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On Almost Feebly Continuous Functions in Bitopological Spaces

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Abstract: As a generalization of feebly p-continuous functions due to Jelic$^{'}$ in 1990, we introduce the notion of almost feebly continuous functions in bitopological spaces and obtain several characterizations and some properties of pairwise almost feebly continuous functions.

Keywords: Feebly open, α-open, (i,j)- feebly open, pairwise almost feebly continuous, bitopolological spaces.
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1 Introduction

The study of bitopological spaces was first initiated by J.C. Kelly [5] and thereafter a large number of papers have been done to generalize the topological concepts to bitopological setting.Maheshwari et al. [8] introduced and studied the notion of almost feebly continuous functions between topological spaces as a generalization of almost continuity in the sense of Singal [14] and feeble continuity [9]. In 1988, Noiri [12] defined the notion of almost α-continuous functions and proved that the concept of almost α-continuity and almost feeble continuity are equivalent. Quite recently, Duszynski, Rajesh and Balabigai [2] have defined the concept of b-continuous functions in bitopological spaces. The purpose of the present paper is to extend the concept of almost feebly continuous functions to the setting of bitopological spaces. The fundamental properties of pairwise almost feeble continuity are investigated as a generalization of feebly p- continuous due to *Jelic*$^{'}$ [4] and pairwise almost continuous due to Bose and Sinha [1].

2 Preliminaries

Throughout the present paper, the spaces (X,τ_1,τ_2) and (Y,σ_1,σ_2) always mean bitopological spaces on which no separation axioms are assumed unless explicitly mentioned. For a subset A of X, $\tau_i - Cl(A)$ (resp. $\tau_i - Int(A)$) denotes the closure (resp. interior) of A with respect to τ_i for $i = 1,2$. However, $\tau_i - Cl(A)$ and $\tau_i - Int(A)$ are briefly denoted by $Cl_i(A)$ and $Int_i(A)$, respectively, for $i = 1,2$ if there is no possibility of confusion.

Definition 1.*A subset A of a bitopological space (X, τ_1,τ_2) is said to be:*
(a)(τ_i,τ_j)- regular-open [3] (briefly (i,j)-regular open) if $A = Int_i(Cl_j(A))$;
(b)(τ_i,τ_j)- semi-open [10] (briefly (i,j)-semi-open) if $A \subset Cl_j(Int_i(A))$;
(c)(τ_i,τ_j)- preopen [4] (briefly (i,j)-preopen) if $A \subset Int_i(Cl_j(A))$;
(d)(τ_i,τ_j)-α-open [4] (briefly $(i,j)\alpha$-open) if $A \subset Int_i(Cl_j(Int_i(A)))$, where $i \neq j, i, j = 1,2$.

Definition 2.*A subsetA of a bitopological space (X, τ_1,τ_2) is said to be:*
(a)(τ_i,τ_j)-feebly open [4] (briefly (i,j)-feebly open) if there exists a τ_i-open set U such that $U \subset A \subset (j,i)-sCl(U)$ where $(j,i)-sCl(U)$ denotes the semi closure [10] of U in topology τ_j, for $i \neq j$; and $i,j = 1,2$.
(b)(τ_i,τ_j)-δ- open [6] (briefly (i,j)-δ- open) if for each $x \in A$, there exists an (i,j)-regular open set W such that $x \in W \subset A$, where $i \neq j$; and $i,j = 1,2$.

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The complement of an (i,j)-regular open (resp. (i,j)-feebly open,(i,j)-δ-open) set is said to be (i,j)-regular closed (resp. (i,j)-feebly closed,(i,j)-δ-closed). The collection of all (i,j)-regular open (resp. (i,j)-feebly open,(i,j)-δ-open, (i,j)-preopen, (i,j)-α-open)sets of a space (X,τ_1,τ_2)will be denoted by $(i,j)-RO(X)$(resp. $(i,j)-FO(X)$,(i,j)-δ $O(X)$, $(i,j)-PO(X)$, $(i,j)-\alpha\ (X)$).

