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Stochastic Network Calculus for Rician Fading in Underwater Wireless Networks

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Abstract: Performance evaluation plays a fundamental role in the design of modern underwater wireless communication systems, which are growing rapidly. In this system, performance evaluation of systems plays a fundamental role in determining the effectiveness of the system. Analysis of backlog and delay in any underwater wireless communication networks becomes a tough task. In past few years, the stochastic growth of modern networks resulted in complexity of analysing the algorithms and application. Since traditional mathematical modeling theories such as queuing theory, effective bandwidth, and deterministic networks calculus are not applicable to analyze the Quality of Service (QoS) for the present day packet switched multimedia networks due to their inherently random behavior. To analyze the present day networks, non-deterministic network calculus is much needed. Stochastic network calculus emerges as an appropriate mathematical tool for modeling and calculating the performance of wireless network and wired networks. While research in stochastic network calculus is in early days, it is still gaining much attention in the research community for analyzing any wireless communication networks. In this research article, the authors have analyzed and created a mathematical model for underwater wireless communication channel. The channel is subjected to Rician fading based on SNC that obtains Stochastic Arrival Curve and Stochastic Service Curve, as an extension to the Rayleigh fading. This model is adopted for deriving the performance of delay and backlog bounds in underwater acoustic Rician fading channels using Stochastic Network Calculus.

Keywords: Rician Fading, Determinstic Network calculus, Stochastic Network Calculus, Underwater wireless,, Backlog.

1 Introduction

Analyzing the performance of a system with its critical parameters plays an important role in the success of the system. There are generally two ways to analyse any system; working with the real set up '(actual system) or experiment with the model of actual system. The latter needs more importance as modelling a system needs to consider the underlying mathematical theories with analytical solutions that can be tested using respective simulations. Fig.1 provides the different ways to analyse system performance. Constructing appropriate mathematical model is very crucial to make meaningful studies, and choosing the right simulation tool to represent our idea would help us arrive at successful modelling.

Conventional telephone networks [1] were based on Circuit switched networks, where there is fixed bandwidth in the communication channel between the sender node and the received node. The allocated bandwidth is available for the whole duration of communication. For the efficient use of bandwidth data

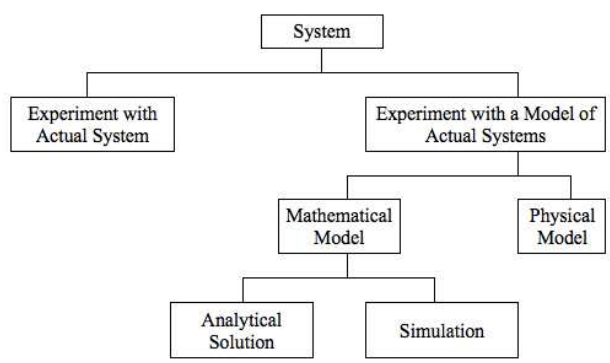

Fig. 1: Ways to Study a System

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networks emerged, where the information is sent as packets and it leads to packet switched networks [2].

To analyze the performance of circuit switched networks, Erlang proposed Queuing Theory [3]. Telephone networks are generally analyzed by Queuing theory with low variability [4]. As the scale of network increases, it?s heterogeneity, users, applications and its traffic increases. In order to analyze the performance of packet switched networks, Theories like; effective bandwidth [5] and network Calculus [6] were introduced. The concept of effective bandwidth motivated the need of resource allocation in packet switched networks that facilitates a traffic model with statistical multiplexing. The sum of individual flows in any network wireless communication system determines the effective bandwidth. The limitations of the theory are; statistical multiplexing may not be accurately calculated. Another limitation in the statistical multiplexing is that, individual flows have different QoS requirements. Then FIFO method is not applicable for scheduling of these flows. The effective bandwidth becomes a challenge due to heterogeneous QoS requirements in individual flows. Then application of effective bandwidth and effective capacity remains a challenge. To overcome the above issue network calculus [7] was developed by Cruz in 1991. Network calculus tool is used to analyse the performance of packet switched networks. The two tracks of Network calculus are; Deterministic Network Calculus and Non-Deterministic or Stochastic Network Calculus (SNC). To date lot of DNC results are available and the summary can be found in [8,9]

Using the input flows to the networks or denotes arrival curves. Arrival curves in deterministic network calculus denotes the individual flows or in other terms as arrival process. Similarly Service curves in deterministic network calculus denote the service process. Statistical properties of arrivals are not determined using deterministic network calculus, as the DNC limitations are exposed. In addition, the deterministic service curves are additive in nature. Additive property states that, when number of flows in the network system is multiplexed then aggregate of flow increases. So the statistical multiplexing gain is not exposed in the Deterministic network calculus. Deterministic arrival curves and deterministic service curves helps in deriving the performance of worst-case analysis bounds. The worst-case analysis bounds are derived for analysis of acoustic underwater communication networks. In[10], where the Rayleigh fading is already attempted stochastic network calculus. In acoustic networks the acoustic data communication is irregular in nature. Further the communication is unstable. So it is difficult to determine the worst-case performance bounds [11]. In order to implement service provisioning of non deterministic service curves, the performance analysis bounds needs to be cooperated with various probabilities. In general any networks or specified networks such as acoustic networks, the stochastic service guarantees can be derived using Stochastic Network Calculus. Moreover, fading channels in underwater acoustic networks may have only stochastic service guarantees due to their time-varying nature. A brief overview of various ways to study analysis of network is given in Fig 2. Arrival curves models and service curves models are derived and extended. The extended models show the probabilistic representation of Deterministic Network Calculus and called as stochastic arrival and stochastic service curves. The basic properties like service guarantees, output characterization, concatenation property, leftover service and superposition property should satisfy the traffic and server models of network calculus

