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Harvesting Strategies for “Bianchetti” and “Blue Fish” in the Ligurian Sea (North Mediterranean)

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Abstract: A three level trophic fish web model is proposed, consisting of clupeiforms pilchard (*Sardina pilchardus*) and anchovy (*Engraulis encrasicolus*), at the bottom, mackerel (*Scomber scombrus*) in the intermediate trophic level and tunny (*Thunnus thynnus*) as top predators. The model includes human harvesting. The clupeiforms population is further partitioned into three age classes, eggs, larvae and the adult fishes to account for possible cannibalism but mainly for the fact that these larvae in the Mediterranean countries are highly appreciated as a delicacy for human consumption. In the absence of fisheries, to prevent total extinction and maintaining coexistence it is important that a sufficient amount of eggs fertilize to form larvae. The predation rate of larvae by other fish populations also has an important role, especially for the occurrence of periodic solutions. Different harvesting policies are then numerically simulated. Independently of the harvesting policy used, even a small amount of selective harvesting of larvae may cause total extinction of the system. To prevent it, fishing that avoids the catch of the larvae should be adopted. With non selective harvesting the system is preserved, but for certain parameter ranges oscillations arise, which under unfavorable environmental perturbations may lead the food chain to collapse.

Keywords: Trophic levels; Food web; Cannibalism; Extinction; Harvesting strategy; Stage-structured model

1 Introduction

Starting from the seventies, researchers in mathematical biology have devoted much effort to the study of food chain systems [17,25,30]. Also models for cannibalism have been investigated, since they frequently occur in natural systems, [2,23,38]. More recently, in the past twenty years, several models for plankton dynamics have been formulated, [9,19,33,34]. This is a relevant research topic because of its importance for human feeding. In fact plankton lies at the bottom of the food chain in the ocean, and thus ultimately also affects fisheries. Some of the models combined also the physical features of the ocean, [31], other recent research efforts have been dedicated to the issue of pattern formation, [16,27,24], and the occurrence of red, or brown, tides, [6,7], which also have a negative effect on important human industries such as fisheries and tourism.

In this investigation we consider a complex situation in which several fish populations interact in a food web, to which is added also the external human intervention, represented by fishing. We mainly concentrate here on the fundamental role that younglings have in shaping the population dynamics and how this ultimately affects the future outcome of the whole aquatic food chain. This is particularly important in a closed sea such as the Mediterranean. In part larvae can be subject of cannibalism by their own adults, [28], and above all they are harvested by man. Both these aspects are suitably taken into account in the model. The aim is the investigation of selective harvesting on the larva of a specific fish population, in presence of its predator and a top predator. We compare the results with unselective fishing on all the fish populations.

In Italy the whitebait of anchovy and pilchard are indicated with the vernacular term of *bianchetti* (or *gianchetti*). As in use for other fish species both in Europe

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and in other countries of the world, such as New Zealand, Australia, China, in Liguria and in other Italian Regions also the whitebait of anchovy and pilchard remains in the human alimentary tradition as a delicacy and expensive food. In fact, while fishing of *bianchetti* of anchovy is forbidden according to rules of the European Community (CE 2550/2000), fishing of pilchard *bianchetti* is still allowed, though regulated by norms and periods which vary every year and depending on geographic locations: usually it is performed for 60 days in the period between January and March [12]. Specifically, in 2010 the period has been from February 15th until April 15th. Considering the size of the young fishes, from 7,000 to 12,000 individuals can be found in 1 kg of catch and during a season of fishing, along the Ligurian coast, the authorized boats can fish up to a few tons of *bianchetti*. Recently, consumption of the blue fish has increased, even if the rate of catch is flatly decreasing. Recent reports, [14], indicate that also *bianchetti* consumption in Italy has increased in the past years, despite a decline of the catch.

The paper is organized as follows. After giving some biological background, in Section 3 we describe the model. In Section 4 we analyze the system with no harvesting, establishing the equilibria and their stability, analytically or by means of simulations. In Section 5 harvesting is taken in consideration, with a comparison of several options for the strategies. A final discussion concludes the paper.

2 Biological background

The term “blue fish” identifies a group of deep-sea species, characterized by a dark blue color, also with green shades, on the back and on part of both sides as well as a silver color on the sides and the abdomen. In water, this color takes on a mimetic effect, since the fish becomes invisible for predators, both fishes and cetaceans, which prey on it from the bottom upwards, as well as horizontally, or from the top, mainly the birds. The blue fish are mostly gregarious, the gathering in form of school reducing for the single specimen the possibility of being predated. This occurs because the predator, in front of a remarkable number of prey, feels difficult to select the individual fish to attack, [20,26,35]. Already almost a century ago it was observed that schools of sardines can confuse Great Northern loons, [1]. In other situations the same phenomenon is observed, for instance Japanese honeybees form a defensive ball around attacking hornets, [22], while flocks of Bush tits detecting hawks make a confusion chorus, [18]. In fact, considering the limited visibility in water, every fish can be seen by a predator in a restricted field, defined by the maximum distance of visibility: if the fishes of the school swim very near each other, they cause an overlapping of fields and the probability by the predator to sight a school on the open sea becomes only slightly greater than that of sighting a single specimen. The predators, like mackerel and tunny, in turn gather into schools with the aim of both increasing their field of vision, therefore the hunting area, and of hunting in a coordinate and cooperative way. For instance tunnies form sets of 10-15 individuals which arrange themselves in a parabolic formation, with the concavity ahead. In this way, the prey are gradually surrounded and the predators save energy. Moreover, among the reasons leading to the formation of wide fish gatherings, there could be a number of advantages, among which energy saving, as mentioned, and also more ease in reproduction [5].

Furthermore, blue fishes include migratory species, moving in the water both vertically and horizontally; the reasons, the duration, and the extent of the migrations vary from species to species but are mostly connected with trophism and reproduction, in addition to environmental reasons, such as salinity and temperature.

In the Mediterranean, the species belonging to blue fish have size ranging from 10 cm up to 1 m; in addition to the clupeiforms pilchard (*Sardina pilchardus*) and anchovy (*Engraulis encrasicolus*), they include different species, such as mackerel (*Scomber scombrus*) and tunny (*Thunnus thynnus*).

The anchovy makes remarkable migrations approaching the coast in the spring while going down, in winter, to depths of more than 100 m; it has trophic diurnal habits and feeds on zooplankton, selecting prey one by one. The reproduction occurs from April until September, with a peak in June-July; its maximum length is about 20 cm, [4]. The pilchard makes vertical and horizontal migrations according to the temperature. It feeds on plankton filtered by means of branchial spines, thin and dense. The reproduction occurs all year round and shows a maximum in winter; the maximum size is about 20 cm [4]. At the hatch the *bianchetti* are 2 mm long and maintain a transparent body till they reach the length of 35-40 mm: at this point, the body begins to pigment and the pigmentation is complete when the size reaches 60 mm (“dressed” *bianchetti*). The *bianchetti* live near the coastline and move to the open sea only when they have reached maturity [5].

The mackerel makes seasonal migrations, being in deep waters during the winter and moving towards the coast during the reproductive period, i.e. around the end of the winter to the beginning of the spring. Since the beginning of autumn, it feeds mainly on small clupeiforms. The maximum size of the species is 30-35 cm, exceptionally up to 50 cm, and the maximum weight is about 1.5 kg [3].