A subset A is said to be pairwise feebly open (resp. pairwise feebly closed) if it is $(1,2)$-feebly open (resp. $(1,2)$-feebly closed) and $(2,1)$-feebly open (resp. $(2,1)$-feebly closed). Pairwise regular open sets, pairwise-δ-open sets, pairwise regular closed sets and pairwise-δ-closed sets are similarly defined.

Definition 3.*Let$A$ be a subset of a bitopological space(X,τ_1,τ_2), the intersection of all (i,j)-feebly closed (resp. (i,j)- δ closed)sets of (X,τ_1,τ_2)containing A is called the (τ_i,τ_j)-feeble closure (resp. (τ_i,τ_j)-δ- closure) of A and is denoted by (τ_i,τ_j)-$fCl(A)$ (briefly (i,j)-$fCl(A)$) (resp. (τ_i,τ_j)-$\delta Cl(A)$ (briefly $(i,j)-\delta Cl(A)$)).*

Definition 4.*A function g:$(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be pairwise continuous [13] (resp. pairwise open) if the induced functions $g:(X,\tau_1)\to(Y,\sigma_1)$ and $g:(X,\tau_2)\to(Y,\sigma_2)$ are both continuous (resp. open).*

Definition 5.*A function g:$(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be feebly p- continuous [4] (resp. pairwise α-continuous [9]) if the inverse image of eachσ_i-open set of Y is (i,j)-feebly open (resp. (i,j)-α-open) in (X,τ_1,τ_2), where $i\neq j$ and $i,j=1,2$.*

Definition 6.*A function g:$(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be pairwise almost continuous [1] if for each $x\in X$ and each σ_i-open neighbourhood V of $f(x)$, there exists a τ_i-open neighbourhood U of x such that $g(U)\subset Int_i(Cl_j(V))$ for $i\neq j$ and $i,j=1,2$.*

Lemma 1.*(Jelic$'$ [4]) Let A be a subset of a bitopological space (X,τ_1,τ_2). Then A is (i,j)-feebly open if and only if it is (i,j)-α-open for $i\neq j$ and $i,j=1,2$.*

3 Characterizations

Definition 7.*A function g:$(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be pairwise almost feebly continuous (briefly pairwise a. f. c.) (resp. pairwise almost-α-continuous (briefly pairwise a. α. c.))if for each $x\in X$ and each(i,j)-regular open set V of Y containing $g(x)$, there exists an (i,j)-feebly open (resp. (i,j)-α-open) set U of X containing x such that $g(U)\subset V$ for $i\neq j$ and $i,j=1,2$.*
By Lemma (2.7), an (i,j)- feebly open set is equivalent to an (i,j)- α-open set and hence pairwise a. f. c. is equivalent to pairwise a. α. c.

Theorem 1.*For a function g:$(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$, the following are equivalent:*
(a) g is pairwise a. f. c.;
(b) For each $x\in X$ and each σ_i-open set V of Y containing $g(x)$, there exists an (i,j)-feebly open set U of X containing x such that $g(U)\subset Int_i(Cl_j(V))$ for $i\neq j$ and $i,j=1,2$.
(c) $g^{-1}(F)$ is (i,j)-feebly closed in X for each(i,j)-regular closed set F of Y;
(d) $g^{-1}(V)$ is (i,j)-feebly open in X for each (i,j)-regular open set V of Y.

*Proof.*This is shown by the usual techniques and is thus omitted.

Theorem 2.*For a function g:$(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$, the following are equivalent:*
(a) g is pairwise a. f. c.;
(b) $g((i,j)-fCl(A))\subset(i,j)-\delta Cl(g(A))$ for each subset A of X;
(c) $(i,j)-fCl(g^{-1}(B))\subset g^{-1}((i,j)-\delta Cl(g(B)))$; for each subset B of Y;
(d) $g^{-1}(F)$ is (i,j)-feebly closed in X for each (i,j)-δ-closed subset F of Y;
(e) $g^{-1}(V)$ is (i,j)-feebly open in X for each (i,j)-δ-open subset V of Y.