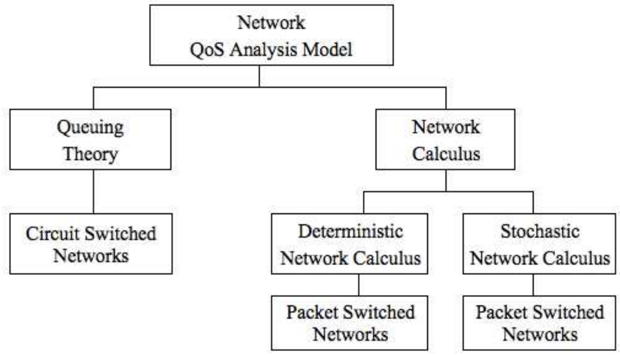

Fig. 2: Network Analysis- Taxonomy

Presently, there is no proper mathematical formulation and proof that demonstrates the Rician fading effects in underwater acoustic channels. From existing literature, we conclude that fading effects were generally modelled using queuing theory and deterministic network calculus [12]. The key challenge in analysing fading an acoustic system is that, temporal uncertainties are inherent in fading channels. In this research study, we have constructed a mathematical model using stochastic network calculus to determine the Rician fading effects in acoustic transmission in underwater wireless sensor networks. This work is an extension to the already available Rayleigh fading model [10]. The model is simulated using scientific network modeling tool, OPNET and conducted performance analysis.

The article is written as follows; the basics of stochastic are explained in next section, followed by, the basics of underwater acoustic fading channels and its performance analysis bounds in Section 4. In Section 5, modelling the acoustic underwater Rician fading channel using stochastic network calculus. In Section 6, presents the performance evaluations, conclusions and further work.

2 Basic Notations of Stochastic Network Calculus

2.1 Basic Notations

Stochastic Network calculus is derived from Erlang's works Queuing Theory [13]. This section explains the basics of stochastic network calculus. Using the time factor represents process in any flow time t. The arrival process is represented as the $A_p(t)$. $A_p(t)$ is defined as the incoming traffic to the communication network. Similarly the network service departing from any communication networks is represented as $D_p(t)$. The arrival process is serviced and the service process is denoted as $S_p(t)$. Impairment process in the network is denoted by $I_m(t)$. In Stochastic Network Calculus, all the process arriving are considered to be non-negative process and all negative process are increasing functions. So flow is given by

$$I\mathscr{F} = \{f\,(.) : \forall 0 \le a \le b, 0 \le f\,(a) \le f(b\,)\}$$

at a given time $t = 0$. i.e,

$$A_p\,(0) = D_p\,(0) = S_p\,(0) = I_m\,(0) = 0.$$

for any, $0 \le a \le b$,

$$Let\ A_p\,(a,\ b)\ \equiv\ A_p\,(b)\ -\ A_p\,(a)$$

$$D_p\,(a,b)\ \equiv\ D_p\,(b)\ -\ D_p\,(a)$$

$$S_p\,(a,b)\ \equiv\ S_p\,(b)\ -\ S_p\,(a)$$

$\mathscr{F}_i$ denotes the set of non-negative wide-sensing increasing function and $\mathscr{F}_d$ denotes the non-negative decreasing functions.

$$\bar{\mathscr{F}_i} = \{p\,(.) :\ \forall 0 \le a \le b, 0 \le f\,(b) \le f\,(a)\}$$

$$\bar{\mathscr{F}_d} = \{p\,(.) :\ \forall 0 \le a \le b, 0 \le f\,(b) \le f\,(a)\}$$

Distribution of any function for random variables R is denoted as $\mathscr{F}_{t1}\,(t) = \Pr ob\,\{t1 \le t\}$, and the necessary distribution function complementary is denoted as,

$$\mathscr{F}_{t1}\,(t) = \Pr ob\,\{t1 \le t\}$$

When the model transforms there is need of stronger requirement. The stronger requirement on the bounding function is denoted as F_b So F_b is denoted as follows

$$F_b = \left\{ x\,(.) : \forall a \ge 0, \left(\int_x^{\infty} dz \right)^a x\,(z) \in \overline{F_b} \right\}$$

2.2 Operators in Stochastic Network Calculus

In $(\min,+)$ algebra, the following properties are defined: The $(\min,+)$ *convolution* of function a and b is

$$(a \otimes b)\,(y) = \inf_{0 \le d \le c} [p]$$

$$p = a\,(d) + b\,(c-d)$$

The $(\min,+)$ *deconvolution* of function a and b is

$$(a \oslash b)\,(y) \equiv \sup_{t \ge 0} \{p\}$$

$$p = a\,(y+s) - b\,(t)$$

Pointwise minimum of a and b is

$$(a \wedge b)\,(y) = \min\,[a\,(y)\,, b\,(y)]$$

Pointwise maximum of a and b is

$$(a \vee b)\,(y) = \max\,[a\,(y)\,, b\,(y)]$$

Normal convolution of function a and b is

$$(a * b)\,(t) = \int_0^t a\,(t-x)\,db\,(x)$$

2.3 Performance Metrics,Traffic and Server Models

In stochastic network calculus backlog is defined as follows;

$$B_l\,(t) = A_p\,(t) - D_p\,(t)$$

In stochastic network calculus the delay $D_l\,(t)$ in the system at time t is represented as;

$$D_l\,(t) = \inf\left\{\tau \ge 0 : A_p\,(t) \le D_p\,(t+\tau)\right\}$$

For traffic arrival models, we have:

