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Plane Waves on Rotating Microstretch Elastic Solid with Temperature Dependent Elastic Properties

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Abstract: Plane waves propagating in generalized thermoelastic half space rotating with specific angular frequency has been investigated. The appropriate model of the problem in context of Green and Naghdi theory is generated, while modulus of elasticity is taken as a linear function of reference temperature. An exact approach of normal mode analysis method is implemented to obtain the expression for the displacement components, stresses and temperature distribution functions. The variations of the considered variables against the vertical distance are illustrated graphically and a comparison is made for results predicted by both the theories in presence and absence of rotation effect we have also encountered the effect because of temperature involved in elastic parameters.

Keywords: Green-Naghdi theory, thermoelasticity, heat dependent elastic constants, microstretch, rotation, normal mode method

1 Introduction

The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent, linear elasticity describes the mechanical behavior of the other common solid materials, e.g. concrete, wood and coal. However, the theory does not apply to the behavior of many of the new synthetic materials of the clastomer and polymer type. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves the influence of the body microstructure becomes significant, this influence of microstructure results in the development of new types of waves is not in the classical theory of elasticity. Metals, polymers, composites, solids, rocks, and concrete are typical media with microstructures. More generally, most of the natural and man-made materials including engineering, geological and biological media possess a microstructure.

Eringen and Suhubi[1] and Eringen[2]- [4]developed the linear theory of micropolar elasticity and microstretch elastic solids. This theory of microstretch thermoelasticity is the generalization of the theory of micropolar elasticity and a special case of the micromorphic theory. The material points of microstretch elastic solids can stretch and contract independently of their translations and rotations. The basic results in the theory of micro stretch elastic solids were obtained in the literature[5]-[8].

The theory of thermo-micro-stretch elastic solids was introduced by Eringen[9] . In the framework of this theory Eringen established a uniqueness theorem for the mixed initial-boundary value problem. The theory was illustrated through the solution of one-dimensional waves and compared with lattice dynamical results. The asymptotic behavior of solutions and an existence result were presented by Bofill and Quintanilla[10] . A reciprocal theorem and a representation of Galerkin type were presented by De Cicco and Nappa[11] . De Cicco and Nappa[12] extended a linear theory of thermo-microstretch elastic solids that permits the transmission of heat as thermal waves at finite speed. The theory is based on the entropy production inequality proposed by Green and Laws[13] . The basic results and an extensive review of the theory of thermo-microstretch elastic solids can be found in the book of Eringen[5].

Most of the investigations were done under the assumption of temperature-independent material properties, which limit the applicability of the solutions obtained to certain ranges of temperature. Modern

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structural elements are often subjected to temperature change of such magnitude that their material properties may be longer be regarded as having constant values even in an approximate sense. At high temperature the material characteristics such as modulus of elasticity, thermal conductivity and the coefficient of linear thermal expansion are no longer constants[14] . The thermal and mechanical properties of the materials vary with temperature, so the temperature-dependent on the material properties must be taken into consideration in the thermal stress analysis of these elements. Othman and Kumar[15] investigated the dependence of modulus of elasticity on reference temperature in generalized magneto-thermo-elasticity and obtained interesting results. Youssef[16] used the equation of generalized thermoelasticity with one relaxation time with variable modulus of elasticity and the thermal conductivity to solve a problem of an infinite material with spherical cavity.

Green and Naghdi[17]-[19] proposed another three models, which are subsequently referred to as GN-I, II and III models. The linearized version of model-I corresponds to the classical thermoelastic model-II the internal rate of production of entropy is taken to be identically zero implying no dissipation of thermal energy. This model admits un-damped thermoelastic waves in a thermoelastic material and is best known as the theory of thermoelasticity without energy dissipation. Model-III includes the previous two models as special cases, and admits dissipation of energy in general. Othman [20] constructed a model of the two-dimensional equations of generalized thermoelasticity with two relaxation times in an isotropic elastic medium with the modulus of elasticity being dependent on the reference temperature. Lotfy and Othman [21] studied the effect of rotation on generalized thermo-microstretch elastic medium under different theories. Recently some authors discussed different types of problems in generalized thermo-microstretch elastic medium (Marin[22]-[23] Baljeet[24] , Kumar[25]-[26] Othman et al. [27]-[28] , Ellahi et al. [29]-[33]).

This paper presents an attempt to examine the temperature dependency of elastic modulus and rotation on the behavior of two-dimensional solution in a generalized thermo-microstretch elastic medium. We have also encountered the fascinating theories of generalized thermoelasticity presented by Green and Naghdi. We have adopted normal mode analysis method and find the expressions for field variables.

