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Control of Quantum and Classical Correlations in Werner-Like States of Two-Qubit Cavity System under Dissipative Environments

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Abstract: Quantum and classical correlations are studied for Werner-Like States of two-qubit cavity system interacting with a thermal reservoir. Starting from Werner-like states, we have shown that entanglement sudden death and decay of both the quantum discord and classical correlation are accelerated by the different factors: thermal photons, cavity decay and the purity of the initial state. By these factors, the death-start points of the correlations can be controlled and the two-qubit states have no correlations that can be determined. There is no sudden death for quantum discord and classical correlation.

Keywords: Quantum discord, Classical correlations, Werner-like state

1 Introduction

The development of the quantum information technology stimulates a deep study of the properties of quantum correlations inherent in a quantum system. Therefore, a noted interest has been devoted to the definitions and understanding of quantum and classical correlations in quantum systems in the last two decades. It is well known that the total correlation in a bipartite quantum system can be measured by quantum mutual information [1], which may be divided into classical and quantum parts [2]-[5]. The quantum part is called quantum discord (QD) which was originally introduced in [4]. Recently, it has become apparent that quantum discord is a more general concept to measure quantum correlation than quantum entanglement(QE) since there is a nonzero quantum discord in some separable mixed states [4]. Recently, a comparison between the dynamics of quantum discord and entanglement has been made under the same conditions, when entanglement dynamic undergoes a sudden death [6,7,8]. The dynamics of quantum discord and entanglement has been recently compared also under the same conditions when entanglement dynamic undergoes a sudden death [9]-[13]. Interestingly, it has been proven both theoretically and experimentally that such states provide computational speedup compared to classical states in some quantum computation models [14, 15]. In these contexts, quantum discord could be a new resource for quantum computation.

The calculation of quantum discord is based on a numerical maximization procedure. Such a procedure does not guarantee exact results and there are few analytical expressions for some special cases [16,17]. To avoid this difficulty, Ref.[18] introduced the geometric measure of quantum discord (GMQD), which measures the quantum correlations through the minimum Hilbert-Schmidt distance between the given state and zero discord state.

On the other hand, realistic quantum systems will inevitably interact with the environments. The interaction between the system and the environment usually leads to a decoherence process [19,20]. This is one fundamental obstacle to reliable quantum computation. Therefore, understanding the dynamics of quantum and classical correlations (CC) is an interesting line of research [21]-[24]. In these references, It is showed that both GMQD and QD die asymptotically with entanglement sudden death, and the discontinuity in the decay rate of GMQD does not always imply the discontinuity in the

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decay rate of QD. The entanglement sudden death [25,26] has been experimentally observed in an implementation using twin photons [27], and atomic ensembles [28].

In this paper the dynamics of quantum and classical correlations of two-qubit cavity system, interacted with a thermal reservoir, are study. Therefore, the quantum correlation via quantum discord and its geometric measure is compared with both quantum entanglement and classical correlation.

2 Measures of quantum and classical correlations

2.1 Quantum discord

To quantify the quantum correlations of a bipartite system, no matter whether it is separable or entangled, one can use the quantum discord [2,4]. Quantum discord measures all nonclassical correlations and is defined as the difference between total correlation and the classical correlation with the following expression

$$D(\rho^{AB}) = \mathscr{I}(\rho^{AB}) - \mathscr{J}(\rho^{AB}), \tag{1}$$

which quantifies the quantum correlations in ρ^{AB} and can be further distributed between entanglement and quantum dissonance (quantum correlations excluding entanglement)[29]. Here the total correlation between two subsystems A and B of a bipartite quantum system ρ^{AB} is measured by quantum mutual information,

$$\mathscr{I}(\rho^{AB}) = \mathscr{S}(\rho^{A}) + \mathscr{S}(\rho^{B}) - \mathscr{S}(\rho^{AB}), \tag{2}$$

where $\mathscr{S}(\rho^{AB}) = Tr(\rho^{AB}\log\rho^{AB})$ is the von Neumann entropy, $\rho^{A} = Tr_B(\rho^{AB})$ and $\rho^{B} = Tr_A(\rho^{AB})$ are the reduced density operators of the subsystems A and B, respectively. The measure of classical correlation is introduced implicitly in Ref.[4] and interpreted explicitly in the Ref.[2]. The classical correlation between the two subsystems A and B is given by

$$\mathscr{J}(\rho^{AB}) = \max_{\{\Pi_k\}}[\mathscr{S}(\rho^{A}) - \sum_k p_k \mathscr{S}(\rho_k)], \tag{3}$$