Definition 1: The traffic-amount-centric *(t.a.c)*

When an arrival process or flow $A_p(t)$ is traffic-amount-centric then the bounding function of $f \in \overline{\mathscr{F}}_d$ for the corresponding stochastic arrival curve $\alpha_i \in \mathscr{F}_i$, denoted as; If for all $a \ge 0$ and $b \ge 0$, it holds

$$Prob\,\{P > b\} \le f\,(b)$$

$$P = A_p\,(s,a) - \alpha_i\,(a-s)$$

Definition 2: The virtual-backlog-centric model (v.b.c)

When an arrival process or flow$A_p\,(t)$ is virtual-backlog-centric then the bounding function of f $\in \overline{\mathscr{F}}_d$for the corresponding stochastic arrival curve $\alpha_i \in \mathscr{F}_i$ is denoted by and given as follows if for all $a \ge 0$

and all $b \geq 0$, it holds $P\{X > b\} \leq f(b)$ $X = \sup_{0 \leq s \leq a} [A_p(s,a) - \alpha_1(a-s)]$

Definition 3: The maximum-virtual-backlog-centric model *(m.v.b.c)*

When an arrival process or flow$A_p(t)$ is maximum virtual backlog centric then the bounding function of $f \in \overline{\mathscr{F}}_d$, For the corresponding stochastic arrival curve is denoted by $A_p \tilde{\ } m_{vbc} \langle f, \alpha_i \rangle$, if for all $a \geq 0$ and all $b \geq 0$, it holds

$$Prob\{X \leq f(b)\}$$

where, $X = \sup_{0 \leq s \leq a} \sup_{0 \leq c \leq s} [A_p(c,s) - \alpha_i(s-u) > b]$

Definition 4: The weak stochastic model *(s.c)*

When server provides a flow $A_p(t)$ is termed as weak stochastic service curve with a bounding function $a \in \overline{\mathscr{F}}_d$ for the corresponding stochastic service serve $\beta_i \in \mathscr{F}_i$ and the weak stochastic curve is denoted as $S_p \tilde{\ } w_{sc} \langle a, \beta_i \rangle$, if for all $a \geq 0$ and all $b \geq 0$, it holds

$$Prob\{X > b\} \leq a(b)$$

where, $X = A_p \otimes \beta_1(b) - D_p(b)$

Definition 5: The stochastic service curve model *(s.s.c)*

When server provides a flow $A_p(t)$ is termed as stochastic service curve with a bounding function $a \in \overline{\mathscr{F}}_d$, for the corresponding stochastic service serve $\beta_1 \in \mathscr{F}_i$ and the stochastic curve is denoted as $S_p \tilde{\ } s_{sc} \langle a, \beta_1 \rangle$, if for all $a \geq 0$ and all $b \geq 0$, it holds

$$Prob\{X > b \leq a(b)\}$$

where, $[X = \sup_{0 \leq s \leq a} [A_P \otimes \beta_1(b) - D_p(b)]$

Definition 6: The strict stochastic service curve model *(s.s.s.c)*

When server provides a flow $A_p(t)$ is termed as weak stochastic service curve with a bounding function $a \in \overline{\mathscr{F}}_d$, for the corresponding stochastic service serve $\beta_1 \in \mathscr{F}_i$ and the strict stochastic curve is denoted as $S_p \tilde{\ } s_{ssc} \langle a, \beta_1 \rangle$, if for all $a \geq 0$ and all $b \geq 0$, it holds

$$Prob\{X - b\} \leq a(b)$$

where, $X = S_P(a, b) < \beta_1(b - a)$

The above definitions listed, proves the properties of backlog and delay bound in stochastic network calculus. In stochastic network calculus$(\mathscr{F}_i, \wedge, \otimes)$ is proved to be a complete dioid and based on the property following Lemma is proved.

(i) The Closure property in stochastic network calculus states:

$$\forall x, \quad y \in \overline{\mathscr{F}}_d, \quad x \wedge y \in \overline{\mathscr{F}_d}; \quad x \wedge y \in \overline{\mathscr{F}}_d$$

(ii)The Associativity property in stochastic network calculus is represented as:

$$\forall x, \quad y \in \overline{\mathscr{F}}_d, (x \wedge y) \wedge z = x \wedge (y \wedge z)$$

$$(x \wedge y) \wedge z = x \wedge (y \wedge z)$$

(iii) The Commutativity property is represented as:

$$\forall x, \quad y \in \overline{\mathscr{F}}_d, \quad x \wedge y = y \wedge x; \quad x \wedge y = y \wedge x$$