The tunny ravenously feeds on all the fishes it meets; the fish interrupts feeding only during the reproductive period, i.e. around May and June, when it moves towards the coast; it begins again the trophic migration to the open sea at the end of summer or beginning of the autumn. The tunny length is well over 1 m and up to 3 m, with a weight from 30-40 kg up to 400 kg [3].

In the Ligurian Sea (Northern Mediterranean), the waters 10 miles off the coast represent a “meeting point” for some species of blue fish, i.e. a trophic and migration area [5]: anchovy and pilchard constitute prey for the mackerel which in turn is prey for the tunny. When the mackerel is in shallower waters, at the end of winter, it feeds also on *bianchetti*. Mainly when it is juvenile, the tunny prey also the clupeiforms. Besides in the gut content of anchovy also the *bianchetti* have been found, [28], evidently selected by the anchovy during its feeding "one by one". During its non selective filter-feeding on planktonic organisms, the pilchard eats its own eggs and small larvae.

The man feeds on all the levels of this trophic chain: in the Ligurian Sea the blue fish represents the predominant part of the superficial fishing, performed by means of seines, having mesh-size and length which vary according to the fish species to be captured. Lately the recommendations relevant to correct human feeding induce an increased consumption of the blue fish, whose meat is rich in unsaturated fatty acids.

3 Model formulation

We begin by illustrating the system characteristics. There are two adult populations involved, the top, P, and intermediate, F, predators, tuna and mackerel respectively, while in view of the interest in the fishing of *bianchetti* we partition anchovies and pilchard, among adults, larvae and eggs, denoted by S, L and E respectively. For mathematical simplicity, we at first assumed the interactions among the populations to be of the form Holling type I, i.e. governed by the mass action law. Later on, to observe the robustness of our results, we numerically tested the model also with a Holling type II functional response.

The model is

$$\begin{aligned}\frac{dE}{dt} &= -\mu E + mS - \theta ES \\ \frac{dL}{dt} &= eSE - \nu L - a_1 LF - H_1(L) \\ \frac{dS}{dt} &= lL - nS - a_2 SF - bSP - H_2(S) \qquad (1) \\ \frac{dF}{dt} &= a_1 k_1 LF + a_2 k_2 SF - cFP - fF - H_2(F) \\ \frac{dP}{dt} &= bhSP + cgFP - pP - H_2(P).\end{aligned}$$

Here, also human interference, i.e. fishing, is accounted for. The harvesting strategies for the larva and different fish populations are modeled by the last terms of each equation, but for the first one, and respectively denoted by H_1 and H_2. In this way we account for the fact that *bianchetti* may or may not be subject to the same fishing strategy as the other fishes, while all the adult fishes are harvested with the same strategy.

The parameters e and l represent the maturation rates from egg to larva and from larva to adult population respectively, μ is the loss rate of E, due to natural mortality, dispersion due to the currents, predation by other species; ν is again the loss rate of individuals in the L population, but in this case in addition to the above mortality factors, also the maturation process into adults is accounted for; $\theta > e$ denotes the net loss in egg population E due the cannibalism of the adult fish S on its own eggs and the fecundation process for which eggs mature to the larval stage. The parameter m is the reproduction rate of the S population; the mackerels F hunt at rate a_1 on larvae L and a_2 on pilchards S with mass conversion factors k_1 and k_2 respectively; similarly b and h respectively denote the predation rates and conversion factors of P on S while c and g are again the predation rate and conversion factor of tunnies P by hunting F's. The natural death rates of S, F and P are respectively n, f and p.

The first, second and the third equations denote the evolution of the various life stages of the prey S. In the first equation for the eggs dynamics, the first term denotes the loss of eggs due to natural mortality, predation by other species and dispersion. The second term represents new recruits due to reproduction of the adults. Their environment is a large area with enough resources, so that Malthus growth can be assumed. The last term models cannibalism by S and maturation to become *bianchetti* due to hatching. Note that both these processes are related to encounters of individuals in the two populations either for predation or for mating, as eggs are fertilized in water.

The second equation shows the larvae dynamics. Individuals enter this class via maturation, first term, and are subject to mortality or mature to become adults: in both cases their numbers decrease, and this is modeled by the second term; finally they can be prey of the mackerels F. The pilchards S are modelled by the third equation. They enter this class from matured larvae, at rate $l < \nu$, are subject to the natural mortality, second term, and are preyed by mackerels and by tunnies. The mackerels dynamics benefits from hunting the larvae and the pilchards, first two terms, but is subject also to hunt by tunnies and to its own natural mortality, third and fourth terms. The tunnies could in principle also feed on other sources, but we disregard this situation here. They feed both on the pilchards and on the mackerels, first two terms of the last equation, but die out exponentially in their absence, third term.

4 The system with no harvesting

4.1 Boundary equilibria and their stability

Proposition 4.1. The system (1) with no harvesting, $H_1 \equiv H_2 \equiv 0$, has the following boundary equilibria in the $E-L-S-F-P$ phase space, in addition to the trivial equilibrium in which each population vanishes,

$V_0 \equiv (0,0,0,0,0)$:

$$V_1 \equiv \left(\frac{\nu n}{el}, \frac{n\Delta}{l}, \Delta, 0, 0\right), \quad \Delta = \frac{\mu \nu n}{mel - \theta \nu n},$$

$$V_2 \equiv \left(\Theta, \frac{pe\Theta}{bh\nu}, \frac{p}{bh}, 0, \frac{le\Theta - n\nu}{\nu b}\right), \quad \Theta = \frac{mp}{\mu bh + \theta p}.$$

There is another boundary equilibrium point $V_3 \equiv (E_3,\ L_3,\ S_3,\ F_3,\ 0)$ with $P = 0$, where

$$E_3 = \frac{mS_3}{\mu + \theta S_3},\ L_3 = \frac{S_3(n + a_2F_3)}{l},$$
$$S_3 = \frac{fl}{a_1k_1(n + a_2F_3) + a_2k_2l}$$

where F_3 is a root of the equation

$$Q_1F^4 + Q_2F^3 + Q_3F^2 + Q_4F + Q_5 = 0 \tag{2}$$

with $Q_1 = a_1^3k_1^2a_2^2\mu$ and

$$\begin{aligned}
Q_2 &= (a_1k_1n + a_2k_2l)a_1^2\mu k_1a_2 + a_1k_1a_2(\nu\mu a_1k_1a_2 \\
&\quad + a_1^2\mu k_1n + a_1\mu a_2k_2l + a_1\theta fl + a_1^2\mu k_1a_2^2 \\
&\quad + a_1^2a_2\mu k_1n), \\
Q_3 &= (a_1k_1n + a_2k_2l)(\nu\mu a_1k_1a_2 + a_1^2\mu k_1n + a_1\mu a_2k_2l \\
&\quad + a_1\theta fl + a_1^2\mu k_1a_2^2 + a_1^2a_2\mu k_1n) + a_1k_1a_2(a_1n\theta fl \\
&\quad + a_1^2a_2\mu k_1n + \nu\mu a_2k_2l + a_1n\mu a_2k_2l + \nu\theta fl \\
&\quad + a_1^2\mu k_1n^2 + \nu a_2\mu a_1k_1n + a_1\mu a_2^2k_2l \\
&\quad + \nu\mu a_1k_1a_2^2 + \nu\mu a_1k_1n + a_1a_2\theta fl), \\
Q_4 &= (a_1k_1n + a_2k_2l)(a_1n\theta fl + a_1^2a_2\mu k_1n + \nu\mu a_2k_2l \\
&\quad + a_1n\mu a_2k_2l + a_1^2\mu k_1n^2 + \nu\theta fl + \nu a_2\mu a_1k_1n \\
&\quad + a_1\mu a_2^2k_2l + \nu\mu a_1k_1a_2^2 + \nu\mu a_1k_1n + a_1a_2\theta fl) \\
&\quad + a_1k_1a_2(\nu n\mu a_2k_2l + \nu a_2\mu a_1k_1n + \nu\mu a_1k_1n^2 \\
&\quad + \nu n\theta fl + \nu\mu a_2^2k_2l - efl^2m + \nu a_2\theta fl), \\
Q_5 &= (a_1k_1n + a_2k_2l)(\nu\mu a_2^2k_2l - efl^2m + \nu n\mu a_2k_2l \\
&\quad + \nu a_2\mu a_1k_1n + \nu n\theta fl + \nu a_2\theta fl + \nu\mu a_1k_1n^2).
\end{aligned}$$