2 Formulation of the Problem

We obtain the constitutive and the field equations for a linear isotropic generalized thermo-microstretch elastic solid in the absence of body forces. We use a rectangular coordinate system(x,y,z) having originated on the surface $y=0$ and z-axis pointing vertically into the medium. The thermoelastic plate is rotating uniformly with an angular frequency $\Omega=\Omega\, n$, where n is a unit vector representing the direction of the axis of rotation. The basic governing equations of linear generalized thermoelasticity with rotation in the absence of body forces and heat sources are

$$\sigma_{ji,j}=\rho[\ddot{u}+\ \Omega\ \times(\Omega\times u)+2\Omega\ \times\dot{u}]_i, \tag{1}$$

$$\varepsilon_{ijr}\sigma_{jr}+m_{ji,j}=j\rho\frac{\partial^2\phi_i}{\partial t^2}, \tag{2}$$

$$\alpha_0\nabla^2\phi^*-\frac{1}{3}\lambda_1\phi^*-\frac{1}{3}\lambda_0(\nabla.u)+\frac{1}{3}\hat{\gamma}_1T=\frac{3}{2}\rho j\frac{\partial^2\phi^*}{\partial t^2}, \tag{3}$$

$$K^*\nabla^2T+K\nabla^2\dot{T}=\rho C_E\ddot{T}+\hat{\gamma}T_0\ddot{u}_{i,i}+\hat{\gamma}_1T_0\frac{\partial\phi^*}{\partial t}, \tag{4}$$

$$\sigma_{il}=(\lambda_0\phi^*+\lambda u_{r,r})\delta_{il}+(\mu+k)u_{l,i}+\mu u_{i,l}-k\varepsilon_{ilr}\phi_r-\hat{\gamma}T\delta_{il}, \tag{5}$$

$$m_{il}=\alpha\phi_{r,r}\delta_{il}+\beta\phi_{i,l}+\gamma\phi_{1,i}, \tag{6}$$

$$\lambda_{\mathrm{i}}=\alpha_0\phi^*_{,i}, \tag{7}$$

$$e=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z} \tag{8}$$

Where T is the temperature above the reference temperatureT_0 chosen so that $\left|(T-T_0)/T_0\right|<<1,\lambda,\ \mu$ are the counterparts of Lame parameters, the components of displacement vector u are u_i, t is the time, σ_{ij} are the components of stress tensor, e is the dilatation, e_{ij} are the components of strain tensor, j the micro inertia moment, ϕ^* is the scalar microstretch, ϕ is the rotation vector m_{ij} is the couple stress tensor, δ_{ij} is the Kronecker delta, ε_{ijr} is the alternate tensor, the mass density is ρ, the specific heat at constant strain is C_E,, the thermal conductivity is K^* and K material characteristic of the theory. The state of plane strain parallel to the xy-plane is defined by

$$u_1=u(x,z,t),u_2=0,u_3=w(x,z,t),\varphi_1=\varphi_3=0, \tag{9}$$

$$\phi_2=\phi_2(x,z,t),\phi^*=\phi^*(x,z,t),\ \ \Omega=(0,\Omega,0).$$

We assume that

$$\lambda=\lambda_2 f(T),\mu=\mu_0 f(T),\gamma=\gamma_0 f(T),\lambda_0=\lambda_0^* f(T),$$
$$k=k_0 f(T),\beta=\beta_0 f(T),\alpha=\alpha_1 f(T),\hat{\gamma}=\hat{\gamma}_0 f(T),$$
$$\alpha_0=\alpha_0^* f(T),\lambda_1=\lambda_3 f(T),\hat{\gamma}_1=\hat{\gamma}_1^* f(T).$$

Where $f(T)$ is a non-dimensional function depending on temperature, during the case of temperature independent modulus of elasticity, $f(T) = 1$ Equations (1)-(9) will become,

$$\rho[\ddot{u} + \Omega \times (\Omega \times u) + 2\Omega \times \dot{u}]_i = f(T)[\lambda_0^* \varphi_{,i}^* + (\lambda_2 + \mu_0) u_{j,ji} + (\mu_0 + k_0) u_{i,jj} + k_0 \varepsilon_{ijl} \varphi_{l,j} - \gamma_0 T_{,i}] + f_{,j}[\lambda_0^* \varphi^* \delta_{ij} + \lambda_2 u_{r,r} \delta_{ij} + \mu_0 u_{j,i} + (\mu_0 + k_0) u_{i,j} + k_0 \varepsilon_{ijl} \varphi_l - \gamma_0 T \delta_{ij}] \quad (10)$$

$$f(T)[\gamma_0\, \phi_{2,jj} - 2k_0 \phi_2 + k_0 (u_{1,3} - u_{3,1})] + \gamma_0 f_{,j} \phi_{2,j} = j\rho \ddot{\phi}_2, \quad (11)$$

$$f(T)(\alpha_0^* \phi_{,jj}^* - \frac{1}{3}\lambda_3 \phi^* - \frac{1}{3}\lambda_0^* u_{j,j} + \frac{1}{3}\gamma_1^* T) = \frac{3}{2}\rho j \ddot{\phi}^*, \quad (12)$$

$$K^* T_{,jj} + K \dot{T}_{,jj} = \rho C_E \ddot{T} + f(T)(\hat{\gamma}_0^* T_0 \ddot{e} + \hat{\gamma}_1^* T_0 \dot{\phi}^*). \quad (13)$$