(iv) The Distributivity property in Stochastic network calculus is as follows:

$$\forall x, \quad y \in \overline{\mathscr{F}}_d, (x \wedge y) \wedge z = (x \wedge z) \wedge \quad (y \wedge z)$$

(v) The Zero element property is given as:

$$\forall x \in \overline{\mathscr{F}_{d},} \, x \wedge \bar{\in} = x$$

(vi) The Absorbing zero element property in stochastic network calculus is given as:

$$\forall x \in \overline{\mathscr{F}_d}, \quad x \wedge \bar{\in} = \bar{\in} \wedge x = \bar{\in}$$

(vii) Idempotency of addition in stochastic network calculus:

$$\forall x \in \overline{\mathscr{F}}_d, x \wedge x = x$$

(viii) Comparison property in stochastic network calculus:

$$x_1 \wedge x_2 \leq x_1 \vee x_2 \leq x_1 \otimes x_2$$

(ix) Monotonicity property in stochastic network calculus:

$$If \ x_1 \leq y_1 \ and \ x_2 \leq y_2, \ then \ x_1 \wedge x_2 \leq y_1 \wedge y_2;$$

$$x_1 \wedge x_2 \leq y_1 \wedge y_2; \ x_1 \vee x_2 \leq y_1 \vee y_2;$$

3 Basics of Underwater Acoustic Fading Channel

Existing models for fading using queuing theory and network calculus are suitable only for circuit switched communication, and do not represent the stochastic nature of present day packet switched communication requirements. The accuracy of the Stochastic Network Calculus (SNC) model and its simulations for underwater acoustic networks was never attempted before. Since the other layers [14] in the network stack and its functionalities depends on the physical layer, there is need to model this layer efficiently and effectively with present day feasible mathematical models like the SNC. SNC based models are now only being recently researched for underwater acoustic communications, hence lack of new constructs and mathematical formulations that satisfy different physical layer constraints is a real challenge. The proposed model serves as a platform or origin for researchers working in acoustic channel modelling especially in propagation of underwater wireless sensor networks. These proposed and tested models can facilitate in the analysis and design of real-time test bed implementation of acoustic channels in the physical layer of the network stack.

Underwater acoustic channel is an invisible path that connects the transmitting and the receiving end in an underwater communication. The propagation of acoustic waves requires a medium (shallow water, deep water, very shallow water, sea water) to transmit data. Regardless of the type of medium, there exists the loss as such as Propagation loss and fading. Fading is further classified as slow fading and fast fading. Propagation loss, which is also termed as path loss is reflected in energy loss, which can be shown by the value of received power at the received end. Path loss is affected by factors like; distance, with or without line of sight, water density, strong currents, waves etc.[15]. There are a few path loss models such as the free space path loss model, one slope model and others [16]. An underwater environment contains various living objects, non-living objects and absorption affects, some part of the acoustic transmitted signal gets reflected, diffracted and scattered and known as slow fading. In underwater acoustics, multipath propagation loss causes fast fading. Amplitude of the received signal based on multipath propagation loss causes fast fading. In addition to slow and fast fading, there is flat fading and frequency selective fading in acoustic propagation. When the transmission bandwidth is less than 10Mbits/sec, the frequency selective effect can be ignored and, in terms of the bandwidth and the fading depth is considered to be at the same level. This is known as flat fading. In Rayleigh fading model [17] for acoustic multi-path propagation, the received power follows an exponential distribution, when there is a line of sight propagation path, Rician fading model [18] is adapted. Nakagami model is a combination of the Rayleigh and Rician channel models. Different values of the channel parameter have the ability to decide whether it is a Rayleigh or a Rician channel. Gilbert-Elliot fading gives the condition of the channel as to whether it is good or bad, based on a two states of Markov chain [19] at the Packet level. In this paper, we have implemented the Rician Fading technique using SNC. We have obtained Stochastic Arrival and Stochastic Service Curve and verified the tightness of the bounds.

4 Performance Bound Analysis

In this section, the performance bound analysis is derived. Performance bound analysis is derived for following test cases. As Deterministic arrival curve and stochastic arrival curve. When the acoustic transmitter propagates the data constantly through acoustic medium or when the acoustic transmitter communicates constantly then the channel is modelled using deterministic arrival curve. When the acoustic transmitter propagates the data randomly through acoustic channel or when the acoustic transmitter communicates randomly, then the channel is modelled using stochastic arrival curve. As mentioned earlier $A_p(t)$ denotes the arrival process and process leaving the system is denoted as departure process $D_p(t)$.

4.1 Deterministic arrival traffic

When the nodes communicate with each other in the acoustic channel, the arrival process for each sender node is represented as $A_p(t)$. Similarly, When the nodes communicate with each other in the acoustic channel then the departure process is denoted as $D_p(t)$. As mentioned earlier the deterministic arrival curves and deterministic service curves in the acoustic channel in our system is termed as $\alpha_1(t)$ and $\beta_i(t)$, ε respectively. The backlog for the stochastic process is represented as $B_l(t)$. The backlog in derived as follows,

$$\Pr ob\{X \geq Y\} \leq \varepsilon,$$

where, $X = B_l(t)$, $Y = \alpha_1 \oslash \beta_1$
The delay bound in the stochastic network calculus in our system is represented as $D_l(t)$. The stochastic upper bound for the delay function is expresses as follows,

