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Generation of Fractals via Self-Similar Group of Kannan Iterated Function System

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Abstract: In this paper, we introduce and investigate some definition and property of self-similar group and strong self-similar group of Kannan contraction. These groups are found as the attractor of a Kannan iterated function system (KIFS). This paper improves the Hutchinson-Barnsley theory in the sense of KIFS. Fractal set can be constructed by self-similar and strong self-similar group in the sense of KIFS of compact topological group. Finally, we prove the relation between profinite group and strong self-similar group of Kannan contraction.

Keywords: Fractals; Kannan Contraction; Kannan iterated function system; self-similar set; Topological group; Profinite group.

1 Introduction

Fractal Analysis was introduced by Mandelbrot in 1975 [1] and popularized by various mathematicians. A fractal is an object which appears self-similar under varying degrees of magnification. Mathematically, sets with non-integral Hausdorff dimension which exceeds its topological dimension, are called Fractals by Mandelbrot. Hutchinson [2] introduced the formal definition of iterated function systems (IFS). Barnsley [3,4,5] developed the theory of IFS called the Hutchinson-Barnsley (HB) theory. In order to define and construct the fractal as a compact invariant (or self-similar) subset of a complete metric space generated by the IFS of Banach contractions. That is, Hutchinson introduced an operator on hyperspace of nonempty compact sets called as a HB operator, which defines a fractal set as a unique fixed point by using the Banach fixed point theorem in the complete metric space.

The concept of self-similarity is very suitable for the study of fractals. In order to understand self-similar sets in depth we must understand their group structure[6]. There is a natural tendency to think that a compact fractal space, full of holes, cannot admit an infinite group of motions. The self-similarity is based on the re-scaling principle and is developed in term of the renorm group. The renorm group has made its appearance in different places; it refers to a set of concepts related to the change of a mathematical model depending on the change of the observation scale [7]. The renorm group is a cyclic group or a continuous one parameter group, such as the group generated by the adding machine or a group of specific transformations of a partial solution of a mathematical problem. As very often these transformations do not have an inverse, so it is a semigroup [7,8].

The class of groups, self-similar groups lies behind the non-cyclic renormalization group. There are many ways to define self-similar groups. One way is, to define them as groups generated by Mealy type automata. This makes self-similar groups suitable for the needs of computer science [7]. Another way to defining the self-similar group is via an iterated function system of compact topological group, which brings them close to various topics in dynamical systems.

Self-similarity is the most fundamental property of the fractals. In order to analyse self-similar sets in depth we must realize their group structure. The self-similarity property of the fractal sets is defined on group structure by S.Kocak and Mustafa Saltan et al [11,12]. The classical examples of the fractals are the Cantor set, Koch curve and Sierpinski triangle. These sets are found as the attractor of an iterated function system (IFS) of Banach contraction. We can construct the fractals by using HB

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theory. In general these resulting constructions are self-similar.

On the basis of iterated function system given by Barnsley [3], Sahu et al [9] introduced the Kannan iterated function system (KIFS) for constructing the fractal sets. In this paper, we introduce the self-similar group and strong self-similar group of Kannan contraction. Moreover, we investigate some properties of strong self-similar group and we also discuss the relation between profinite group and strong self-similar group in the sense of KIFS.

If G is a self-similar group (strong self-similar group) of Kannan contraction, then G is also describable as the attractor of a KIFS and one of the Kannan contractions of KIFS is a group homomorphism (isomorphism). The image of G under this Kannan contraction map is its proper subgroup H being homomorphic (isomorphic) to G. This paper improves the HB theory in the sense of KIFS. Fractal set can be defined as a self-similar and strong self-similar group in the sense of KIFS of compact topological space.

2 Preliminary

2.1 Metric Fractals

In this section, we recall the HB theory to define and construct the fractals in the complete metric space.