Proof.(a) $\Rightarrow(b)$: Let A be any subset of X. Since $(i,j)-\delta Cl(g(A))$ is $(i,j)-\delta$-closed in Y, it is denoted by $\cap\{F_\alpha:F_\alpha$ is (i,j) regular closed in Y, $\alpha\in\bigtriangledown\}$. We have $A\subset g^{-1}((i,j)-\delta Cl(g(A)))=\cap\{g^{-1}(F_\alpha):(F_\alpha)$is (i,j) regular closed in Y, $\alpha\in\bigtriangledown\}$. Therefore, by theorem 3.1 $g^{-1}((i,j)-\delta Cl(g(A)))$ is (i,j)- feebly closed in X and hence $(i,j)-fCl(A)\subset g^{-1}((i,j)-\delta Cl(g(A)))$. This implies that $g((i,j)-fCl(A))\subset(i,j)-\delta Cl(g(A))$.

(b) $\Rightarrow$(c): Let B be any subset of Y. We have $g((i,j)-fCl(g^{-1}(B))\subset(i,j)-\delta Cl(g(g^{-1}(B)))\subset(i,j)-\delta Cl(B)$ and hence $(i,j)-fCl(g^{-1}(B))\subset g^{-1}((i,j)-\delta Cl(B))$.

(c) $\Rightarrow$(d): Let F be any $(i,j)-\delta-$closed subset of Y. Then we have $(i,j)-fCl(g^{-1}(F))\subset g^{-1}((i,j)-\delta Cl(F))=g^{-1}(F)$ and hence $g^{-1}(F)$ is (i,j)- feebly closed in X.

(d) $\Rightarrow$(e): Let $V\in(i,j)-\delta O(Y)$, then by (d) we have $g^{-1}(Y-V)=X-g^{-1}(V)$ is (i,j)- feebly closed in X and hence $g^{-1}(V)\in(i,j)-FO(X)$.

(e) $\Rightarrow$(a): Let $V\in(i,j)-RO(Y)$. Since V is $(i,j)-\delta$-open in Y, then $g^{-1}(V)$ is (i,j) feebly open in X and hence by theorem 3.2 g is pairwise a. f. c.

The following theorem characterizes pairwise a. f. c. functions whose proof is immediate from Lemma 2.7 and Theorem 2.1 of [1].

Theorem 3.*A function g:$(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pairwise a. f. c. if and only if g:$(X,\tau_1^\alpha,\tau_2^\alpha)\to(Y,\sigma_1,\sigma_2)$ is pairwise almost continuous where $\tau_i^\alpha=(i,j)-\alpha(X)$ for $i\neq j$ and $i,j=1,2$.*

Lemma 2.*For a bitopological space (X,τ_1,τ_2), if $U\in\tau_i$, then $Int_i(Cl_j(U))=(i,j)-sCl(U)$, for $i\neq j$ and $i,j=1,2$.*

*Proof.*The first inclusion is clear. To prove the second inclusion, let $x \notin Int_i(Cl_j(U))$. Then $x \in X - Int_i(Cl_j(U)) = Cl_j(Int_i(X - U))$ and $Cl_j(Int_i(X - U))$ is (i,j) semi- open. Since $U \in \tau_i, U \subset Int_i(Cl_j(U))$ and thus, $\phi = U \cap (X - Int_i(Cl_j(U)) = U \cap Cl_j(Int_i(X-U))$. Thus, we have $x \notin (i,j) - sCl(U)$. Hence the lemma holds.

Theorem 4.*A function $g:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is pairwise a. f. c. if and only if for each $x \in X$ and each $V \in \sigma_i$ containing $g(x)$, there exists an (i,j)-feebly open set U of X containing x such that $g(U) \subset (i,j) - sCl(U)$, for $i \neq j$ and $i,j = 1,2$.*

*Proof.*Let $V \in \sigma_i$ containing $g(x)$, then by Theorem 3.2, there exists an (i,j)-feebly open set U of X containing x such that $g(U) \subset Int_i(Cl_j(V))$. By lemma 3.5 we have $g(U) \subset (i,j) - sCl(V)$.
Conversely, let $V \in \sigma_i$ containing $g(x)$, there exists an (i,j)-feebly open set U of X containing x such that $g(U) \subset (i,j) - sCl(V) = Int_i(Cl_j(V))$, by lemma 3.5. Therefore, by Theorem 3.2, g is pairwise a. f. c.

