\{3,4,5,6\},\{4,5\},\{5\}\}$.

iv)The number of all hnT_0-topologies on X_6 is $N_6(hnT_0) = 22238$ and:
$\beta(22237) = \{\{1,3,4,5\}, X_6, \{3,4,5\},\{4,5\},\{5\}\}$.

v)The number of all $nhnT_0$-topologies on X_6 is $N_6(nhnT_0) = 57266$ and:
$\beta(57266) = \{\{1,3,4,5\}, X_6, \{3,4,5\},\{4\},\{5\}\}$.

vi)The number of all $EhnT_0$-topologies on X_6 is $N_6(EhnT_0) = 8643$ and:
$\beta(8642) = \{\{1,3,4,5\}, X_6, \{3,4,5\},\{4,5\},\{5\}\}$.

vii)The number of all $nEnhnT_0$-topologies on X_6 is $N_6(nEnhnT_0) = 43671$ and:
$\beta(43671) = \{X_6-\{2\}, X_6-\{1\},\{3,4,5,6\},\{4\},\{5\}\}$.

viii)The number of all $EnhnT_0$-topologies on X_6 is $N_6(EnhnT_0) = 13595$ and:
$\beta(13595) = \{\{1,3,4,5\}, X_6, \{3,4,5\},\{4\},\{5\}\}$.

ix)The number of all $hnEnT_0$-topologies on X_6 is $N_6(hnEnT_0) = 13595$ and:
$\beta(13595) = \{X_6-\{2\}, X_6-\{1\},\{3,4,5\},\{4,5,6\},\{4,5\},\{5\}\}$.

x)The number of all $nEhnT_0$-topologies on X_6 is $N_6(nEhnT_0) = 70861$ and:
$\beta(70861) = \{X_6-\{2\}, X_6-\{1\},\{3,4,5\},\{4,5,6\},\{4,5\},\{5\}\}$.

5 Conclusion

There is a well-known correspondence between the topologies on finite set X_n of n points and preorders relations on those sets. For our approach, we characterized the principal hyperconnected T_0-topologies on a nonempty set X. We established an algorithms in Fortran 77 for construction and enumeration of all strictly weaker and all weaker T_0 and nT_0-topologieson on X_n. We applied these algorithms to different finite sets and bases.

References

[1] Y. Chen, Fuzzy Sets and System **84**, 97-102 (1996).
[2] J.W Evans, F. Harry and M.S Lynn, Commur ACM **10**, 295-297 (1967).
[3] A.S Farrag, On *S*- topological spacses, Ph.D. Thesis Assiut Unv., (1983).
[4] A.S Farrg and A.A Sewisy, IJCM. **72**, 433-440 (1999).
[5] A.S Farrg and A.A Sewisy, IJCM. **74**, 471-482 (2000).
[6] O. Frohlich, Math. Ann. **156**, 76-95 (1964).
[7] I. Jason Brown and Stephen Watson, Discrete Mathematics, 15427-1539 (1996).
[8] V. Krischnamurthy, Amer.Math. Monthly **3**, 154-157 (1966).
[9] A.S Mashhour and A.S Farrag, "Simple topological space", In 14^{th} An.Conf. In Stat. Comp. Sci. Res. Math., Cairo University (1979),78-85.
[10] A.A Nabiha and A.G Naoum, "On maximal irreducible space", Bull. Coll. Sci. (Baghdad), no.2, 17(1976), 477-490.
[11] T. Noiri, Math. Appl. **24**, 182-190 (1979).
[12] A.A Ramadan, Fuzzy Sets and System **48**, 371-375 (1992).
[13] H.R Warren, Houston Journal of Mathematics **8**(2), 297-301 (1982).
[14] A.K Steiner, Trans. Amer. Math. Soc. **122**, 379-398 (1966).

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