Definition 21 ([2,3]). *Let (X,d) be a metric space and $\mathscr{K}(X)$ be the collection of all non-empty compact subsets of X. Define, $d(x,B) = \inf_{y\in B} d(x,y)$ and $d(A,B) = \sup_{x\in A} d(x,B)$ for all $x \in X$ and $A,B \in \mathscr{K}(X)$. The Hausdorff metric or Hausdorff distance (H_d) is a function $H_d : \mathscr{K}(X) \times \mathscr{K}(X) \longrightarrow \mathbb{R}$ defined by $H_d\Big(A,B\Big) = \max\Big\{d(A,B), d(B,A)\Big\}$. Then H_d is a metric on $\mathscr{K}(X)$ and hence $(\mathscr{K}(X), H_d)$ is called a Hausdorff metric space.*

Theorem 21 ([2,3]). *If (X,d) is complete metric space, then $(\mathscr{K}(X), H_d)$ is also a complete metric space.*

Definition 22 ([2,3]). *The function $f : X \longrightarrow X$ is said to be a contraction or Banach contraction mapping on a metric space (X,d), if there exists $k \in [0,1)$ such that*

$$d(f(x), f(y)) \leq kd(x,y), \quad \text{for all } x,y \in X.$$

Here k is called a contractivity ratio of f.

Definition 23 ([2,3]). *Let (X,d) be a metric space and $f_n : X \longrightarrow X,\ n = 1,2,3,\ldots,N\ (N \in \mathbb{N})$ be N - Banach contraction mappings with the corresponding contractivity ratios $k_n,\ n = 1,2,3,\ldots,N$. Then the system $\{X; f_n,\ n = 1,2,3,\ldots,N\}$ is called an iterated function system (IFS) or hyperbolic iterated function system with the ratio $k = \max_{n=1}^{N} k_n$.*

Definition 24 ([2,3]). *Let (X,d) be a metric space. Let $\{X; f_n,\ n = 1,2,3,\ldots,N; N \in \mathbb{N}\}$ be an IFS of Banach contractions. Then the Hutchinson-Barnsley (HB) operator of the IFS is a function $F : \mathscr{K}(X) \longrightarrow \mathscr{K}(X)$ defined by*

$$F(B) = \bigcup_{n=1}^{N} f_n(B), \quad \text{for all } B \in \mathscr{K}(X).$$

Theorem 22 ([2,3]). *Let (X,d) be a metric space. Let $\{X; f_n,\ n = 1,2,3,\ldots,N; N \in \mathbb{N}\}$ be an IFS of Banach contractions. Then, the HB operator (F) is a Banach contraction mapping on $(\mathscr{K}(X), H_d)$.*

Theorem 23 ([2,3]HB Theorem for Metric IFS). *Let (X,d) be a complete metric space and $\{X; f_n,\ n = 1,2,3,\ldots,N; N \in \mathbb{N}\}$ be an IFS of Banach contractions. Then, there exists only one compact invariant set $A_\infty \in \mathscr{K}(X)$ of the HB operator (F) or, equivalently, F has a unique fixed point namely $A_\infty \in \mathscr{K}(X)$.*

Definition 25 ([2,3]Metric Fractal). *The fixed point $A_\infty \in \mathscr{K}(X)$ of the HB operator F described in the Theorem 23 is called the Attractor (Fractal) of the IFS of Banach contractions. Sometimes $A_\infty \in \mathscr{K}(X)$ is called a Metric Fractal generated by the IFS of Banach contractions.*

2.2 Attractor of Kannan iterated function System

Definition 26 ([10]). *The function $f : X \longrightarrow X$ is said to be a Kannan contraction mapping on a metric space (X,d), if there exists $k \in (0,1/2)$ such that*

$$d(f(x), f(y)) \leq k[d(x, f(x)) + d(y, f(y))], \quad \text{for all } x,y \in X.$$

Here k is called a Kannan contractivity (K-contractivity) ratio of f.

On the basis of definition of (hyperbolic) iterated function system given by Barnsley [3], Sahu et al. [9] introduce attractor of Kannan iterated function system (KIFS) as follows.

Definition 27 ([9]). *Let (X,d) be a complete metric space and $f_n : X \longrightarrow X,\ n = 1,2,3,\ldots,N\ (N \in \mathbb{N})$ be N - Kannan contraction mappings with the corresponding contractivity ratios $k_n,\ n = 1,2,3,\ldots,N$. Then the system $\{X; f_n,\ n = 1,2,3,\ldots,N\}$ is called a Kannan iterated function system (KIFS).*

Lemma 21 ([9]). *Let $f : X \longrightarrow X$ be a continuous Kannan mapping on the metric space (X,d) with K-contractivity ratio k. Then $f : \mathscr{K}(X) \longrightarrow \mathscr{K}(X)$ defined by*

$$f(B) = \{f(x) : x \in B\}, \quad \text{for every } B \in \mathscr{K}(X)$$

is a Kannan mapping on $(\mathscr{K}(X), H_d)$ with contractivity ratio k.