*Remark.*From Definition 2.4, 2.5 and 3.1, it follows immediately that we have the following implications:
pairwise continuity $\to$feebly p- continuity $\to$ pairwise almost feebly continuity.
However, none of these implications can be reversed, which we can see in the following examples.

Example 1.(Jelic$'$ [3])Let $X = \{a,b,c\}$, $\tau_1 = \{\phi, X, \{a\}, \}$, $\tau_2 = \{\phi, X, \{a,b\}\}$, $Y = \{1,2\}$, $\sigma_1 = \{\phi, Y, \{1\}\}$, $\sigma_2 =$ be the indiscrete topology and $g:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ such that $g(c) = 2$ and $g(a) = g(b) = 1$. Then g is feebly p-continuous but not pairwise continuous.

*Example 2.*Let $X = \{a,b,c\}$, $\tau_1 = \{\phi, X, \{b\}, \{c\}, \{b,c\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{b,c\}\}$, $Y = \{1,2,3\}$, $\sigma_1 = \{\phi, Y, \{2\}, \{1,2\}\}$ and $\sigma_2 = \{\phi, Y, \{2\}, \{3\}, \{2,3\}\}$. Define $g:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ by $g(a) = 2 = g(c)$, and $g(b) = 1$. Then g is pairwise a.f.c. but not feebly p-continuous, since $\{2\} \in \sigma_1$ and $g^{-1}(\{2\})$ is not $(1,2)$-feebly open in X.

Lemma 3.*Let A and B be a subset of a bitopological space $(X,\tau_1,\tau_2$).Then the following hold.*
(a) If $A \in (i,j) - PO(X)$ and $B \in (i,j) - FO(X)$, then$A \cap B \in (i,j) - FO(A)$, for $i \neq j$ and $i,j = 1,2$.
(b) If $A \in (i,j) - FO(B)$ and $B \in (i,j) - FO(X)$, then $A \in (i,j) - FO(X)$, for $i \neq j$ and $i,j = 1,2$.

Proof.(a) Follows from Lemma 2.1 and Lemma 3.11 of [7].
(b)Follows from Lemma 2.1 and Lemma 1.9 of [4].

Theorem 5.*If a function $g:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is pairwise a. f. c. Then the restriction $g|X_0 : (X_0, \tau_1|X_0, \tau_2|X_0) \to (Y,\sigma_1,\sigma_2)$ is pairwise a. f. c. for $(i,j)-$ preopen subset X_0 of X.*

*Proof.*Let $V \in (i,j) - RO(Y)$. Since g is a pairwise a. f. c. then $g^{-1}(V) \in (i,j) - FO(X)$ and by Lemma 3.10, $(g|X_0)^{-1}(V) = g^{-1}(V) \cap X_0$ is (i,j)-feebly open in X_0. This shows that $(g|X_0)$ is pairwise a. f. c.

Theorem 6.*Let $g:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a function and $\{U_\alpha : \alpha \in \bigtriangledown\}$ a cover of X by (i,j)-feebly open sets of X. If the restriction $g|U_\alpha : (U_\alpha, \tau_1|U_\alpha, \tau_2|U_\alpha) \to (Y,\sigma_1,\sigma_2)$ is pairwise a. f. c. for each $\alpha \in \bigtriangledown$, then g is pairwise a.f.c.*

*Proof.*Let V be any (i,j)-regular open set in Y. Then we have $g^{-1}(V) = X \cap g^{-1}(V) = \cup\{U_\alpha \cap g^{-1}(V) : \alpha \in \nabla\} = \cup\{(g|U_\alpha)^{-1}(V) : \alpha \in \nabla\}$. Since $g|U_\alpha$ is pairwise a. f. c., then $(g|U_\alpha)^{-1}(V)$ is $(\tau_i|U_\alpha, \tau_j|U_\alpha)$-feebly open in U_α for each $\alpha \in \nabla$. It follows from Lemma 3.10 that $(g|U_\alpha)^{-1}(V) \in (i,j) - FO(X)$ for each $\alpha \in \nabla$ and hence $g^{-1}(V) \in (i,j) - FO(X)$. Therefore, g is pairwise a. f. c.