Proof. Since Q_i s’, $i = 1,2,3$ are positive, equation (2) has a unique positive root if and only if $Q_5 < 0$. This is only possible if e or m or both are very large so that the term efl^2m becomes greater than the other positive terms present in the expression of Q_4 and Q_5. Hence, the necessary and sufficient condition for the feasibility of a unique boundary equilibrium point V_3 is that *em* should exceed a threshold value. Further, from the analysis of V_3 we need either $a_1 \neq 0$ or $a_2 \neq 0$ for the feasibility of S_3, i.e., F has to feed on either L or S. This is obvious, since no other food sources are present for the mackerels, and in their absence the latter would disappear. □

Remark 4.1. The feasibility condition for V_1 is given by $mel > \theta\nu n$, for V_2 feasibility is ensured by $le\Theta > n\nu$.

Generally it is the size of the predator that selects the size of the prey. In the autumn, rather than pilchards’ eggs or zooplankton, mackerel prefer, or find more abundant, other types of food. At the peak of the summer instead, after the reproduction season, the feeding of the young mackerel is rather voracious and based on everything they can find, whether be it pilchards’ eggs, pelagic crustaceans, clupeiforms or zooplankton.

Proposition 4.2. Exploring the case in which mackerel divert their attention from *bianchetti*, i.e. setting $a_1 = 0$, the equilibrium point V_3 can be explicitly evaluated,

$$\widehat{V}_3 \equiv \left(\frac{mf}{a_2k_2\mu + \theta f}, \frac{f(n + a_2\Omega)}{a_2k_2l}, \frac{f}{a_2k_2}, \Omega, 0\right),$$

where,

$$\Omega = \frac{eflm - \nu\mu a_2^2k_2 - \nu n\mu a_2k_2 - \nu n\theta f - \nu a_2\theta f}{\nu\theta f + \nu\mu a_2k_2}.$$

Remark 4.2. In this case, i.e. for $a_1 = 0$, V_3 is feasible for $eflm > \nu\mu a_2^2k_2 + \nu n\mu a_2k_2 + \nu n\theta f + \nu a_2\theta f$.

Proposition 4.3. Next if we assume that F does not feed on S, i.e., $a_2 = 0$ the equation (2) becomes,

$$\begin{aligned}
&(a_1^2\mu k_1n + a_1\theta fl)F^2 + (\nu\theta fl + a_1n\theta fl + a_1^2\mu k_1n^2 \\
&\quad + \nu\mu a_1k_1n)F - efl^2m + \nu n\theta fl + \nu\mu a_1k_1n^2 = 0.
\end{aligned} \tag{3}$$

The equation (3) has a unique positive real root if and only if $efl^2m > \nu n\theta fl + \nu\mu a_1k_1n^2$. Thus when $a_2 = 0$, the equilibrium point V_3 is feasible for

$$efl^2m > \nu n\theta fl + \nu\mu a_1k_1n^2. \tag{4}$$

The stability properties of these equilibrium points are studied through the Jacobian matrix analysis. The Jacobian J at an arbitrary point is given by the matrix below with $J_{44} = a_1k_1L + a_2k_2S - cP - f$, $J_{55} = bhS + cgF - p$,

$$\begin{pmatrix}
-\mu - \theta S & 0 & m - \theta E & 0 & 0 \\
eS & -\nu - a_1F & eE & -a_1L & 0 \\
0 & l & -n - bP - a_2F & -a_2S & -bS \\
0 & a_1k_1F & a_2k_2F & J_{44} & -cF \\
0 & 0 & bhP & cgP & J_{55}
\end{pmatrix}. \tag{5}$$

Proposition 4.4. V_0 is always stable.

Proof. The eigenvalues associated with the matrix (5) evaluated at the origin are $-\mu,\ -\nu,\ -n,\ -f,\ -p$. Since all the eigenvalues are negative real numbers, the claim follows. □

Proposition 4.5. V_1 is always unstable.

Proof. Two eigenvalues associated with the matrix (5) at V_1 are explicitly given by $a_1k_1n\Delta l^{-1} + a_2k_2\Delta - f$, $bh\Delta - p$ and the remaining ones are the roots of the cubic equation

$$Y^3 + \omega_1Y^2 + \omega_2Y + \omega_3 = 0, \tag{6}$$

where $\omega_1 = n + \nu + \mu + \theta\Delta$, $\omega_2 = (\mu + \theta\Delta)(n + \nu)$, $\omega_3 = \Delta(elm - \theta\nu n)$. From the existence criteria we have $mel > n\nu\theta$, so $\omega_3 < 0$. Hence, by the Routh-Hurwitz criterion the claim follows. □

Proposition 4.6. V_2 is stable if and only if the following conditions hold: $\eta_4 > 0$, $\eta_2 > \eta_3\eta_1\eta_4$ and $a_1k_1pme\Theta + a_2k_2pv < cmleh\Theta + cnhv + fbhv$.

Proof. The eigenvalues of the Jacobian (5) at V_2 are

$$\frac{a_1k_1pe\Theta + a_2k_2pv - cleh\Theta - cnhv - fbhv}{bhv}$$

and the roots of the quartic equation

$$\eta_1Z^4 + \eta_2Z^3 + \eta_3Z^2 + \eta_4Z + \eta_5 = 0, \tag{7}$$

with coefficients

$$\begin{aligned}
\eta_1 &= vbh,\\
\eta_2 &= le\Theta bh + v^2bh + \mu vbh + \theta pv,\\
\eta_3 &= pbhle\Theta - pbhvn + le\Theta\mu bh + le\Theta\theta p\\
&\quad + v^2\mu bh + v^2\theta p,\\
\eta_4 &= p^2le\Theta\theta - pv^2nbh - p\mu vnbh - p^2\theta vn - plemv\\
&\quad + le\Theta\theta pv + ple\Theta vbh + ple\Theta\mu bh,\\
\eta_5 &= pvle\Theta\mu bh + p^2vle\Theta\theta - pv^2\mu nbh - p^2v^2\theta n.
\end{aligned}$$

Here η_1 and η_2 are always positive. From the existence criteria of V_2, η_3 and η_5 are also positive. The Routh Hurwitz conditions indicate that all the roots of the equation (7) have negative real parts if and only if $\eta_1 > 0,\ \eta_3 > 0,\ \eta_4 > 0$ and $\eta_2 > \eta_3\eta_1\eta_4$. □

Since it was not possible to find a closed form for V_3, it is not possible to find the stability conditions explicitly for V_3 and hence this analysis is omitted. However, $\widehat{V}_3$ takes a simplest form with $a_1 = 0$ and so the stability property at $\widehat{V}_3$ is studied with $a_1 = 0$.

