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New Exact Solitary Wave Solutions to the Coupled Nonlinear System of Schrodinger Equations by Using the HBM

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Abstract: In this present study, we aim to use of the new extension of the generalized and improved homogeneous balance method for constructing new structure of rich class of exact traveling wave solutions of nonlinear evolution equations by using the Maple package. To demonstrate the novelty and motivation of the proposed method, we implement it to the coupled nonlinear system of Schrodinger equations.It is shown that the method provides a powerful mathematical tool for solving nonlinear evolution equations in mathematical physics.

Keywords: coupled nonlinear system of Schrodinger equations, homogeneous balance method, traveling wave solutions.

1 Introduction

The importance of obtaining the exact solutions, if available, of those nonlinear equations facilitates the verification of numerical solutions and aids in the stability analysis of solutions. The investigation of exact solutions of nonlinear equations plays an important role in the study of nonlinear physical phenomena. Recently, many approaches have been suggested to solve the nonlinear equations, such as the spectral collocation method [1,2,3,4,5], the homogeneous balance method [6,7], the F-expansion method [8]. It provides the generalized solitary solutions and periodic solutions, as well. Taking advantage of the generalized solitary solutions, we can recover some known solutions obtained by existing methods.

In this paper we extend the homogeneous balance method to a class of nonlinear evolution equations with imaginary number and modulus. We consider the coupled (2+1)-dimensional nonlinear system of Schrodinger equations as

$$\begin{cases} iE_t - E_{xx} + E_{yy} + |E|^2 E - 2NE = 0, \\ N_{xx} - N_{yy} - \left(|E|^2\right)_{xx} = 0, \end{cases} \tag{1}$$

where$E(x,y,t)$and $N(x,y,t)$are complex-valued functions. Nonlinear partial differential equation systems of the type given by (1) play an important role in atomic physics, and the functions $E(x,y,t)$ and $N(x,y,t)$have different physical meanings in different branches of physics [9,10,11,12,13,14,15,16] Well-known applications are, for instance, in fluid dynamics [9]and plasma physics [11]. In the context of water waves, $E(x,y,t)$is the amplitude of a surface wave packet while $N(x,y,t)$ is the velocity potential of the mean flow interacting with the surface waves [10]. However, in the hydrodynamic context, $E(x,y,t)$ is the envelope of the wave packet and $N(x,y,t)$ is the induced mean flow [9]. In addition, equations (1) are relevant in a number of different physical contexts, describing slow modulation effects of the complex amplitude$N(x,y,t)$, due to a small nonlinearity, on a monochromatic wave in a dispersive medium.

2 Basic definitions for the homogeneous balance method

For a given partial differential equation

$$G(u, u_x, u_t, u_{xx}, u_{tt}, \ldots.), \tag{2}$$

Our method mainly consists of four steps:

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Step 1: We seek complex solutions of Eq. (2) as the following form:

$$u = u(\xi), \quad \xi = ik(x - ct), \tag{3}$$

Where k and c are real constants. Under the transformation (3), Eq. (2) becomes an ordinary differential equation

$$N(u, iku', -ikcu', -k^2u'', ...), \tag{4}$$

Where$u' = \frac{du}{d\xi}$.
Step 2: We assume that the solution of Eq. (4) is of the form

$$u(\xi) = \sum_{i=0}^{n} a_i \phi^i(\xi), \tag{5}$$

Where $a_i (i = 1, 2, .., n)$ are real constants to be determined later and ϕsatisfy the Riccati equation

$$\phi' = a\phi^2 + b\phi + c \tag{6}$$

Eq. (6) admits the following solutions:
Case1: Let $\phi = \sum_{i=0}^{n} b_i \tanh^i \xi$,Balancing $\phi'$with ϕ^2 in Eq.(6) gives$m = 1$so

$$\phi = b_0 + b_1 \tanh \xi, \tag{7}$$

Substituting Eq. (7) into Eq. (6), we obtain the following solution of Eq. (6)

$$\phi = -\frac{1}{2a}(b + 2\tanh \xi), \quad ac = \frac{b^2}{4} - 1. \tag{8}$$

Case2:when$a = 1, b = 0$,the Riccati Eq. (6) has the following solutions

$$\begin{aligned} &\phi = -\sqrt{-c}\tanh\left(\sqrt{-c}\xi\right), \quad c < 0 \\ &\phi = -\tfrac{1}{\xi}, \quad c < 0 \\ &\phi = \sqrt{c}\tan\left(\sqrt{c}\xi\right), \quad c > 0. \end{aligned} \tag{9}$$