Corollary 1.*Let $g:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a function and $X = \cup\{U_\alpha \in \tau_1 \cap \tau_2 : \alpha \in \nabla\}$. Then g is pairwise a. f. c. if and only if the restriction $g|U_\alpha : (U_\alpha, \tau_1|U_\alpha, \tau_2|U_\alpha) \to (Y,\sigma_1,\sigma_2)$ is pairwise a. f. c. for each $\alpha \in \nabla$.*
Let $\{(X_\alpha, \tau_1\{\alpha\}, \tau_2\{\alpha\} : \alpha \in \bigtriangledown\}\}$ be a family of bitopological spaces. Let (X,τ_1,τ_2) be the product space, where $X = \Pi X_\alpha$, and τ_i denotes the product topology of $\{\tau_i(\alpha) : \alpha \in \nabla\}$ for $i = 1,2$.

Lemma 4.*(Nasef and Noiri [11]) Let A_α be a nonempty subset of X_α for each $\alpha = \alpha(1), \alpha(2), ..., \alpha(n)$. Then $A = \Pi$*
$\{A_{\alpha(k)} : 1 \leq k \leq n\} \times \Pi\{X_\alpha : \alpha \neq \alpha((k), 1 \leq k \leq n\}$ is (τ_i,τ_j)- feebly open if and only if $\{A_{\alpha(k)}$ is $(\tau_i, (\alpha(k)),$
$\tau_i(\alpha(k)))$-feebly open in $X_{\alpha(k)}$ for each $k = 1,2,...,n$. Let $\{(X_\alpha, \tau_1(\alpha), \tau_2(\alpha) : \alpha \in \nabla\}$
and $\{(Y_\alpha, \sigma_1(\alpha), \sigma_2(\alpha) : \alpha \in \nabla\}$ be two arbitrary families of bitopological spaces with the same set of indices.
Let g_α: $(X_\alpha, \tau_1(\alpha), \tau_2(\alpha)) \to (Y_\alpha, \sigma_1(\alpha),$
$\sigma_2(\alpha))$ be a function for each$\alpha \in \nabla$. And let $g:(X,\tau_1,\tau_2)$ $\to$ (Y,σ_1,σ_2) be the product function defined by $g(\{x_\alpha\}) = \{g_\alpha(x_\alpha)\} = \Pi X_\alpha$ for each$\{X_\alpha\} \in X$, where τ_i and σ_i denote the product topologies for $i = 1,2$.

Theorem 7.*The product function $g:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is a pairwise a. f. c. if and only if g_α: $(X_\alpha, \tau_1(\alpha), \tau_2(\alpha)) \to (Y_\alpha, \sigma_1(\alpha), \sigma_2(\alpha))$ is pairwise a. f. c. for each $\alpha \in \nabla$.*

*Proof.*Necessity. Let α be an arbitrary fixed index and V_α a $(\sigma_i(\alpha), \sigma_j(\alpha))$- regular open set of Y_α. Then $V_\alpha \times \Pi\{Y_\beta : \beta \in \nabla - \{\alpha\}\}$ is (σ_i, σ_j)-regular open in ΠY_α and hence $g^{-1}(V_\alpha \times \Pi Y_\beta) = g_\alpha^{-1}(V_\alpha) \times \Pi Y_\beta$ is (τ_i,τ_j)- feebly open in ΠX_α and by Lemma 3.14 we have $g_\alpha^{-1}(V_\alpha)$ is $(\tau_i(\alpha), \tau_j(\alpha))$- feebly open in X_α. Therefore, g_α is pairwise a. f. c.
Sufficiency. Let $\{x_\alpha\}$ be any point of ΠX_α and W be a