Proposition 4.7. The equilibrium point $\widehat{V}_3$ (assuming $a_1 = 0$) is stable if and only if both the following conditions hold

$$\frac{bhf}{a_2k_2} + cg\Omega < p,\ \zeta_4 > 0, \quad \zeta_2 > \zeta_1\zeta_3\zeta_4.$$

Proof. The eigenvalues of the Jacobian (5) at $\widehat{V}_3$ with $a_1 = 0$ are

$$\frac{bhf}{a_2k_2} + cg\Omega - p,$$

and the roots of the quartic equation,

$$\zeta_1V^4 + \zeta_2V^3 + \zeta_3V^2 + \zeta_4V + \zeta_5 = 0 \tag{8}$$

where $\zeta_1 = a_2k_2$, $\zeta_2 = va_2k_2 + na_2k_2 + a_2^2\Omega k_2 + \rho_1$,

$$\begin{aligned}
\zeta_3 &= a_2^2\Omega fk_2 + na_2k_2v + a_2^2k_2\Omega v\\
&\quad + \rho_1(n + a_2\Omega + v) - \frac{lemfa_2k_2}{\rho_1},\\
\zeta_4 &= a_2^2k_2f\Omega v + \rho_1(a_2f\Omega + vn + a_2\Omega v)\\
&\quad - lemf - \frac{lemfa_2k_2\mu}{\rho_1},
\end{aligned}$$

$\zeta_5 = fa_2\Omega\rho_1v$ and $\rho_1 = \mu a_2k_2 + \theta f$.

Since $\zeta_2 > 0$ and $\zeta_5 > 0$, the claim follows. □

4.2 The coexistence equilibrium

We now turn to simulations. Fecundity of pilchards depends on geographical area, ranging from 50000 to 60000 and from 76000 to 490000 eggs, with spawning all year round once or twice per year. The larval stage duration from egg to adult lasts 40 days, [11,37]. Based on this information we consider two possible scenarios, one with low reproduction rate and one with a higher one. In the first case, assuming 72000 eggs per year mature, we get $m = 200$ per day, while in the latter one we have taken 360000 eggs per year to mature, giving $m = 1000$ per day. Splitting evenly the maturing duration among egg to larvae and larvae to adult, we take $e = 0.05$ and $v = l = 0.05$. For the mortality, [10], the natural mortality amounts to 0.64 per year, slightly lower, 0.5, in the Adriatic, [29], giving $n = 0.002$ per day. For *Tunnus thynnus* natural mortality is 0.14 per year for all ages, [15], giving $p = 0.0004$ per day. We use the same values also for mackerel, $f = 0.0004$.

Table 1: The parameter values: those that are found in the literature, [11,37,10,29], are reported with a star, the remaining ones are hypothetical.

Name	values	Name	values
$*m$	200 day^{-1}	a_2	0.1 kg^{-1}day^{-1}
g	0.01	$*v$	0.05 day^{-1}
μ	0.3 day^{-1}	b	0.0015 kg^{-1}day^{-1}
$*e$	0.05 kg^{-1}day^{-1}	c	0.27 kg^{-1}day^{-1}
$*n$	0.002 day^{-1}	$*f$	0.0004 day^{-1}
$*l$	0.05 day^{-1}	k_1	0.6
θ	0.2 kg^{-1}day^{-1}	k_2	0.5
a_1	2.3 kg^{-1}day^{-1}	$*p$	0.0004 day^{-1}
h	0.0001	–	–

The set of parameter values taken from literature are summarized in Table 1. For further analysis the interior equilibrium is investigated by means of numerical simulations using the Matlab built-in routine ode45 with the parameter values given in Table 1. Under these conditions, the coexistence equilibrium is stable, see Figure 1.

Taking now the conversion rate from egg to larva as a parameter to study, we find that when e is reduced to 0.000071, all populations vanish and the system collapses, see Figure 2. Thus total disappearance is possible if a large enough amount of eggs does not hatch to become larvae, supporting and furthering our analytical results of the former Subsection on the local stability of the origin.

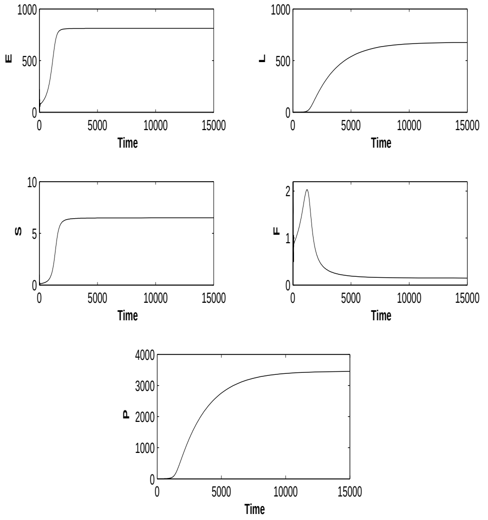

Fig. 1: For the parameters given in Table 1, the system (1) without harvesting shows a stable interior equilibrium steady state.

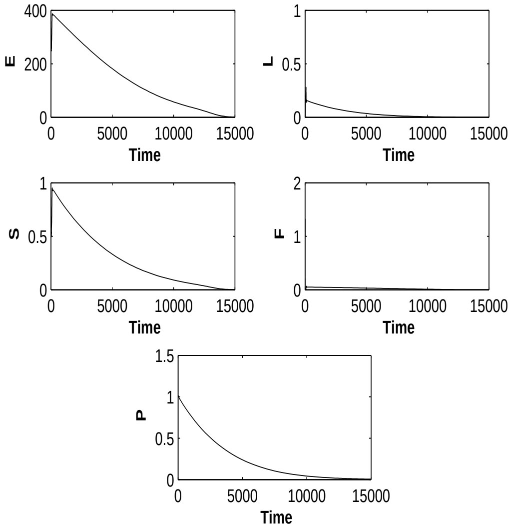

Fig. 2: Total extinction of all the populations in system (1) with no harvesting for a very low egg maturation rate $e = 0.000071$.

5 The various harvesting policies

Here we study the actual system (1) with different harvesting policies, for the prey fish at the larval stage and for all the adult fish populations. This means that we take two functional forms for H_1 and H_2 and combine them also with selective and unselective harvesting, i.e. when the latter is performed only on adult fish or indiscriminately on all the web populations, respectively. The case of fishing on larvae only is also contemplated although less realistic, as the adult fish catching in general continues the whole year round.

5.1 *Linear harvesting policies*

We study the system with the assumption that the harvesting H_1 is done on the larvae L via the function $H_1 = q_1 L$, q_1 being the harvesting rate of the larvae, while the adult fish populations S, F and P are harvested at a different rate q_2, via the functions $H_2(S) = q_2 S$, $H_2(F) = q_2 F$, $H_2(P) = q_2 P$.

With these assumption on H_1 and H_2 the system (1) again has three boundary equilibrium points in addition to the origin. These equilibria are

$$U_1 \equiv \left(\frac{(\nu+q_1)(n+q_2)}{el}, \frac{(n+q_2)\Phi}{l}, \Phi, 0, 0 \right),$$

$$\Phi = \frac{\mu(\nu+q_1)(n+q_2)}{mel - \theta(\nu+q_1)(n+q_2)},$$

$$U_2 \equiv \left(\Psi, \frac{(p+q_2)e\Psi}{bh(\nu+q_1)}, \frac{(p+q_2)}{bh}, 0, \frac{le\Psi}{(\nu+q_1)b} - \frac{n+q_2}{b} \right),$$

$$\Psi = \frac{m(p+q_2)}{\mu bh + \theta(p+q_2)},$$

U_1 and U_2 are feasible respectively for

$$mel > \theta(n+q_2)(\nu+q_1), \quad le\Psi > (n+q_2)(\nu+q_1).$$

If we compare the values of the equilibria and their feasibility conditions for this case with those for the model in absence of harvesting, we note that the only difference is in the scale, since here the constants ν, n, f, p of the model with no harvesting get here replaced by $\nu+q_1$, $n+q_2$, $f+q_2$, $p+q_2$. The same changes occur for the feasibility of U_3, similar to the one of V_3, and for the stability of all the equilibria. Thus we omit the analysis.