Lemma 22 ([9]). *Let (X,d) be a metric space. Let f_n, $n=1,2,3,\ldots,N$ be continuous Kannan mappings on $(\mathscr{K}(X),H_d)$. Let K-contractivity ratio for f_n be denoted by k_n, $n=1,2,3,\ldots,N$. Define $F:\mathscr{K}(X)\longrightarrow\mathscr{K}(X)$ by*

$$F(B)=\bigcup_{n=1}^{N} f_n(B), \quad \text{for each} \quad B\in\mathscr{K}(X).$$

Then F is a Kannan mapping with K-contractivity ratio $k=\max_{n=1}^{N} k_n$.

Theorem 24 ([9]). *If $\{X:(f_0),f_1,f_2,\ldots,f_n\}$, where f_0 is the condensation mapping, is a KIFS with K-contractivity ratio k, then $F:\mathscr{K}(X)\longrightarrow\mathscr{K}(X)$ defined by*

$$F(B)=\bigcup_{n=1}^{N} f_n(B), \quad \text{for all} \quad B\in\mathscr{K}(X).$$

is a continuous Kannan mapping on the complete metric space $(\mathscr{K}(X),H_d)$ with contractivity ratio k. If $A\in\mathscr{K}(X)$ is a unique fixed point of F, which is also called an attractor, then A obeys

$$F(A)=A=\bigcup_{n=1}^{N} f_n(A),$$

and is given by $A=\lim_{n\to\infty}F^{on}(B)$ for any $B\in\mathscr{K}(X)$. F^{on} denotes the n-fold composition of F.

Theorem 25. *Let $\{X;f_0,f_1,\ldots,f_n\}$ be a KIFS with attractor A. If the Kannan contraction mappings $f_0,f_1,\ldots,f_n$ are one-to-one on A and*

$$f_i(A)\cap f_j(A)=\phi \quad \text{for all} \quad i,j\in\{0,1,2,\ldots,n\} \text{ with } i\neq j,$$

then A is totally disconnected set.

2.3 Self-similar and Profinite groups in the sense of IFS

Topological groups can be defined concisely as group objects in the category of topological spaces, in the same way that ordinary groups are group objects in the category of sets. Now we recall the definition of self-similar group of IFS of compact topological space and profinite group.

Definition 28 ([11,12]). *Let (G,d) be a compact topological group with a translation-invariant metric d. G is called a self-similar group, if there exists a proper subgroup H of finite index and a surjective homomorphism $\phi:G\longrightarrow H$, which is a contraction with respect to d.*

Definition 29 ([12]). *Let (G,d) be a compact topological group with a translation-invariant metric d. G is called a strong self-similar group, if there exists a proper subgroup H of finite index and a group isomorphism $\phi:G\longrightarrow H$, which is a contraction with respect to d.*

Definition 210 ([12,13]). *A topological group G is profinite, if it is topologically isomorphic to an inverse limit of finite discrete topological groups. Equivalently, a profinite group is a compact, Hausdorff and totally disconnected topological group.*

3 Self-similar and Profinite groups in the sense of KIFS of compact topological groups

In this section, we introduce the definition and property of self-similar group and strong self-similar group of Kannan contraction. Then, we investigate some properties of strong self-similar and profinite group.

Definition 31. *Let (G,d) be a compact topological group with a translation-invariant metric d. G is called a self-similar group of Kannan contraction, if there exists a proper subgroup H of finite index and a surjective homomorphism $\phi:G\longrightarrow H$, which is a Kannan contraction with respect to d.*

Definition 32. *Let (G,d) be a compact topological group with a translation-invariant metric d. G is called a strong self-similar group of Kannan contraction, if there exists a proper subgroup H of finite index and a group isomorphism $\phi:G\longrightarrow H$, which is a Kannan contraction with respect to d.*

Proposition 31. *A strong self-similar group of Kannan contraction is the attractor of KIFS.*

Proof.