Case3:We suppose that the Riccati Eq. (6) have the following solutions of the form:

$$\phi = A_0 + \sum_{i=1}^{n} \sinh^{i-1}(A_i \sinh\omega + B_i \cosh\omega), \tag{10}$$

Where $\frac{d\omega}{d\xi} = \sinh\omega$ or$\frac{d\omega}{d\xi} = \cosh\omega$. It is easy to find that $m = 1$by Balancing ϕ'withϕ^2. So we choose

$$\phi = A_0 + A_1 \sinh\omega + B_1 \cosh\omega, \tag{11}$$

Where $\frac{d\omega}{d\xi} = \sinh\omega$, we substitute (11) and $\frac{d\omega}{d\xi} = \sinh\omega$, into (6) and set the coefficients of $\sinh^i\omega, \cosh^i\omega\,(i = 0, 1, 2; j0, 1)$ to zero. We obtain a set of algebraic equations and solving these equations we have the following solutions

$$A_0 = -\frac{b}{2a}, A_1 = 0, B_1 = \frac{1}{2a} \tag{12}$$

Where $c = \frac{b^2-4}{4a}$ and

$$A_0 = -\frac{b}{2a}, A_1 = \pm\sqrt{\frac{1}{2a}}, B_1 = \frac{1}{2a} \tag{13}$$

Where$c = \frac{b^2-1}{4a}$. To $\frac{d\omega}{d\xi} = \sinh\omega$ we have

$$\sinh\omega = -\operatorname{csc}h\xi, \cosh\omega = -\coth\xi \tag{14}$$

From (12)–(14), we obtain

$$\phi = -\frac{b + 2\coth\xi}{2a} \tag{15}$$

Where $c = \frac{b^2-4}{4a}$ and

$$\phi = -\frac{b \pm \operatorname{csc}h\xi + \coth\xi}{2a} \tag{16}$$

Where$c = \frac{b^2-1}{4a}$.
Step3. Substituting (7-16) into (4) along with (6), then the left hand side of Eq. (4) is converted into a polynomial in$F(\xi)$; equating each coefficient of the polynomial to zero yields a set of algebraic equations.
Step4. Solving the algebraic equations obtained in step 3, and substituting the results into (5), then we obtain the exact traveling wave solutions for Eq. (2).
Remark 1: If$c = 0$, then the Riccati Eq. (6) reduces to the Bernoulli equation

$$\phi' = a\phi^2 + b\phi, \tag{17}$$

The solution of the Bernoulli Eq. (17) can be written in the following form [23]:

$$\phi = b \times \left[\frac{\cosh[b(\xi+\xi_0)] + \sinh[b(\xi+\xi_0)]}{1 - a\cosh[b(\xi+\xi_0)] - a\sinh[b(\xi+\xi_0)]}\right] \tag{18}$$

Where $\xi_0$is integration constant.
Remark 2: If$b = 0$, then the Riccati Eq. (6) reduces to the Riccati equation

$$\phi' = a\phi^2 + c$$

Which the equation aboveis the special case of the Riccati Eq. (6).

Remark 3: Also, the Riccati Eq. (6) admits the following exact solution [**?**]:

$$\phi = -\frac{b}{2a} - \frac{\theta}{2a}\tanh\left(\frac{\theta}{2}\xi\right) + \frac{\operatorname{sec}h\left(\frac{\theta}{2}\xi\right)}{C\cosh\left(\frac{\theta}{2}\xi\right) - \frac{2a}{\theta}\sinh\left(\frac{\theta}{2}\xi\right)}, \tag{19}$$

Where $\theta^2 = b^2 - 4ac$ and C is a constant of integration.

3 Application to the coupled nonlinear system of Schrodinger equations:

To obtain the exact solutions of (1), we use the transformations

$$\begin{aligned} &E(x,y,t) = u(\xi)\exp(i\eta),\\ &N(x,y,t) = v(\xi)\\ &\xi = k(x+ly+2(\alpha-\beta l)t),\\ &\eta = \alpha x+\beta y+\gamma t, \end{aligned} \tag{20}$$