From the available literature, [10], the fishing mortality in the Adriatic, [29], is 0.30 per year so that $q_1 = 0.001$ per day. The fishing mortality for *Tunnus thynnus* depends on the age, and we use an average value of $q_2 = 0.0004$ per day. So, for further analysis we simulate the system (1) numerically with the parameter values given in Table 1 with $q_1 = 0.001$ and $q_2 = 0.0004$ as found in the literature. In these conditions the populations coexist, see Figure 3. But if the harvesting rates q_1 and q_2, are both increased to the level 0.02, we observe total extinction of the system.

5.1.1 Harvesting only of larvae

Although not much realistic, since fisheries operate continually in time, we study here the food web with the assumption that the harvesting H_1 is done only on the larvae. Using again the set of parameter values given in

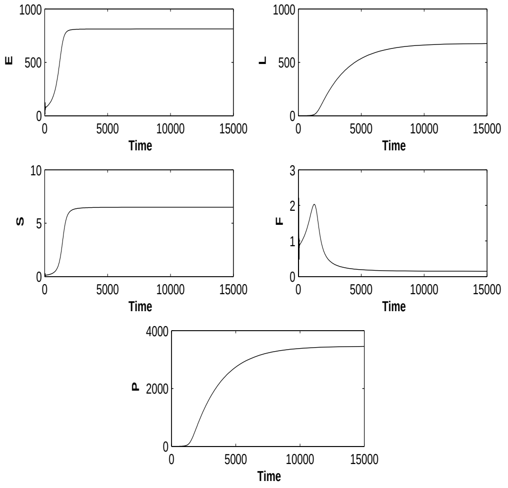

Fig. 3: Coexistence for the system (1) is attained when the fishing efforts are $q_1 = 0.001$ and $q_2 = 0.0004$, as given in the literature. Here the linear form of harvesting is used. The other parameters are the same as in Figure 1.

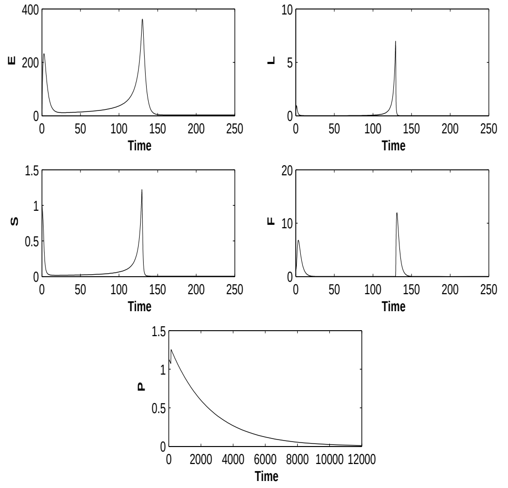

Fig. 4: Starting from the coexistence state of Figure 1 with selective linear harvesting on L at rate $q_1 = 2.1$, $q_2 = 0$, the other parameters being the same as in Figure 1, the food web collapses.

Table 1, if we introduce fisheries with removal rate $q_1 = 2.1$, the whole system is driven to extinction, as depicted in Figure 4. Thus increase in harvesting on larvae can drive the system to total extinction but that need almost a 2000 fold increase than normal rate and that too with the top predator taking much larger time than the others species. Thus in case of linear harvesting strategy only on the larval population, it is very difficult to cause total collapse of the food web. Recall however that this situation is quite unrealistic, since adult fishes are caught generally the whole year round.

5.1.2 Harvesting only adult fish

We consider the system (1) with the assumption that the harvesting is forbidden on the younglings, i.e. $H_1 = 0$ and H_2 is performed only on the adult fish populations S, F and P. For small values of q_1 the system is stable at the coexistence steady state, but we do not report graphically this result here. As the value is increased to $q_2 = 0.01$ we observed total collapse of the system, Figure 5.

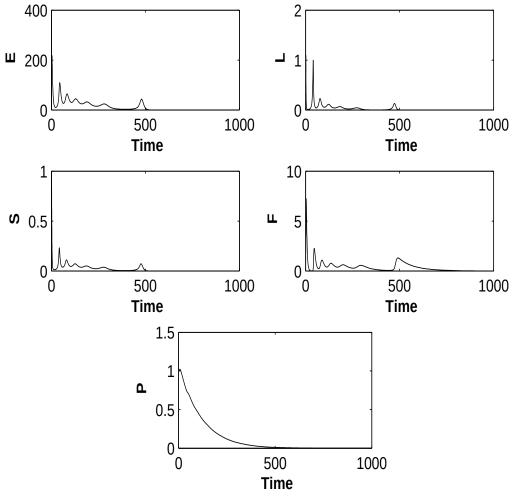

Fig. 5: Again for linear selective harvesting on adult fish only, starting from the condition and parameter values of Figure 1, we obtain total extinction with fishing efforts $q_1 = 0$, $q_2 = 0.01$.

5.2 Harvesting with bounded maximal return

We now consider a more realistic description of the fish catches, expressed by the Holling type-II function. For illustrating the features of this function with respect to the Holling type-I one can consult any standard text in mathematical biology or even in operations research, where the gain expressed by this function is known as the law of diminishing returns, [8,13,16,21], but see also [32]. The two functions for the larvae and for the adult fishes are respectively expressed by

$$H_1(L) = \frac{q_1 L}{1 + r_1 L}, \quad H_2(x) = \frac{q_2 x}{1 + r_2 x}.$$

The model (1) with these functions becomes too complex to be studied analytically. We rather investigate it only by means of numerical simulations. For the usual set of parameter values of Table 1, and new parameter values for the harvesting functions given by $q_1 = 0.001$, $q_2 = 0.0004$, $r_1 = 0.1$ and $r_2 = 0.4$, the system shows stable coexistence. The system remains unchanged for a large range of r_1 and r_2. But, as expected, upon increasing the values of q_i's, like in linear harvesting, here too the system becomes totally extinct.

6 A possibly endangered situation

We now consider a set of hypothetical values and perform bifurcation analysis to understand the possible ecological danger associated with possible shifts of some parameter values due possibly to relevant exogenous changes in the environmental conditions. The hypothetical set of parameter values, given in Table 2, are chosen so that the system shows stable coexistence. We start by searching periodic solutions.

Table 2: A hypothetical set of parameter values.