Let G be a strong self-similar group of Kannan contraction. So there is a proper subgroup H of G with finite index such that the mapping $\phi_o:G\longrightarrow H$ is a group isomorphism and is a Kannan contraction. Let $[G:H]=n$ and let $x_o=e$ be the identity element of G. For all $i,j\in\{0,1,2\ldots,n-1\}$ and $i\neq j$, there are cosets of H in G such that $(H*x_i)\cap(H*x_j)=\phi$ and $G=H\cup(H*x_1)\cup(H*x_2)\cup\ldots\cup(H*x_n-1)$. Define $\phi_i:G\longrightarrow G$ by $\phi_i(g)=\phi_o(g)*x_i$, $i=1,2,3,\ldots,n-1$. Now, we show that $\phi_i(g)$ is a Kannan contraction for each i. It is obvious that

$$\phi_i(G)=H*x_i$$

because of ϕ_o is surjective. Since ϕ_o is a Kannan contraction mapping with contraction ratio k and d is a translation invariant metric, we obtain that

$$\begin{aligned} d(\phi_i(g),\phi_i(h)) &= d(\phi_o(g)*x_i,\phi_o(h)*x_i)\\ &= d(\phi_o(g),\phi_o(h))\\ &\leq \alpha[d(g,\phi_o(g))+d(h,\phi_o(g))] \end{aligned}$$

for all $g,h\in G$. Therefore ϕ_i is a Kannan contraction mapping with contraction ratio k for $i=1,2,3,\ldots,n-1$ and

$$\begin{aligned} G &= H\cup(H*x_1)\cup(H*x_2)\cup\ldots\cup(H*x_n-1)\\ &= \phi_o(G)\cup\phi_1(G)\cup\phi_2(G)\cup\ldots\cup\phi_{n-1}(G) \end{aligned}$$

$$G=\bigcup_{n=0}^{n-1}\phi_i(G).$$

Thus, G is the attractor of the KIFS $\{G;\phi_o,\phi_1,\dots,\phi_{n-1}\}$. Proposition 31 explains the reason why the groups defined in definition 31 and definition 32 are self-similar group.

Proposition 32. *Let $(G,*,d)$ and $(G^{'},*^{'},d^{'})$ be compact topological groups. If G is a strong self-similar group of Kannan contraction and $f:G\longrightarrow G^{'}$ is both an isometry map and a group isomorphism, then $G^{'}$ is also a strong self-similar group of Kannan contraction.*

Proof.

f is surjective and isometry, so there exists $x,y,z\in G$ such that $f(x)=x^{'}$, $f(y)=y^{'}$ and $f(z)=z^{'}$ for all $x^{'},y^{'},z^{'}\in G^{'}$. d is translation-invariant metric, we compute

$$\begin{aligned}
d'(x'*'z',y'*'z') &= d'(f(x)*'f(z),f(y)*'f(z))\\
&= d'(f(x*z),f(y*z))\\
&= d(x*z,y*z)\\
&= d(x,y)\\
&= d'(f(x),f(y))\\
&= d'(x',y')
\end{aligned}$$

G is a strong self-similar group of Kannan contraction, there exists a subgroup H of finite index and a group isomorphism $\phi:G\longrightarrow H$. Let $f(H)=H'$. f is a group isomorphism, it is obvious that H' is a subgroup of G' with finite index.

Define $f_{|H}:H\longrightarrow H'$ by $f_{|H}(x)=f(x)$ for all $x\in H\subseteq G$. Now we prove that $\phi'=f_{|H}\circ\phi\circ f^{-1}:G'\longrightarrow H'$ is both a Kannan contraction mapping and a group isomorphism. f, $f_{|H}$ and ϕ are group isomorphisms, it is clear that ϕ' is also a group isomorphism. ϕ is a Kannan contraction mapping with contraction ratio k and f, $f_{|H}$ are isometries, we get

$$\begin{aligned}
d'(\phi'(g'),\phi'(h')) &= d'(f_{|H}\circ\phi\circ f^{-1}(g'),f_{|H}\circ\phi\circ f^{-1}(h'))\\
&= d'(f_{|H}(\phi\circ f^{-1})(g'),f_{|H}(\phi\circ f^{-1})(h'))\\
&= d(\phi\circ f^{-1}(g'),\phi\circ f^{-1}(h'))\\
&\leq k[d'(f^{-1}(g'),\phi(f^{-1}(g')))\\
&\quad +d'(f^{-1}(h'),\phi(f^{-1}(h')))]\\
&= k[d'(f_{|H}\circ f^{-1}(g'),f_{|H}\circ\phi(f^{-1}(g')))\\
&\quad +d'(f_{|H}\circ f^{-1}(h'),f_{|H}\circ\phi(f^{-1}(h')))]\\
&= k[d'(g',\phi'(g'))+d'(h',\phi'(h')]
\end{aligned}$$

for all $g',h'\in G'$. It gives that ϕ' is Kannan contracion mapping on G'.