where $\{\Pi_k\}$ is a complete set of projectors to measure the subsystem B, and $\rho_k = Tr_B[(I^A \otimes \Pi_k)\rho_{AB}(I^A \otimes \Pi_k)]/p_k$ is the state of the subsystem A after the measurement resulting in outcome k with the probability $p_k = Tr_{AB}[(I^A \otimes \Pi_k)\rho^{AB}(I^A \otimes \Pi_k)]$, and I^A denotes the identity operator for the subsystem A. Here, maximizing the quantity represents the most gained information about the system A as a result of the perfect measurement $\{\Pi_k\}$. It can be shown that quantum discord is zero for states with only classical correlations and nonzero for states with quantum correlations. Note that discord is not a symmetric quantity, i.e., its amount depends on the measurement performed on the subsytem A or B [18].

2.2 Geometric measure of quantum discord

The geometric measure of quantum discord quantifies the quantum correlation through the nearest Hilbert-Schmidt distance between the given state and the zero discord state [18,21], which is given by

$$D_A^g = \min_{\chi\in S} \|\rho^{AB} - \chi\|^2, \tag{4}$$

where S denotes the set of zero discord states and $\|A\|^2 = Tr(A^\dagger A)$ is the square of Hilbert-Schmidt norm of Hermitian operators. The subscript A of D_A^g implies that the measurement is taken on the system A. A state χ on $H^A \otimes H^B$ is of zero discord if and only if it is a classical-quantum state [30]. An easily computable exact expression for the geometric measure of quantum discord is obtained in Ref.[18] for a two qubit system.

2.3 Entanglement via negativity

Here, one uses the negativity[31] to measure the entanglement, i.e., the negative eigenvalues of the partial transposition of ρ^{AB} are used to measure the entanglement of the qubits system. Therefore, the negativity of a state ρ^{AB} is defined as

$$N(\rho) = \max(0, -2\sum_j \mu_j), \tag{5}$$

where μ_j are the negative eigenvalues of $(\rho^{AB}(t))^{T_B}$, and T_B denotes the partial transpose with respect to the second system.

3 The model and quantum and classical correlations

Here, one considers a qubit passes consecutively through cavity A, a field damping region and cavity B. One of the pioneering potential applications of this qubit in the context of quantum information is "artificial atoms"(qubits), *e.g.*, the system of the Cooper pair box [32]. The qubit is initially prepared in $|1\rangle$ and the decay of the radiation field inside a cavity may be described by a model in which the field is coupled to a whole set of reservoir modes. If the two cavities are initially in vacuum state and the qubit always leaves the setup in $|0\rangle$, the interaction Hamiltonian in the rotating-wave approximation is given by [33]

$$\begin{aligned} H = &\sum_j (g_j^A \hat{b}_j^\dagger \hat{a}_A + g_j^B \hat{c}_j^\dagger \hat{a}_B) e^{-i(\omega-\nu_j)t} \\ &+ (g_j^{*A} \hat{b}_j \hat{a}_A^\dagger + g_j^{*B} \hat{c}_j \hat{a}_B^\dagger) e^{i(\omega-\nu_j)t}, \end{aligned} \tag{6}$$

where $\hat{a}_{A(B)}$ and $\hat{a}_{A(B)}^\dagger$ are annihilation and creation operators of the mode of the electromagnetic field $A(B)$ of frequency ω. $\hat{b}(\hat{c})_j$ and $\hat{b}^\dagger(\hat{c}^\dagger)_j$ are the modes of cavity

$A(B)$. These modes have frequencies ν_j and damp the field. $g_j^{A(B)}$ are the coupling constants between the field and the cavity. According to the general quantum reservoir theory, with the Hamiltonian (Eq.6), the master equation for the reduced density matrix for the filed in the cavities is given by [34]

$$\frac{\partial\rho(t)}{\partial t} = \frac{\gamma}{2}\sum_j(\bar{n}_j+1)[2\hat{a}_i\rho(t)\hat{a}_i^\dagger - \hat{a}_i^\dagger\hat{a}_i\rho(t) - \rho(t)\hat{a}_i^\dagger\hat{a}_i] + \frac{\gamma}{2}\sum_j\bar{n}_j[2\hat{a}_i^\dagger\rho(t)\hat{a}_i - \hat{a}_i\hat{a}_i^\dagger\rho(t) - \rho(t)\hat{a}_i\hat{a}_i^\dagger], \quad (7)$$

where the cavities are assumed to be identical, i.e., the cavities have the same decay rates, $\gamma_A = \gamma_A = \gamma$, and also the cavities A and B have the same thermal reservoir, i.e, $\hat{n}_A = \hat{n}_A = \hat{n}$. If the reservoir is at zero temperature ($\bar{n} = 0$) and the remaining terms are due to vacuum fluctuations, the solutions of Eq.7 depend on the initial state of the cavities. In the following, one will focus on the dynamics of entanglement, quantum discord and classical correlations for different initial cavity states, including pure and mixed states.