Names	values	Names	values
m	0.35 day^{-1}	a_2	0.2 $\text{kg}^{-1}\text{day}^{-1}$
g	0.6	v	0.3 day^{-1}
μ	0.3 day^{-1}	b	0.6 $\text{kg}^{-1}\text{day}^{-1}$
e	0.8 $\text{kg}^{-1}\text{day}^{-1}$	c	0.7 $\text{kg}^{-1}\text{day}^{-1}$
n	0.14 day^{-1}	f	0.4 day^{-1}
l	0.5 day^{-1}	k_1	0.7
θ	0.2 $\text{kg}^{-1}\text{day}^{-1}$	k_2	0.2
a_1	0.65 $\text{kg}^{-1}\text{day}^{-1}$	p	0.5 day^{-1}
h	0.5	–	–

The values of different parameters are varied one by one, keeping all the other ones fixed. It is observed that when the predation rate a_1 on L by the predators F increases to 0.75, the system shows coexistence through periodic oscillations, see Figure 6. This is clearer from the bifurcation diagram obtained using AUTO, Figure 7, where we have taken the parameter a_1 as the bifurcation parameter. It is observed that for low value of a_1, the population F goes to extinction, while all the other ones attain a steady state, with constant values independent of the values of the parameter a_1. Past the branch point in the terminology of AUTO, which in this case is rather a transcritical bifurcation point, all the populations coexist at a stable steady state with F increasing as a_1 increases, and all the other populations instead diminishing. When the value crosses some threshold value a_{1c}, specifically here it is $a_{1c} = 0.67$, at which a Hopf bifurcation occurs, the populations coexist through periodic oscillations. Thus we have proven by means of simulations the existence of a critical threshold a_{1c} where the Hopf bifurcation occurs around the positive steady state, thereby inducing oscillations of the populations. The hunting rate of F's on larvae therefore plays an important role in the dynamics of the whole system and consequently on the evolution of larvae and their predators. The same bifurcation is obtained for a_2, i.e., predation rate on S by the predators F, not reported here.

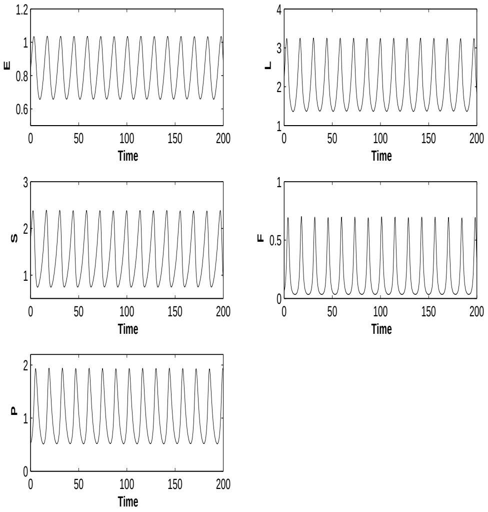

Fig. 6: Coexistence of all the species via oscillations in system (1) with no harvesting when the F's hunting rate a_1 increases here from 0.65 to 0.75.

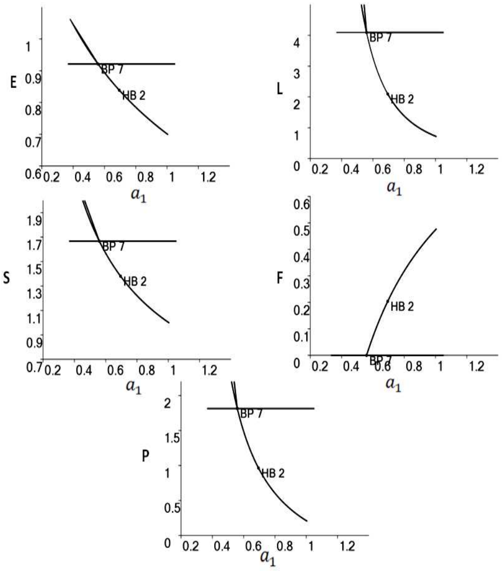

Fig. 7: For the system (1) without harvesting, using AUTO, we plot a bifurcation diagram taking the F's hunting rate a_1 as bifurcation parameter. At the critical point a_{1c} there occurs a Hopf bifurcation. BP stands for branch point, i.e. a transcritical bifurcation, and HB means Hopf-bifurcation.

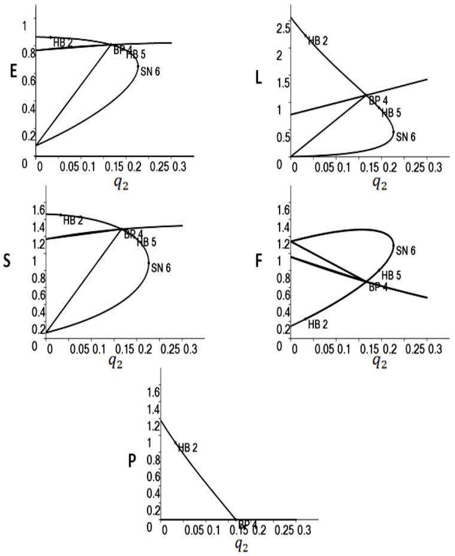

Fig. 8: Bifurcation diagram, obtained using AUTO, for selective adult fish linear harvesting, i.e. $q_1 = 0$, taking q_2 as bifurcation parameter. Note the Hopf bifurcation at a lower critical value, here the value is very small, and total extinction when a higher threshold is reached. BP stands for branch point, i.e. transcritical bifurcation, and HB means Hopf-bifurcation, SN is for saddle node bifurcation.

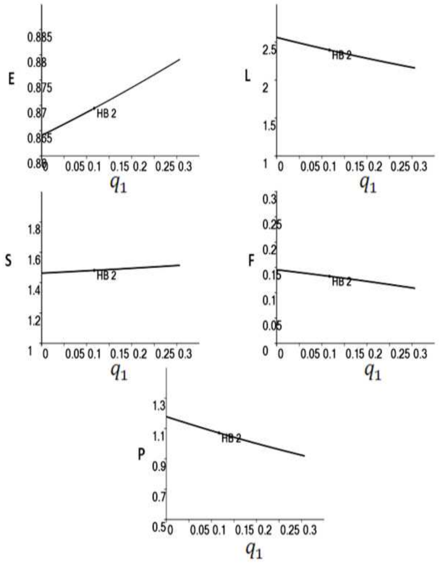

Fig. 9: Bifurcation diagram, obtained using AUTO, as function of the effort q_1 for the Holling type II selective harvesting on larvae only. HB means Hopf-bifurcation.

Interesting findings on bifurcation are observed in the model with harvesting, for example linear harvesting on adult fish only. To understand the changes in the dynamical behaviour of the species as functions of the changes in the value of q_2 we plot the corresponding bifurcation diagram in Figure 8. The structure of this picture is rather complicated, showing a branch point, i.e. a transcritical bifurcation, two Hopf bifurcations and a saddle-node bifurcation. For a certain range of q_2 all the populations coexist, then a bifurcation occurs and oscillations arise. In Figure 8 at the branch point, i.e. transcritical bifurcation, BP4, P vanishes while the other populations remain in the system. As q_2 increases past the branch point, i.e. transcritical bifurcation, BP4 there occurs a Hopf-bifurcation, HB5, among the remaining populations and then a saddle-node bifurcation, SN6, leading all the population to collapse, i.e. causing total extinction of the system. But already an important remark here is that even for a not so large value of the harvesting rate, $q_2 \approx 0.17$, the top predator population disappears.

Next we consider the model in the presence of harvesting with bounded maximal return. We consider this type of harvesting only on the larvae, i.e., we have system (1) with $q_2 = 0 \equiv H_2$. With the parameter values given in Table 1 and $r_1 = 0.7$, the bifurcation diagram is plotted in Figure 9 taking q_1 as the bifurcating parameter. The changes in the dynamical behaviour of the species as functions of the larvae capturing rates show an initial steady state, which exists for a small range of q_1, followed by a range of q_1 for which all the populations coexist via limit cycles, past the Hopf bifurcation point located at $q_1 \approx 0.12$. In fact for this small value of q_1, a Hopf-bifurcation, HB2, occurs and sustained periodic solutions arise.