Proposition 33. *If $G_1,G_2,\dots,G_n$ are strong self-similar group of Kannan contraction, so is $G_1\times G_2\times\dots\times G_n$.*

Proof.

Since $(G_1,*_1,d_1),(G_2,*_2,d_2),\dots,(G_n,*_n,d_n)$ are compact topological groups, $G_1\times G_2\times\dots\times G_n$ is a compact topological group. Moreover, there are subgroups $H_1,H_2,\dots,H_n$ of $G_1,G_2,\dots,G_n$ respectively such that $[G_i:H_i]=m_i$, and the mappings

$$\phi_i:G_i\longrightarrow H_i$$

are Kannan contractions with corresponding contraction ratios k_i and also group isomorphisms for $i=1,2,\dots,n$, since these groups are strong self-similar in the sense of Kannan contraction. Define the mapping

$$\phi:G_1\times G_2\times\dots\times G_n\longrightarrow H_1\times H_2\times\dots\times H_n$$

$$\phi(g_1,g_2,\dots,g_n)=(\phi_1(g_1),\phi_2(g_2),\dots,\phi_n(g_n))$$

It is obvious that $H_1\times H_2\times\dots\times H_n$ is a subgroup of $G_1\times G_2\times\dots\times G_n$ and $[G_1\times G_2\times\dots\times G_n:H_1\times H_2\times\dots\times H_n]$ $=m_1m_2\dots m_n$. $\phi_1,\phi_2,\dots,\phi_n$ are group homomorphisms, we compute

$$\begin{aligned}
&\phi(g*h)\\
&=\phi((g_1,g_2,\dots,g_n)*(h_1,h_2,\dots,h_n))\\
&=\phi((g_1*_1h_1,g_2*_2h_2,\dots,\phi_n(g_n*_nh_n))\\
&=(\phi_1(g_1)*_1\phi_1(h_1),\phi_2(g_2)*_2\phi_2(h_2),\dots,\phi_n(g_n)*_n\phi_n(h_n))\\
&=(\phi_1(g_1),\dots,\phi_n(g_n))*(\phi_1(h_1),\dots,\phi_n(h_n))\\
&=\phi((g_1,g_2,\dots,g_n))*\phi((h_1,h_2,\dots,h_n))\\
&=\phi(g)*\phi(h).
\end{aligned}$$

It is clear that ϕ is bijective due to the definitions of $\phi_1,\phi_2,\dots,\phi_n$. Hence ϕ is a group homomorphism. Let $k=max\{k_1,k_2,\dots,k_n\}$ for $i\in\{1,2,\dots,n\}$. Then, we obtain that

$$d(\phi(g),\phi(h))=d(\phi(g_1,g_2,\dots,g_n)*\phi(h_1,h_2,\dots,h_n))$$

$$\begin{aligned}
&\phi(g*h)\\
&= \phi((g_1,g_2,\dots,g_n)*(h_1,h_2,\dots,h_n))\\
&= d((\phi_1(g_1),\dots,\phi_n(g_n))*(\phi_1(h_1),\dots,\phi_n(h_n)))\\
&= max\{d_1(\phi_1(g_1),\phi_1(h_1)),\dots,d_n(\phi_n(g_n),\phi_n(h_n))\}\\
&\leq max\{k_1[d_1(g_1,\phi_1(g_1))+d_1(h_1,\phi_1(h_1))],\\
&\quad \cdots,k_n[d_n(g_n,\phi_n(g_n))+d_n(h_n,\phi_n(h_n))]\}\\
&\leq max\{k[d_1(g_1,\phi_1(g_1))+d_1(h_1,\phi_1(h_1))]\\
&\quad \cdots,k[d_n(g_n,\phi_n(g_n))+d_n(h_n,\phi_n(h_n))]\}\\
&= k\,max\{[d_1(g_1,\phi_1(g_1))+d_1(h_1,\phi_1(h_1))],\\
&\quad \cdots,[d_n(g_n,\phi_n(g_n))+d_n(h_n,\phi_n(h_n))]\}\\
&= k\{[d_1(g_1,\phi_1(g_1))+d_1(h_1,\phi_1(h_1))],\\
&\quad \cdots,[d_n(g_n,\phi_n(g_n))+d_n(h_n,\phi_n(h_n))]\}\\
&= k\{(d_1(g_1,\phi_1(g_1)),\dots,d_n(g_n,\phi_n(g_n)))\\
&\quad +(d_1(h_1,\phi_1(h_1)),\dots,d_n(h_n,\phi_n(h_n)))\}\\
&= k[d(g,\phi(g)+d(h,\phi(h))]
\end{aligned}$$