$$\Pr ob\{X > Y\} \leq \varepsilon,$$

where, $X = D_l(t), Y = h(\alpha_i, \beta_i)$

$$\Pr ob\{X \geq Y\}$$

where, $X = B_l(t), Y = \alpha_1 \oslash \beta_1(0)$

$$\leq \Pr ob\{A_p(t) - D_p(t)\ \} \geq \sup_{0\leq\tau\leq t} (A_p(t))$$

$$A_p(t) = [A_p(t) - A_p(t-\tau) - \beta_1(\tau)]$$

$$\leq \Pr ob\{A_p(t) - D_p(t)\ \} \geq \sup_{0\leq\tau\leq t} (A_p(t))$$

$$= Pr\{X - Y\}$$

where, $X = A_l(t) - D_l(t) \geq A_l(t)$

$$Y = \inf_{0\leq\tau\leq t} [A_l(t-\tau) + \beta_1(\tau)\ ]$$

$$= Pr\{X \leq Y\}$$

where, $X = D_l(t), Y = A_p \wedge \beta_1(t) \leq \varepsilon$

$$D_l(t) = inf\{\tau \geq 0 : X \leq Y\}$$

where, $X = A_l(t), Y = D_l(t+\tau)$

Using this derivation, we get

$$Pr\{D_l(t) \geq \tau_0\} \leq Pr\{X \geq Y\}$$

where, $X=A_p(t), Y=D_l(t+\tau_0)$
The performance bound can be proved as follows,

$$Pr\{X\geq Y\}$$

where, $X=A_l(t), Y=D_l(t+\tau_0)$

$$X-Y=\ A_p(t)-A_p\otimes S_{sc}+S_{ac}S_{sc}=\beta_1(t+\tau_0)$$
$$S_{ac}=A_p\otimes\beta(t+\tau_0)-S_d$$
$$S_d=D_l(t+\tau_0)$$
$$=A_l(t)-In_{pr}+In_{sp}$$
$$In_{pr}=\inf_{0\leq s\leq t+\tau_0}[A_p(s)+\beta_1(t+\tau_0-s)]$$
$$In_{sp}=[A_p\otimes\beta_1(t+\tau_0)-D_1]\,D_1=D_l(t+\tau_0)$$

The maximum horizontal difference between the stochastic arrival curve and the stochastic service curve is represented as τ_0. The stochastic performance bound for the arrival curve is represented

$$Pr\{D_l(t)\geq\tau_0\}\leq XX=Pr\left\{D_l(t)\leq A_p\otimes\beta(t)\right\}\leq\varepsilon$$

4.2 Stochastic arrival traffic

The exponentially bounded burstiness process provides the arrival process in stochastic arrival curve. The property of stochastic network calculus represents the arrival process in a stochastically bounded process. Consider the stochastic arrival curve process as following

$$Pr\{X\geq 0\}Y$$
$$X=\sup_{0\leq s\leq t}[A_p(t)-A_p(s)-\alpha_1(t-s)], Y=b_1e^{-b_2\sigma}$$

where $0\leq s\leq t, \sigma>0$, and $\alpha_1(t)=A+B$

$$A=\rho.t, B=\sigma$$

When the nodes communicate with each other in the acoustic channel, the arrival process for each sender node is represented as $A_p(t)$. Similarly, When the nodes communicate with each other in the acoustic channel then the departure process is denoted as $D_p(t)$. As mentioned earlier the stochastic arrival curves and stochastic service curves in the acoustic channel in our system is termed as $A_p(t)\tilde{}\alpha_1(t)$, $f_1(\sigma)$, and $\alpha(t)=\rho.t+\sigma$ respectively, where $A_p(t)$ receives the service curve$\beta(t), \varepsilon$. The backlog in derived as follows,
(i) Backlog bound: $[Pr\{X\geq Y\}\leq\varepsilon+Z$
where, $X=B_l(t), Y=\alpha_1\oslash\beta_1(0)Z=f_1(\sigma)$
(ii) Delay Bound: $Pr\{X\geq Y\}\leq\varepsilon+Z$
where, $X=D_l(t), Y=h(\alpha_1,\beta_1), Z=f_1(\sigma)$
Since probability of any event in the space is less than or equal to 1, by applying the probability property, we get

$\varepsilon+f_1(\sigma)=P_{pr}$

$$P_{pr}=\min(\varepsilon+f_1(\sigma),\ 1)$$
$$B_l(t)=A_p(t)-D_l(t)$$
$$=A_p(t)-X+Y-Z$$
$$X=A_p\otimes\beta_1(t), Y=A_p\otimes\beta_1(t), Z=D_l(t)$$
$$=A_p(t)-\inf_{0\leq s\leq t}X+Y$$
$$X=[A_p(s)+\beta_1(t-s)], Y=[A_p\otimes\beta_1(t)-D_l(t)]$$
$$\leq\sup_{0\leq s\leq t}[A(t)-X]\ +Y$$
$$X=A_p(s)-\beta_1(t-s)-\alpha_1(t-s)+\alpha_1(t-s),$$
$$Y=[A_p\otimes\beta_1(t)-D_l(t)]$$

$$\leq\sup_{0\leq s\leq t}X+\sup_{t\geq 0}[Y]+[Z]$$
$$X=[A_p(t)-A_p(s)-\alpha_1(t-s)],$$
$$Y=\alpha_1(t)-\beta_1(t), Z=A_p\otimes\beta_1(t)-D_l(t)$$