Again to investigate the relation between the capturing rate q_1 and the saturation constant r_1, we plot in the two dimensional $r_1 - q_1$ parameter space a bifurcation diagram, Figure 10. For low values of r_1 and q_1, we observe a saddle-node bifurcation SN5. For very low values of r_1 and q_1 the system attains a stable coexisting state, but by increasing either r_1 or q_1, the stability of the system is destroyed and the system shows periodic oscillations.

We finally consider the harvesting with bounded maximal return for all the adult fishes, i.e. we set $q_1 = 0$. With the parameter values given in Table 1 and fixing $r_2 = 1.2$, we plot the bifurcation diagram, Figure 11, taking q_2 as the bifurcating parameter. The structure of this diagram is again rather complicated, showing a Hopf bifurcation HB2 for very low values of q_2, around 0.04, leading to an unstable manifold for which the system populations start all to oscillate giving rise to limit cycles in the phase space. With an increase in q_2, we observe a saddle-node bifurcation (SN6) and a branch point, i.e. transcritical bifurcation, located at $q_2 \approx 0.22$. Past this value, it is seen from the figure that the top predator population gets extinguished. But the presence of the saddle-node bifurcation itself ultimately drives the whole system to extinction.

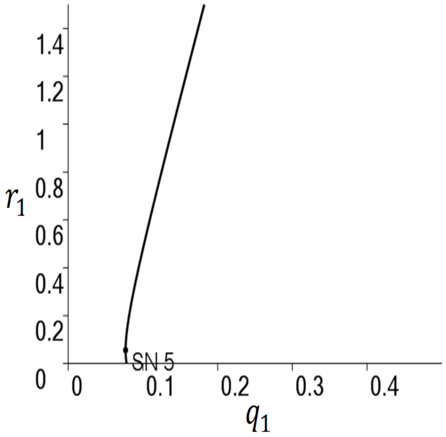

Fig. 10: Two parameter space diagram, obtained using AUTO, showing the stability regions as functions of q_1 and r_1 for the Holling type II selective harvesting on larvae only. A saddle-node (SN) bifurcation is observed for low values of q_1 and r_1.

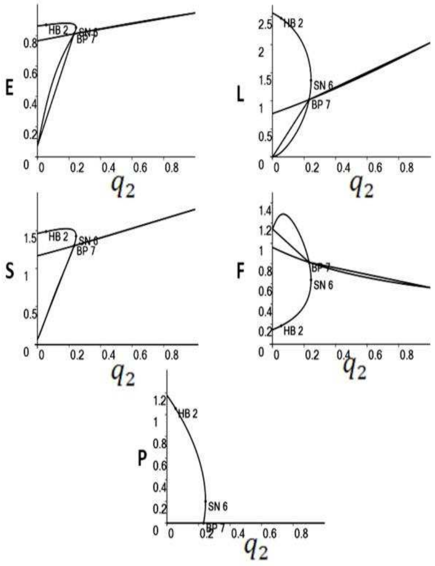

Fig. 11: Bifurcation diagram, obtained using AUTO, as function of the effort q_2 for the Holling type II selective harvesting only on adult fishes. BP stands for branch point, i.e. transcritical bifurcation, and HB means Hopf-bifurcation, SN is for saddle node bifurcation.

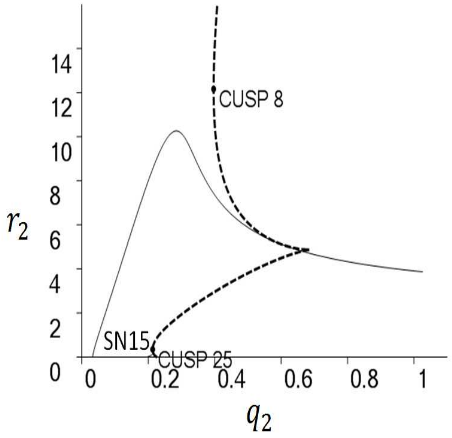

Fig. 12: Two parameter space diagram, obtained using AUTO, showing the stability regions as functions of q_2 and r_2 for the Holling type II selective harvesting only on adult fishes. Two CUSP bifurcation are observed, both for low values of q_2, but one for low value of r_2 and the other one for a high value of r_2.

We also provide a two-parameter space diagram in terms of q_2 and r_2. In addition to the saddle node bifurcation, located on the bottom left and almost overlapping a cusp bifurcation, we observe the presence of a second cusp bifurcation for a moderate value of q_2, see Figure 12. The cusp bifurcation implies the presence of a hysteresis phenomenon. For the first one of the two, this is also clear e.g. from Figure 11 looking at the behavior of the top predator population P near $q_2 = 0.22$.

6.1 Numerical Result with Holling type-II interaction

To understand the effect of predator saturation, we now perform a numerical test on the model (1) modified with Holling type-II functional response as the interaction term and then compare the result with the original model (1). With this assumption, the model (1) becomes

$$\begin{aligned}
\frac{dE}{dt} &= -\mu E + mS - \theta ES \\
\frac{dL}{dt} &= eSE - \nu L - \frac{a_1 LF}{1+\beta_1 F} - H_1(L) \\
\frac{dS}{dt} &= lL - nS - \frac{a_2 SF}{1+\beta_2 F} - \frac{bSP}{1+\gamma_1 P} - H_2(S) \\
\frac{dF}{dt} &= \frac{a_1 k_1 LF}{1+\beta_1 F} + \frac{a_2 k_2 SF}{1+\beta_2 F} - \frac{cFP}{1+\gamma_2 P} - fF - H_2(F) \\
\frac{dP}{dt} &= \frac{bhSP}{1+\gamma_1 P} + \frac{cgFP}{1+\gamma_2 P} - pP - H_2(P).
\end{aligned} \tag{9}$$

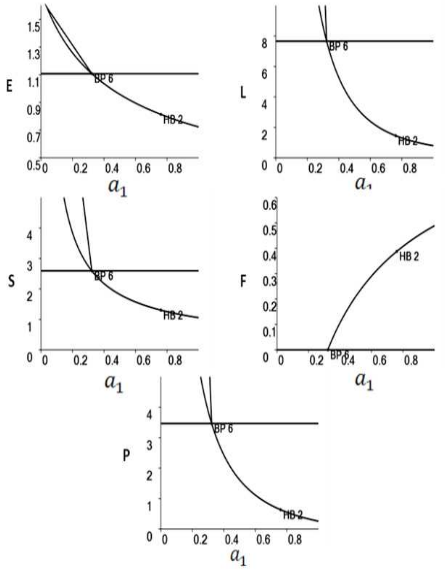

Fig. 13: For the system (9) without harvesting, using AUTO, we plot a bifurcation diagram taking the F's hunting rate a_1 as bifurcation parameter. Here also, like in case of system (1), there is a critical point a_{1c} where Hopf bifurcation occurs.

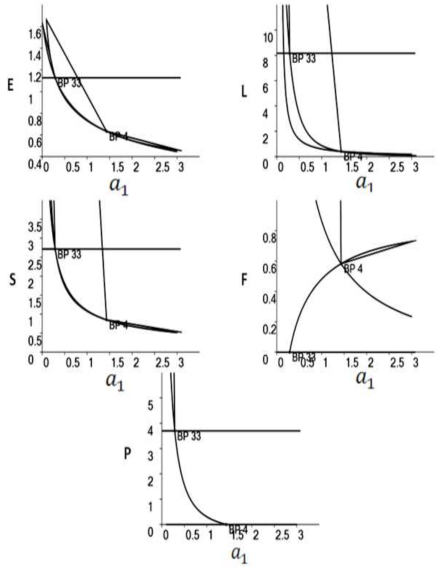

Fig. 14: We plot a bifurcation diagram for the system (9) with a_1 as bifurcation parameter for a higher predator saturation constant $\beta_1 = 0.2$. In this case no Hopf bifurcation occurs.