There are some interesting initial entangled states for the two-bipartite which can be prepared and have potential applications in the quantum information processing tasks. One of them the extended Werner-like state [35], which is defined by

$$\rho^{AB}(0) = p|\varphi\rangle\langle\varphi| + \frac{1}{4}(1-p)\hat{I}, \quad (8)$$

with p is the purity of the initial state of the cavities AB, $\hat{I}$ is a 4×4 identity matrix and $|\varphi\rangle = \sin\theta|01\rangle + \cos\theta|10\rangle$ is the NOON state, which can be generated by passing a qubit is initially in the upper state through the two empty high-Q cavities. In this case, the interaction times of a qubit with two cavities are chosen to be such that one has a $\frac{\pi}{2}$-pulse in the first cavity and a π-pulse in the second cavity [36]. Its potential application in Heisenberg-limited metrology and quantum lithography [37]. This class of mixed state (8) arises naturally in a wide variety of physical situations. The state in Eq.8 reduces to the standard Werner mixed state when $\theta = \frac{\pi}{4}$ and to NOON pure state when $p = 1$. By dealing with the above extended Werner-like state, the effect of mixedness of the initial entangled state is studied. Both the NOON state and Werner state, and the extended Werner-like state belong to the so-called X-class state [38] whose density matrix is given by

$$\rho^{AB}(t) = x|00\rangle\langle00| + y|01\rangle\langle01| + z|10\rangle\langle10| + w|11\rangle\langle11| + d(|01\rangle\langle10| + |10\rangle\langle01|), \quad (9)$$

with the abbreviation

$$x = \left[\frac{2+2\bar{n}-e^{-\gamma(2\bar{n}+1)t}}{4\bar{n}+2}\right]^2 - \frac{1}{4}pe^{-2\gamma(2\bar{n}+1)t},$$

$$y+z = -2\left[\frac{1-e^{-\gamma(2\bar{n}+1)t}}{4\bar{n}+2}\right]^2 + \frac{1}{2}pe^{-2\gamma(2\bar{n}+1)t}$$

$$z-y = p\cos2\theta e^{-\gamma(2\bar{n}+1)t}, d = p\cos\theta\sin\theta\, e^{-\gamma(2\bar{n}+1)t}$$

$$w = 1-x-y-z.$$

The dynamics of GMQD of the density matrix (9) is given by [18]

$$D_A^g = \frac{1}{2}\left[4|d|^2 + (x-z)^2 + (y-w)^2 - max\{2|d|^2, (x-z)^2 + (y-w)^2\}\right] \quad (10)$$

But QD for the density matrix (9) is calculated by the method in Refs.[17,39]. To calculate the negativity, one finds the partial transposition of ρ^{AB}, with respect to the second system B, is given by

$$(\rho^{AB})^{T_B} = x|00\rangle\langle00| + y|01\rangle\langle01| + z|10\rangle\langle10| + w|11\rangle\langle11| + d(|00\rangle\langle11| + |11\rangle\langle00|). \quad (11)$$

One can get the eigenvalues of the density matrix $(\rho^{AB})^{T_B}$ as: $\lambda_1 = y, \lambda_2 = z$ and $\lambda_{3,4} = \frac{1}{2}[(x+w) \pm \sqrt{(x-w)^2 - 4d^2}]$. These eigenvalues are numerically used to calculate the negativity. After simple calculations, one can get the reduced density matrices associated with the above states as

$$\rho^A(t) = (x+y)|00\rangle\langle00| + (z+w)|11\rangle\langle11|, \quad (12)$$

$$\rho^B(t) = (x+z)|00\rangle\langle00| + (y+w)|11\rangle\langle11|. \quad (13)$$

These reduced density matrices of the qubits are represented in diagonal matrices, i.e., the states of $\rho^A(t)$ and $\rho^B(t)$ are classical states.