Delay bound is similar to the deterministic arrival curve,

$$Pr\ \{X\geq 0\}=0;$$
$$X=\mathop{sup}_{0\leq s\leq t}[A_p(t)-A_p(s)-\alpha_1(t-s)]$$

Delay bound is derived as,

$$Pr\{d(t)\geq h(\alpha_1,\ \beta)_1\}\leq\varepsilon+X$$
$$X=f_1(\sigma)$$

5 Modeling Acoustic Rician Fading

Stochastic service guarantees for packet switched networks is calculated for underwater acoustic communication using research in stochastic network calculus [20]. In stochastic service curve the aggregation of flows or individual flows represents the probabilistic bound [21] in SNC. Stochastic end-to-end delay and stochastic end-to-end backlog bounds are distributive in stochastic network calculus [22]. By deriving the network service curve stochastic end-to-end delay and stochastic backlog bounds are derived and calculated. The other properties in stochastic network calculus like delay and analysing the service guarantee in the server model facilitates backlog delay guarantee [23]. This concept is applicable to other properties in stochastic network calculus such as output characterization and concatenation. Even though there are various researches on the theories of SNC, only a few study the mapping of theory to real-time network applications. In [24], a Markov chain model of a wireless channel is provided; it doesn't provide a closed-form service curve, whereas we need a stochastic service quality for fading that uses closed-form service curves.

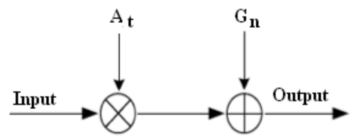

Fig. 3: System Modelling of Acoustic Rician Fading Channel

5.1 Acoustic Channel Model

In Rican fading the transmitted acoustic signals travels through the acoustic medium. When the acoustic signal arrives at the acoustic receiver node with atleast two different paths, then fading occurs. When there are more than two paths from acoustic transmitter and acoustic receiver fading like Rayleigh, Rician, Weibull occurs in the channel. When one of the path in the channel has line-of-sight (LOS) and the LOS signal received in acoustic receiver is stronger then other signals, then it is termed to be Rician Fading [25]. In Rician fading the amplitude gain is denoted as Rician Factor In this fading technique the amplitude gain is characterized by a Rician factor X. Consider a system model of a discrete-time flat-fading acoustic channel

$$C_o = C_g + C_i + G_n$$

where C_i denotes the acoustic channel input and C_o denotes the acoustic channel output. G_n denotes the identically Gaussian noise. Gaussian noise is independent and distributed in the system as $|A_t|\,e^{j\phi}$

Channel gain is expressed as C_g with amplitude $|A_t|$ and phase is denoted as ϕ. Rayleigh fading is uniformly distributed in the region $[0,\ 2\pi]$. Fig. 3 provides the systems model of a fading acoustic channel.

$$C_c = C_b T$$

$$T = \log_2[1+10^{\gamma/10}] = \log_2[1+\frac{P_{at}|A_t|^2}{C_b P_d}]$$

where, C_c= Channel Capacity C_b= Channel Bandwidth P_{at}= Average Transmission power P_d= Power spectral density

The acoustic transmitter is not aware of the instantaneous signal-to-noise ratio (SNR) γ_t in the outage probability O_p of an acoustic fading channel can be expressed

$$0_p\,(T_d) = P_{rx} = \{C_c < T_d\}$$

$$= P_{rx}\left\{\log_2[1+\frac{P_{at}|A_t|^2}{C_b P_d}] < T_d\right\}$$

$$= P_{rx}\{C_c T < T_d\}$$

$$= P_{rx}\left\{|A_t|^2 < \left(2^{T_d/C_b}-1\right).\frac{P_d C_b}{P_{at}}\right\}$$

where, channel gain $|A_t|$ has a distribution with probability density function

$$h(t) = F * G$$

$$F = t.\exp(\frac{-t^2}{2}+X), G = B_f(tc), X = RicianFactor = \frac{c^2}{2}$$

where B_f is the modified Bessel Function of order zero and Rician Factor $X = \frac{c^2}{2}$. It is the relation between the Line-of-sight components. When Rician factor $X \to \infty$, there exists no Line-of-sight component and hence Rayleigh and Rician fading are the same. By transformation theorem, $|A_t|^2$ has an exponential distribution with probability density function,

$$f(t) = \frac{1}{2}\exp\left(-\frac{t}{2}\right)$$

$$0_p(T_d) = 1 - D$$

$$D = \exp(\frac{1-2^{T_d/C_b}}{2.G})$$

$$G = 10^{SNR/10}$$

where $SNR = 10\log_{10}[P_{at}/(P_{sd}C_b]$

5.2 Stochastic Acoustic Service Curve

In acoustic wireless communication, the data traffic is random and irregular. In order to capture the characteristics of acoustic channel, deterministic service curve is not applicable as the acoustic channel is random and irregular. Stochastic service curve is useful in determining the acoustic channel characteristics. The channel capacity $C_c$is described by the two following parameters. Stochastic service curve determines the service capability using the data transmission rate that is denotes as T_r ε denotes the error function in the system. From the previous analysis of the fading channel, we can model the stochastic service curve $\beta\,(t)\,,\ \varepsilon,$

$$S_{sc}(t) = T_d.t$$

$$E_f(T_d) = 1 - \exp\left(\frac{1-2^{T_d/C_b}}{2.SNR}\right)$$

6 Simulation and Performance Evaluation

In this section, we present the performance evaluation of the derived mathematical models using simulations. In order to validate the tightness of the bound, we have simulated using the well-known commercial network simulation software tool OPNET, and compared the results of the simulation with their respective analytical results. The simulation parameters used are mentioned in