To capture the robustness of the results we start with the parameter values corresponding to Table 2, with $\beta_1 = 0.1$, $\beta_2 = 0.2$, $\gamma_1 = 0.16$, $\gamma_2 = 0.2$. The system (9) without harvesting shows a stable interior equilibrium steady state. Next we plot the bifurcation diagram with the hunting rate a_1 as bifurcation parameter and observe the same qualitative result, see Figure 13. But when we increase β_1 to 0.2, the effect of predator saturation is immediately observed, with no Hopf-bifurcation occurring in the bifurcation diagram, see Figure 14.

To observe the effect of harvesting we first simulate the system (9) with linear harvesting with $q_1 = 0.07$ and $q_2 = 0.06$ keeping the other parameters fixed and observe periodic oscillations. We again observe periodic solutions when we simulate the system (9) with Holling type II indiscriminate harvesting where $q_1 = 0.07$, $q_2 = 0.06$, $r_1 = 0.6$, $r_2 = 0.9$. These results are in agreement with the results we obtained for the model (1) with a difference in the quantitative value of q_1 and q_2. Finally we perform a bifurcation diagram taking q_2 as the bifurcation parameter, see Figure 15. The structure here is the same and as complicated as we observe in case of the linear interaction term. Thus we may conclude that the result we obtain from the linear interaction term model (1) is robust enough to hold true for other models with different functional responses.

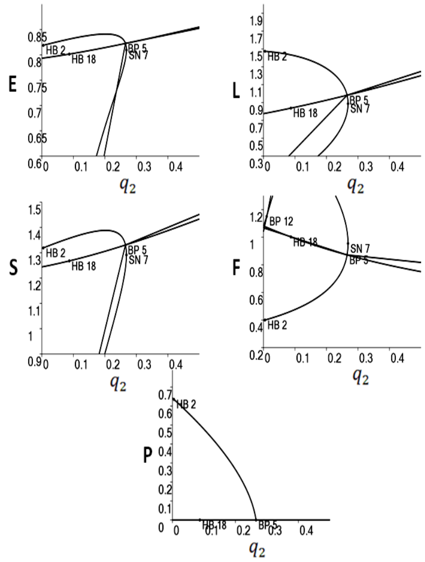

Fig. 15: Bifurcation diagram for the system (9) as function of the effort q_2 for the Holling type II selective harvesting only on adult fishes. BP stands for branch point, i.e. transcritical bifurcation, and HB means Hopf-bifurcation, SN is for saddle node bifurcation.

7 Discussion

Here we considered a five dimensional population model, modelling the situation in the Ligurian sea. This for two main reasons. The Ligurian sea is one of the most important places in the Mediterranean where the catch of *bianchetti* is exerted. Secondly, this catch is not present in all the Mediterranean, since in other cultures perhaps the larvae are less palatable, or due to the local environmental conditions, *bianchetti* are seldom present. The food web accounts for the fact that clupeiforms constitute the prey for mackerels and both are predated by tunnies. The model is set up also to investigate how the possible cannibalism of clupeiforms on their larval offsprings and the latter being subject to capture by selective fishing to get the valuable *bianchetti* affects the long term system dynamics. In the absence of this selective harvesting, the predators-free equilibrium is never stable, so that clupeiforms with their eggs and the *bianchetti* never settle to constant values. The conditional stability of the other two boundary equilibria shows instead that one of the intermediate predators may disappear, under certain conditions on the model parameters. This is certainly not a good implication, since it negatively affects the fishing industry.

Adding also a linear fishing effort in the system, the few analytical results indicate that the model retains the same qualitative behavior as in its the absence, with only quantitative differences, depending on the value of the harvesting rates q_i, $i = 1, 2$.

We consider a set of parameter value representing the real world situation, taken from different literature. In the absence of harvesting, numerical experiments exhibit total extinction if a large enough amount of eggs does not hatch to become larva, in agreement with the analytical result. The same result is observed for the model with high harvesting rates. In the presence of linear harvesting affecting both the larvae and the fishes, the system may go toward extinction even for small values of the harvesting rates of *bianchetti*. With selective fishing performed only on the larvae, there is a high chance of total extinction. Harvesting only on larvae is however a rather unrealistic situation, as fisheries operate continually and therefore the harvest on the adult fish populations in reality never ceases. But such result should warn us of its possible undesirable consequences.

We also considered the Holling type-II form of harvesting, which is more realistic as the Ligurian sea is rather small, so that for increasing fishing efforts and limited resources, diminishing returns are expected.

Assuming to hunt all the populations in the web, the same phenomenon occurring for the linear harvesting arises in this case as well. Changes are here observed only in the values of the fishing rates for which the system behaves differently, i.e. there are quantitative but not qualitative differences with the previous situations.

Therefore, since selective harvesting only on larvae, independently of the specific harvesting policy, may cause total extinction, this kind of selective harvesting must be avoided to prevent extinction of the food web.

With the hypothetical less realistic set of parameter values for endangered ecological situation, we observed that for too low value of the hunting rate a_1 of the mackerels on larvae, the former become extinct. For other situations in nature leading to such outcome, see [36]. This is understandable since larvae and pilchards in this model are the only food source of the mackerels F and the combination of a low a_1 with a low hunting rate a_2 of the mackerels on pilchards S (Table 2) causes the extinction of mackerels. For a moderate value of a_1, all the populations coexist at a stable steady state, bur if the value of this removal rate further increases and crosses a threshold value a_{1c}, a Hopf bifurcation occurs and the populations now coexist by exhibiting persistent periodic oscillations. These remarks enlighten the role of the removal rate a_1 in shaping the dynamics of the food chain.

Under the same ecological endangered situation, when harvesting only on adult fish, the saddle-node bifurcation occurs for values of q_2 and r_2 that are larger if compared with the parameter values q_1 and r_1 at which it occurs when only larvae are harvested. Thus the chance of extinction is higher for the latter case than in the former. In other words, selective harvesting affecting only the adult fishes has a lower environmental impact on the whole food chain compared to selective harvesting performed only on larvae. The chance of collapse of all the populations is smaller, although the coexistence equilibrium may become unstable and limit cycles around it may well arise for smaller value of q_1, see the Hopf bifurcation point HB2 in Figure 11.

To assess the robustness of the model, we performed numerical simulations for the same model in which the Holling type II functional response is used. The result obtained for the original model holds true also for the Holling type II functional response when the predator saturation is kept below a certain value. For higher values of predator saturation, the system never reaches periodic coexistence, rather switches between stable coexistence and extinction.

The fishing of the different species of both the blue fish, in general, and *bianchetti*, in particular, can thus affect the various links of the natural trophic chain, with consequences which could become remarkable or actually irreparable. In spite of its economic palatability, then it seems that fishing of *bianchetti* should be avoided to preserve the long term survival of the food chain resource.

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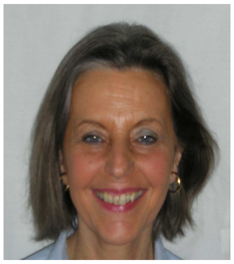

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