In Fig.1, the dynamics of GMQD, QD, QE and CC are ploted as functions of the time t for different values of γ and $\bar{n}$ when $\theta = \frac{\pi}{4}$ and $p = 1$. From Fig.1a, one can easily find common features of GMQD, QD, QE and CC for vacuum reservoir, $\bar{n} = 0$. One can observe that these quantities asymptotically decay with the time t, where QD and QE approximately have the same behavior, but the values of QD are less than of QE in small interval in the first. After that the values of QD always exceed the values of the QE (see Fig.1a). So, QD are more general than QE. On the other hand, GMQD, QE and CC exponentially decay and their decay are faster than the decay of QD. While for the case of the thermal reservoir with nonzero mean photo number, GMQD, QD, QE and CC are plotted in Fig.1b, with $(\bar{n},\gamma) = (0.8, 1.0)$. One sees that all the decays of GMQD, QD, QE and CC will be enhanced as the mean thermal photon number becomes large. However, introducing the nonzero mean photon number results in disappearance of the entanglement completely after a finite time, termed entanglement sudden death [25,26], and the death time

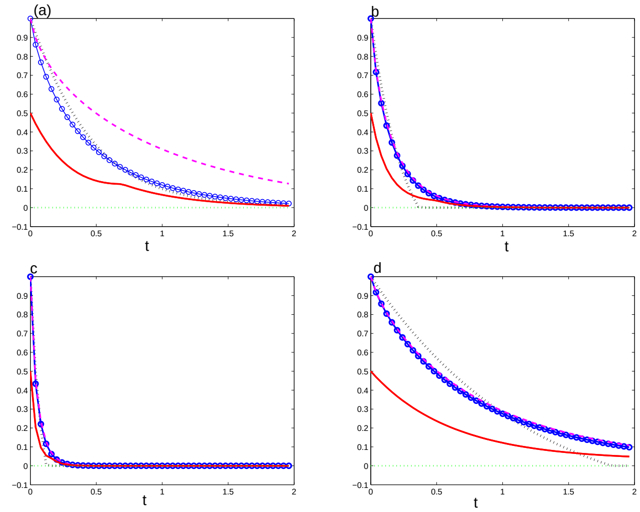

Fig. 1: Time evolutions of QD (dash plots), GMQD (sold plots), QE (dotted plots) and CC (dash-circle plots) for $(\bar{n},\gamma)=(0,1)$ in (a) and $(\bar{n},\gamma)=(0.8,1.0)$ in (b), $(\bar{n},\gamma)=(0.8,3)$ in (c) and $(\bar{n},\gamma)=(0.8,0.2)$ in (d)for $\theta=\frac{\pi}{4}$ and $p=1$.

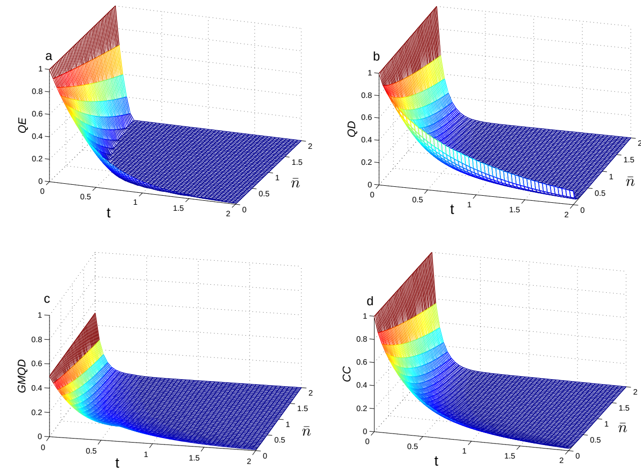

Fig. 2: The negativity in (a), quantum discord in (b), geometric measure of QD in (c) and classical correlation in (d) with $\bar{n}\in[0,2]$ for $p=1$ and $\theta=\frac{\pi}{4}$.

decreases with the increase of $\bar{n}$. This means that QE attains constant values while the quantum correlations of GMQD and QD vary. That is to say, even in the region where the entanglement is zero, the quantum discord can reveal the quantum correlation between the two cavities. In this sense, the quantum discord is more robust than entanglement against decoherence induce by the mean photon number of the thermal reservoir. However, the thermal reservoir also leads to death both the quantum discord and the classical correlation after some time points, these points are called death-start points (DSPs).