Table 1: Simulation Parameters

Channel Width	40 kHz
Transmit Power	2 W
Noise Power Spectral Density	1 dB
Carrier Frequency	40 kHz
Node Count	2
Delay	2s
Speed of Sound	1500 m/s
Rician Fading Parameter	0.5
Transmission Rate	0.09190 pkts/s
Transmission Time	7.75 s
Transmission Range	700 m
Packet Size	1024B
$\sigma(s), \rho(s)$	Bounds on rate and burst

Table 1. A simulation setup for analyzing the effects of fading in an underwater acoustic network is deployed using a single transmitter and receiver node. Fig. 4 shows the OPNET simulation environment with two nodes one acts as a transmitter and another as receiver. Fig. 5 shows the cumulative density function of the delay. From the simulation mode we can conclude that the delay varies between 0.1 to 1.1 seconds. We have compared the simulated results with the analytical results.Fig. 6 explains the comparison between the Signals to Noise ratio with bit error rate. It proves that our simulated model of Rician fading channel using OPNET matches the analytical results of the Rician fading channel modeled in the previous section. The acoustic sender or source sends the data in the acoustic medium to the acoustic receiver and the data rate is *r=40 kbps*.The data is send using the acoustic medium of the rician fading channel. The acoustic receiver, when receives data known as arrival process is modeled by using the stochastic arrival curve.

The arrival process in the channel is determined by using the arrival curve, which is deterministic in nature. Due to this the channel provides small processing delay in acoustic channel medium. The stochastic rate is derived with the small processing delay in the channel. Since the channel is said to have zero processing delay, the backlog bound in the stochastic arrival curves becomes independent of the channel. Violation probability in the channel becomes independent and backlog bound becomes constant factor. In Fig. 7 the loss probability we depict the loss probability model as a function of average channel SNR. The buffer size is fixed to *300kb* and the traffic rate σ is set to *15 kbps*. We have calculated for each node individually. In the Figs. 8 and 9 the stochastic delay for underwater acoustic transmitter is calculated. The violation probability is calculated for individual nodes. As mentioned already we have set the simulation in OPNET to two nodes one as transmitter and another as receiver. The traffic parameters bound rate as $\sigma(s)$ as *100 kb* and burst rate as $\sigma(s)$ *30 Kbps*. The graph illustrates that at sufficiently high SNR values, the delays are achieved even when traffic traverses multiple acoustic propagation links.

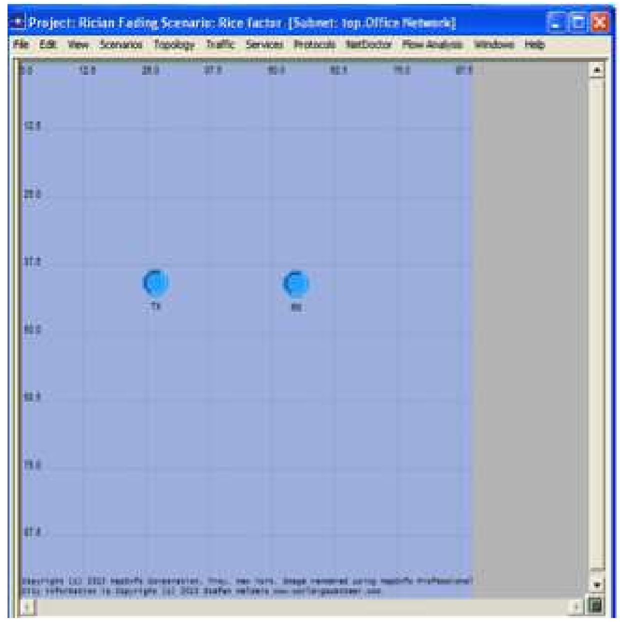

Fig. 4: Simulation Set up

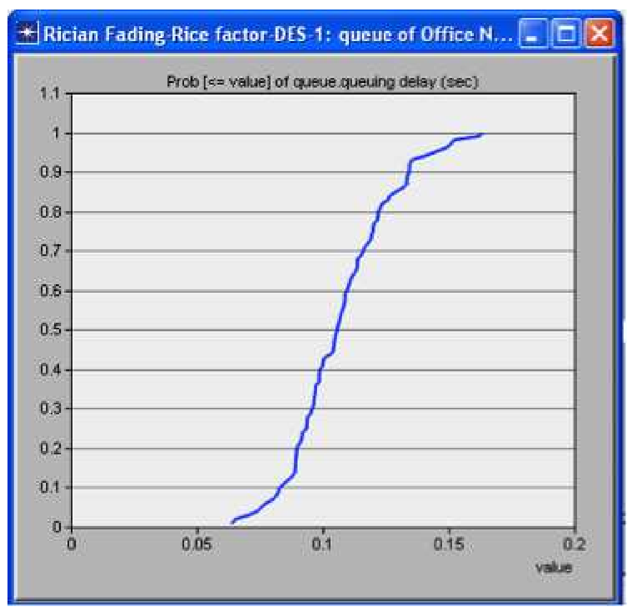

Fig. 5: Cumulative Density Function

The graphs shows here can be used for deploying multiple hop underwater acoustic nodes. Since the average channel SNR largely depends on the signal loss due to path loss, shadowing, which in turn is the function of the transmission circle