Hence, ϕ is a Kannan contraction with contraction ratio k consequently, $G_1\times G_2\times\dots\times G_n$ is a strong self-similar group of Kannan contraction.

The Proposition 33 shows that, the finite product of strong self-similar groups of Kannan contraction is also a strong self-similar group of Kannan contraction.

Proposition 34. *A self-similar group of continuous Kannan contraction is a disconnected set.*

Proof.

Let G be a self-similar group of Kannan contraction. Then G is a topological group. Proposition 31 shows that G is the attractor of the KIFS $\{\phi_0, ..., \phi_{n-1}\}$. For every $i = 1, 2, ..., n-1$, the mappings

$$\phi_i : G \longrightarrow \phi_i(G)$$

are Kannan contractions. Furthermore, we have

$$G = \phi_0(G) \cup \phi_1(G) \cup ... \cup \phi_{n-1}(G)$$
$$\Phi = \phi_i(G) \cap \phi_j(G)$$

for all $i, j \in \{0, 1, 2, ..., n-1\}$ and $i \neq j$. It is well known that the image of a compact set under a continuous map is compact and every compact subspace of a Hausdorff space is closed. Therefore, $\phi_i(G)$ is a closed set for $i = 0, 1, 2, ..., n-1$. Due to the fact that

$$G = \phi_0(G) \cup [\phi_1(G) \cup ... \cup \phi_{n-1}(G)]$$
$$\Phi = \phi_0(G) \cap [\phi_1(G) \cup ... \cup \phi_{n-1}(G)])$$

we obtain that $\{\phi_0(G), [\phi_1(G) \cup ... \cup \phi_{n-1}(G)]\}$ is a closed separation of G. That is G is disconnected set.

Proposition 35. *A strong self-similar group of continuous Kannan contraction is a totally disconnected set.*

Proof.

Let G be a strong self-similar group of continuous Kannan contraction. Proposition 31 shows that G is the attractor of a KIFS $\{\phi_0, ..., \phi_{n-1}\}$. Since $\phi_0 : G \longrightarrow H$ is one-to-one, we get

$$\begin{aligned} \phi_i(g) &= \phi_i(h) \\ \phi_0(g) * x_i &= \phi_0(h) * x_i \\ \phi_0(g) &= \phi_0(h) \\ g &= h \end{aligned}$$

for all $g, h \in G$. This shows that ϕ_i is one-to-one for $i = 1, 2, ..., n-1$. In addition that, $\phi_i(G) \cap \phi_j(G) = \Phi$, for all $i, j \in \{0, 1, 2, ..., n-1\}$ and $i \neq j$. Consequently, G is a totally disconnected set on account of Theorem 25.

The following theorem gives the relation between profinite group and strong self-similar group of Kannan contraction.

Theorem 31. *A strong self-similar group of Kannan contraction is a profinite group.*

Proof.

Let A be a strong self-similar group of Kannan contraction. By the Definition 32. A is a compact topological group and also A is Hausdorff since every metric space is Hausdorff. Proposition 35 shows that A is totally disconnected set. Finally we get A is compact, Hausdorff and totally disconnected. Thus, we have the properties which characterize profinite groups. This shows that a strong self-similar group of Kannan contraction is a profinite group.

4 Conclusion

In this study, we have introduced the self-similar group and strong self-similar group of Kannan contraction. We have proved the strong self-similar group of Kannan contraction is expressed as the attractor of a KIFS and one of the Kannan contractions of KIFS is a group isomorphism. There are many ways to constructing the fractals. In this paper, we have constructed fractals by KIFS of compact topological groups. In addition, we have proved the relation between profinite group and strong self-similar group of Kannan contraction.

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