From Figs.1a-d, one notes that these DSPs can be controlled by the different factors: $\bar{n}$, p, and γ. If one wants to approximately destroy the quantum or classical correlations of the system, the large values of the mean photon number should are used (see Fig.1b). Therefore, the parameter $\bar{n}$ eventually drives the quantum correlation of the system to be zero. One can note that the negativity experiences a sudden transition at particular times changes from a finite to zero value, while GMQD, QD and CC evolve continuously with respect to time even they tend to be zero. This means that no sudden transition occurs for GMQD, QD and CC and they have no phenomenon of sudden death. In Figs.1a,b, the values of DSPs of all the quantities depend on the $\bar{n}$. These DSPs can be delayed and come early by changing the cavity decay rate parameter γ at fixed values for $\bar{n}$ and p. These DSPs come early with large values for the cavity decay rate, $\gamma>1$, while they delay with the small values for $\gamma<1$.

In Figs.2a-d, one examines the effect of the thermal photon parameter on the dynamics of the previous measures of quantum and classical correlations with the fixed values of $\gamma=p=1$ and $\theta=\frac{\pi}{4}$. It is clear that $D_{\underline{A}}^{g}$, $D(\rho)$, $N(\rho)$ and $\mathscr{J}(\rho)$ decrease with the increase of $\bar{n}$, and they have zero values when $\bar{n}>0$. Precisely, one can note that the thermal parameter leads to exponential decay for the values of all the quantities and with larger the value of $\bar{n}$ is, the more rapid they reach its zero asymptotic values. Therefore, a particular region for each measure in which there is no state has quantum or classical correlations can be determined by $\bar{n}$. All the decays of the quantum and classical correlation will be enhanced as the mean thermal photon number becomes large. With the increase of $\bar{n}$, the DSPs of all the quantities exponentially decay. The figures show that the exponentially decay of the QE is faster than that for GMQD, QD and CC. However, one can find that, from Figs.2, the increase of $\bar{n}$ accelerates the entanglement sudden death and the decay of both the correlations, but there is no sudden death of D_A^g, $D(\rho)$ and $\mathscr{J}(\rho)$. This means taht the entanglement is more fragile than quantum discord against $\bar{n}$.

In Figs.3a-d, the effect of the purity of the initial states on the dynamics of the measures is examined with the fixed values of $\gamma=\bar{n}=1$ and $\theta=\frac{\pi}{4}$. It is clear that the dynamics of the measures decrease with the decrease of the purity p. When the purity p is zero, all the measures vanish, this shows that the purity affects the quantum discord and classical correlation in a similar way. One sees that the influence of the purity leads to: the amplitudes of the local maxima of the D_A^g, $D(\rho)$, $N(\rho)$ and $\mathscr{J}(\rho)$ have exponential decay with the decrease of the parameter p. When the correlations measures quite vanish, the states ρ^{AB} completely lose its correlations.

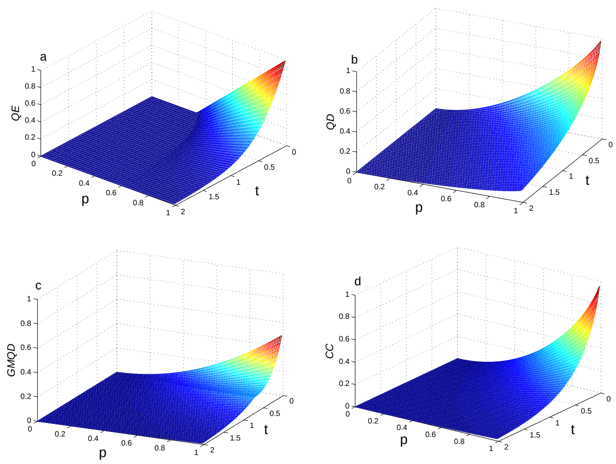

Fig. 3: The negativity in (a), quantum discord in (b), geometric measure of QD in (c) and classical correlation in (d) with $p \in [0,1]$ for vacuum reservoir and $\theta = \frac{\pi}{4}$.

This means that after a particular time, the purity destroys the correlations of the cavities. Therefore, in the presence of the purity one can determine a particular region in which there is no state has any correlation.

4 Conclusions

The dynamics of quantum correlations (including entanglement and quantum discord with its geometric measure) and classical correlation are compared in a two-qubit cavity system interacted with a thermal reservoir. It is found that the different factors: thermal photons, cavity decay rate and purity accelerate lead to the entanglement sudden death and the decay of both the quantum discord and classical correlation, but there is no sudden death of quantum discord and classical correlation. The death-start points of the quantum and classical correlations can be controlled by these factors. These factors also lead to destroy the quantum and classical correlations, therefore, a particular region in which there is no state have any correlation can be determined.

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