Where$k,l,\alpha$and $\beta$are constants to be determined. Note that $\xi$and $\eta$are travellingwave variables, not necessarily in the same direction. That is, $\xi$and $\eta$are independent linear functions of $x,y$and$t$. Then $u$and v are assumed to be rational functions of$\exp(\xi)$. When $u$is positive real, $u$is the modulus of the complex function$E$, and $N$is the argument. The modulus and argument are travelling waves but the two waves may be in different directions.
From (1), we may obtain the system of ordinary differential equations

$$k^2l^2u'' + \left(\alpha^2-\beta^2-\gamma\right)u + u^3 - 2uv = 0, \tag{21}$$

$$\left(1+l^2\right)v'' - \left(u^2\right)'' = 0. \tag{22}$$

Integrating (22) with respect to $\xi$and setting the constants of integration equal to zero yields

$$v = \frac{u^2}{1+l^2} \tag{23}$$

Substituting (23) into (21), we obtain

$$\begin{aligned} &k^2\left(l^2-1\right)u'' +\\ &\left(\alpha^2-\beta^2-\gamma\right)u + \tfrac{l^2-1}{l^2+1}u^3 = 0. \end{aligned} \tag{24}$$

For the solutions of Eq. (24), with the aid of homogeneous balance method we make the following ansatz

$$u(\xi) = \sum_{i=0}^{n} a_i\phi^i(\xi), \tag{25}$$

where $a_i$are all real constants to be determined, n is a positive integer which can be determined by balancing the highest order derivative term with the highest order nonlinear term, then gives$n = 1$. Therefore, we may choose

$$u(\xi) = a_1\phi + a_0 \tag{26}$$

Substituting (26) along with (6) in Eq. (24) and then setting the coefficients of $\phi^j(j = 0,1,2,3,4,5)$to zero in the resultant expression, we obtain a set of algebraic equations and solving these equations with the aid of Maple we have

$$\begin{aligned} &a_1 = \tfrac{\sqrt{2}}{4}\sqrt{-k^2(l^2+1)}\times\\ &\left[\tfrac{k^2b^2l^6-11k^2b^2l^4+11k^2b^2l^2}{ck^2(l^2+1)^2(l^2-1)} - \right.\\ &\tfrac{k^2b^2+2\alpha^2l^4+4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)} +\\ &\tfrac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)} +\\ &\left.\tfrac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\right], \end{aligned} \tag{27}$$

$$\begin{aligned} &a = \tfrac{1}{4}\tfrac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)} +\\ &\tfrac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{4ck^2(l^2+1)^2(l^2-1)}\\ &\tfrac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)} +\\ &\tfrac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)} \end{aligned} \tag{28}$$

$$a_0 = \frac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}, \tag{29}$$

Case 1: By substituting (27-29) in (26) along with (8) we have

$$\begin{aligned} &u(x,y,t) = -\tfrac{\sqrt{2}}{2}\sqrt{-k^2(l^2+1)}\times\\ &(b+2\tanh k(x+ly+2(\alpha-\beta l)t)) +\\ &\tfrac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}. \end{aligned}$$

And from (23) $v$obtained as follow

$$\begin{aligned} &v(x,y,t) = \tfrac{1}{1+l^2}\times\\ &\left[-\tfrac{\sqrt{2}}{2}\sqrt{-k^2(l^2+1)}\times\right.\\ &(b+2\tanh k(x+ly+2(\alpha-\beta l)t))\\ &\left.+\tfrac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\right]^2 \end{aligned}$$

So from (20) we obtain solutions of Eq. (1)

$$\begin{aligned} &E(x,y,t) = \left[-\tfrac{\sqrt{2}}{2}\sqrt{-k^2(l^2+1)}\times\right.\\ &(b+2\tanh k(x+ly+2(\alpha-\beta l)t))\\ &\left.+\tfrac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\right]\times\\ &\exp(i(\alpha x+\beta y+\gamma t)). \end{aligned}$$

And

$$\begin{aligned} &N(x,y,t) = \tfrac{1}{1+l^2}\times\\ &\left[-\tfrac{\sqrt{2}}{2}\sqrt{-k^2(l^2+1)}\times\right.\\ &(b+2\tanh k(x+ly+2(\alpha-\beta l)t))\\ &\left.+\tfrac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\right]^2 \end{aligned}$$

Case 2:

By substituting (27-29) in (26) along with (9) we have solution of the Eq. (1) as follows:

$$E(x,y,t)=\Bigg[\frac{-\sqrt{-2c}}{4}\sqrt{-k^2(l^2+1)}\times \Bigg[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)}+ \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)}+ \frac{+2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\Bigg]\times \tanh\left(\sqrt{-c}k\left(x+ly+2\left(\alpha-\beta l\right)t\right)\right)+ \frac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\Bigg]\exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right),$$