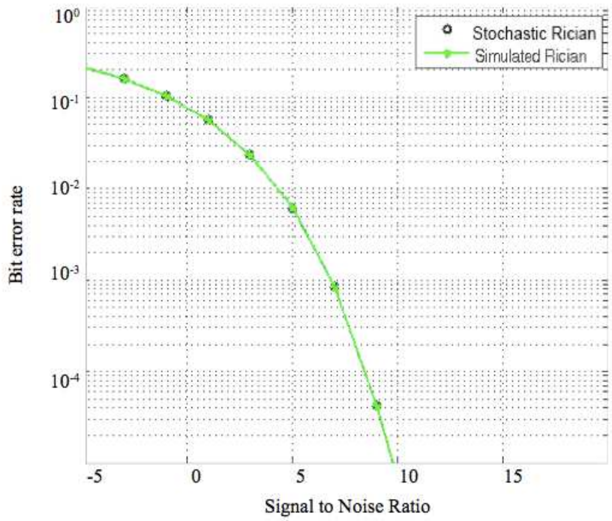

Fig. 6: SNR vs Bit Error Rate

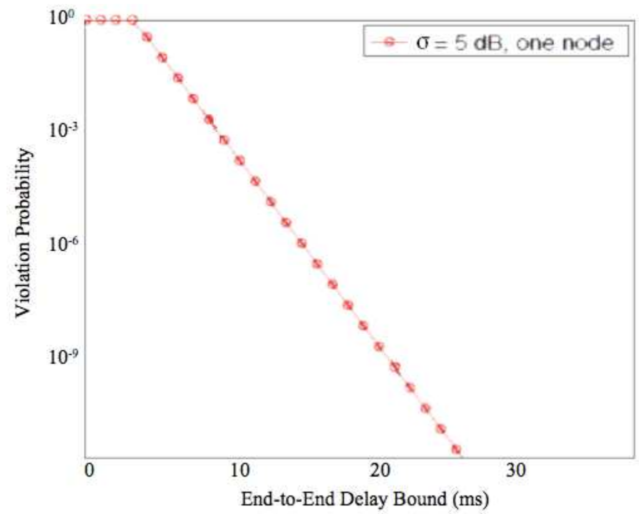

Fig. 8: Delay Bound Vs Violation Probability (for 5 dB)

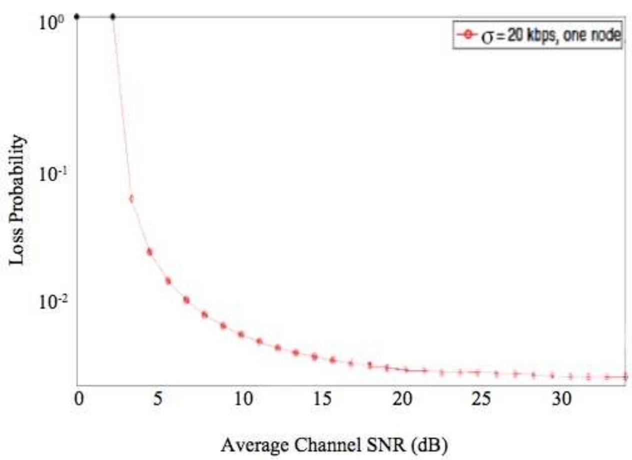

Fig. 7: SNR Vs Loss Probability

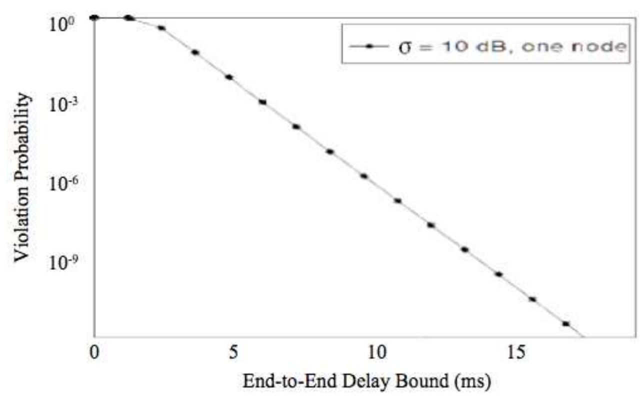

Fig. 9: Delay Bound Vs Violation Probability (for 10 dB)

7 Conclusion and Future Work

In this research article, we have made theoretical and practical contributions in understanding the fading effects in underwater wireless communication. The proposed mathematical model is analyzed and validated with respective simulation methods in OPNET. The results are satisfactory to the extent that our analytical model closely represents the real-time fading effects in underwater acoustic transmission. In our future work we would focus on increasing the network size with different nodes and varying bandwidth of the channel and transmission power. We would also be modeling acoustic channels with other known types of fading like Gilbert-Elliot, Weibull, AWGN, and Nakagami models

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