(σ_i,σ_j)- regular open in ΠY_α containing $f(\{x_\alpha\})$. There exists a finite subset ∇_0 of ∇ such thatV_λ is $(\sigma_i(\lambda),\sigma_j(\lambda))$- regular open in Y_λ for each $\lambda \in \nabla_0$ and $\{g_\alpha(x_\alpha)\} \in \Pi\{V_\lambda : \lambda \in \nabla_0\} \times \Pi\{Y_\beta : \beta \in \nabla - \nabla_0\} \subset W$. For each $\lambda \in \nabla_0$, there exists a $(\tau_i(\lambda),\tau_j(\lambda))$- feebly open set U_λ of X_λ containing x_λ such that $g_\lambda(U_\lambda) \subset V_\lambda$, By Lemma 3.14 $U = \Pi\{U_\lambda : \lambda \in \nabla_0\} \times \Pi\{X_\beta : \beta \in \nabla - \nabla_{0\}}$ is $(\tau_i,\tau_j$-feebly open in ΠX_α containing $\{x_\alpha\}$ and $g(U) \subset W$. This shows that g is pairwise a. f. c.
Recall that a function g:$(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is called pairwise R-map[5] if the inverse image of each (i,j)-regular open set of (Y,σ_1,σ_1) is (i,j)-regular open in (X,τ_1,τ_2) for $i \neq j$ and $i,j = 1,2$.

Theorem 8.*Let* $\{(Y_\alpha,\sigma_1(\alpha),\sigma_1(\alpha)) : \alpha \in \nabla\}$ *be a family of bitopological spaces. If* g:$(X,\tau_1,\tau_2) \to (\Pi Y_\alpha,\Pi\sigma_1(\alpha),\Pi\sigma_2(\alpha))$ *is pairwise a. f. c. then* $P_\alpha \circ g$:$(X,\tau_1,\tau_2) \to (Y_\alpha,\sigma_1(\alpha),\sigma_2(\alpha))$ *is pairwise a. f. c. for each* $\alpha \in \nabla$*, where,* P_α *is the projection of* $(\Pi Y_\alpha,\Pi\sigma_1(\alpha),\Pi\sigma_2(\alpha))$ *onto* $(Y_\alpha,\sigma_1(\alpha),\sigma_2(\alpha))$.

*Proof.*Let α be an arbitrary fixed index and V_α be any $(\sigma_i(\alpha),\sigma_j(\alpha))$-regular open set of Y_α. Since P_α is pairwise continuous and pairwise open, it is a pairwise R-map and hence $p_\alpha^{-1}(V_\alpha)$ is a $(\Pi\sigma_i(\alpha),\Pi\sigma_j(\alpha))$-regular open in $\Pi Y\alpha$.By Theorem 3.2
$g^{-1}(p_\alpha^{-1}(V_\alpha)) = (p_\alpha \circ g)^{-1}(V_\alpha)$ is (τ_i,τ_j)-feebly open in (X,τ_1,τ_2). Hence, $(p_\alpha \circ g)$ is pairwise a. f. c. for each $\alpha \in \nabla$.

Theorem 9.*Let h*:$(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *be a function and* g:$(X,\tau_1,\tau_2) \to (X \times Y,\Omega_1,\Omega_2)$ *be the graph function given by* $g(x) = (x,h(x))$ *for each* $x \in X$ *and* $\Omega_k = \tau_k \times \sigma_k$*, for* $k = 1,2$. *Then h is pairwise a. f. c. if and only if g is pairwise a. f. c..*

*Proof.*This is an immediate consequence of Theorem 3.16.
Recall that a function g:(X,τ_1,τ_2) $\to (Y,\sigma_1,\sigma_2)$ is called pairwise α-irresolute [6] if the inverse image of each $(i,j)-\alpha-$open of Y is an $(i,j)-\alpha-$open set in (X,τ_1,τ_2) for $i \neq j$ and $i,j = 1,2$.

Theorem 10.*Let g*:$(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *and h:* $(Y,\sigma_1,\sigma_2) \to (Z,\theta_1,\theta_2)$ *be functions. Then the composition* $h \circ g : (X,\tau_1,\tau_2) \to (Z,\theta_1,\theta_2)$ *is pairwise a. f. c. if h and g satisfy one of the following conditions:*
(a) g is pairwise a. f. c. and h is pairwise R-map.
(b) g is feebly p-continuous and h is pairwise almost continuous;
(c) g is pairwise α*-irresolute and h is pairwise a.* α*. c.*

*Proof.*It is straightforward and is thus omitted.

4 Conclusion

The study of bitopological spaces generalized most concepts of near open sets and near continuous functions which is applicable in most areas of pure and applied mathematics. This study would open up the academic flood gates and new vistas in the field of fuzzy topology and multi functions for further research studies.

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