And

$$N(x,y,t)=\frac{1}{1+l^2}\Bigg[\frac{-\sqrt{-2c}}{4}\sqrt{-k^2(l^2+1)}\times \Bigg[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)}+ \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}+ \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\Bigg]\times \tanh\left(\sqrt{-c}k\left(x+ly+2\left(\alpha-\beta l\right)t\right)\right)+ \frac{-k^2(l^2-1)b}{(1+l^2)\sqrt{-2k^2(l^2+1)}}\Bigg]^2$$

Case 3:

$$E(x,y,t)=\Bigg[\frac{-\sqrt{2}}{4}\frac{\sqrt{-(l^2+1)}}{x+ly+2(\alpha-\beta l)t}\times \Bigg[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)}+ \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\Bigg] + \frac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\Bigg]\times \exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right),$$

and

$$N(x,y,t)=\frac{1}{1+l^2}\times \Bigg[\frac{-\sqrt{2}}{4}\frac{\sqrt{-(l^2+1)}}{x+ly+2(\alpha-\beta l)t}\times \Bigg[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)}+ \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\Bigg]+ \frac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\Bigg]^2$$

Case 4:

$$E(x,y,t)=\Bigg[\frac{\sqrt{2c}}{4}\sqrt{-k^2(l^2+1)}\times \Bigg[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)}+ \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)}+ \frac{+2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\Bigg]\times \tanh\left(\sqrt{c}k\left(x+ly+2\left(\alpha-\beta l\right)t\right)\right)+ \frac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\Bigg]\times \exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right),$$

And

$$N(x,y,t)=\frac{1}{1+l^2}\times \Bigg[\frac{\sqrt{2c}}{4}\sqrt{-k^2(l^2+1)}\times \Bigg[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)}+ \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\Bigg]\times \tanh\left(\sqrt{c}k\left(x+ly+2\left(\alpha-\beta l\right)t\right)\right) + \frac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\Bigg]^2$$

Case 5: By substituting (27-29) in (26) along with (15) we have

$$E(x,y,t)= \Bigg[-\frac{1}{2}\sqrt{-2k^2(l^2+1)}\times \left(b+2\coth k\left(x+ly+2\left(\alpha-\beta l\right)t\right)\right)\times \Bigg[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)}+ \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\Bigg]\times \Bigg[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)}+ \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)}+ \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\Bigg]^{-1}+ \frac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\Bigg]\times \exp\left(i\left(\alpha x+\beta y+\gamma t\right)\right),$$

And

$$N(x,y,t) = \frac{1}{l^2+1} \times \left[\frac{1}{2}\sqrt{-2k^2(l^2+1)} \times (b+2\coth k(x+ly+2(\alpha-\beta l)t)) \times \left[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)} + \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\right] \times \left[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)} + \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\right]^{-1} + \frac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\right]^2$$

Case 6: By substituting (27-29) in (26) along with (16) we have

$$E(x,y,t) = \left[\frac{1}{2}\sqrt{-2k^2(l^2+1)} \times (b \pm \operatorname{csc}hk(x+ly+2(\alpha-\beta l)t) + \coth k(x+ly+2(\alpha-\beta l)t)) \left[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)} + \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)}\right] \times \left[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)} + \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\right]^{-1} + \frac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\right] \times \exp(i(\alpha x+\beta y+\gamma t)),$$

And

$$N(x,y,t) = \frac{1}{l^2+1} \times \left[\frac{1}{2}\sqrt{-2k^2(l^2+1)} \times (b \pm \operatorname{csc}hk(x+ly+2(\alpha-\beta l)t) + \coth k(x+ly+2(\alpha-\beta l)t)) \left[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)} + \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\right] \times \left[\frac{k^2b^2l^6-11k^2b^2l^4}{ck^2(l^2+1)^2(l^2-1)} + \frac{11k^2b^2l^2-k^2b^2-2\alpha^2l^4-4\alpha^2l^2}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2+2\gamma l^4+4\gamma l^2+2\gamma}{ck^2(l^2+1)^2(l^2-1)} + \frac{2\beta^2l^4-2\alpha^2+4\beta^2l^2}{ck^2(l^2+1)^2(l^2-1)}\right]^{-1} + \frac{-k^2(l^2-1)b}{\sqrt{-2k^2(l^2+1)}}\right]^2,$$

4 Conclusions

In summary in this paper, the homogeneous balance method is employed along with a computerized symbolic computation to obtain the single and combined generalized solutions of coupled (2+1)-dimensional nonlinear system of Schrodinger equations. We have also constructed the extremum point and points of inflection in order to address the general description of the solutions obtained. The results show that the homogeneous balance method is a powerful and promising new method to solve nonlinear evolution equations.

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