Where

$$\hat{\gamma} = (3\lambda + 2\mu + k)\alpha_{t_1}, \quad \hat{\gamma}_1 = (3\lambda + 2\mu + k)\alpha_{t_2} \; and, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \quad (14)$$

The constants $\hat{\gamma}$ and $\hat{\gamma}_1$depend on the mechanical as well as the thermal properties of the body and the dot denote the partial derivative with respect to time, $\alpha_{t_1}, \alpha_{t_2}$are the coefficients of linear thermal expansions. For convenience, the following non-dimensional variables are used:

$$\bar{x}_i = \frac{\omega^*}{c_2} x_i,\ \bar{u}_i = \frac{\rho c_2 \omega^*}{\gamma_0 T_0} u_i, \quad \bar{t} = \omega^* t,\ \bar{T} = \frac{T}{T_0},$$
$$\bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\gamma_0 T_0},\ \bar{m}_{ij} = \frac{\omega^*}{c_2 \gamma_0 T_0} m_{ij},\ \bar{\phi}_2 = \frac{\rho c_2^2}{\gamma_0 T_0}\phi_2,$$
$$\bar{\lambda}_3 = \frac{\omega^*}{c_2 \gamma_0 T_0}\lambda_3, \quad \bar{\phi}^* = \frac{\rho c_2^2}{\gamma_0 T_0}\phi^*,\ \omega^* = \frac{\rho C_E c_2^2}{K^*},$$
$$\bar{\Omega} = \frac{\Omega}{\omega^*},\ c_2^2 = \frac{\mu_0}{\rho}. \quad (15)$$

Using (15), Eqs. (10)-(13) become (dropping the bar for convenience)

$$[\ddot{u} + \Omega \times (\Omega \times u) + 2\Omega \times \dot{u}]_i = \frac{f(T)}{c_2^2}[\lambda_0^* \varphi_i^* + \lambda_2 + \mu_0 u_{j,ji} + (\mu_0 + k_0) u_{i,jj}] + k_0 \varepsilon_{ijl} \varphi_{l,j} - \rho c_2^2 T_{,i}] + \frac{f_{,j}}{\rho c_2^2}[\lambda_0^* \varphi^* \delta_{ij} + \lambda_2 u_{r,r} \delta_{ij} + \mu_0 u_{j,i} + (\mu_0 + k_0) u_{i,j} + k_0 \varepsilon_{ijl} \varphi_l - \rho c_2^2 T \delta_{ij}] \quad (16)$$

$$\frac{f(T)}{j\rho c_2^2}[\gamma_0\, \phi_{2,jj} - \frac{2k_0 c_2^2}{\omega^{*2}}\phi_2 + \frac{k_0 c_2^2}{\omega^{*2}}(u_{1,3} - u_{3,1})] + \frac{\gamma_0}{j\rho c_2^2} f_{,j} \phi_{2,j} = \ddot{\phi}_2, \quad (17)$$

$$f(T)[\frac{\alpha_0}{c_2^2}\phi_{,jj}^* - \frac{1}{3}\frac{\lambda_3}{\omega^{*2}}\phi^* - \frac{1}{3}\frac{\lambda_0^*}{\omega^{*2}} u_{j,j} + \frac{1}{3}\frac{\gamma_1^* \rho c_2^2}{\gamma_0 \omega^{*2}} T] = \frac{3}{2}\rho j \ddot{\phi}^*, \quad (18)$$

$$\varepsilon_2 (\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}) + \varepsilon_3 (\frac{\partial^2 \dot{T}}{\partial x^2} + \frac{\partial^2 \dot{T}}{\partial z^2}) = \ddot{T} + f(T)(\varepsilon_1 \ddot{e} + \varepsilon_4 \frac{\partial \phi^*}{\partial t}). \quad (19)$$

Where

$$\varepsilon_1 = \frac{\gamma_0^2 T_0 (1 - \beta^* T_0)}{\rho^2 C_E c_2^2}, \quad \varepsilon_2 = \frac{K^*}{\rho C_E c_2^2}, \quad \varepsilon_3 = \frac{K\omega^*}{\rho C_E c_2^2}$$

and $\varepsilon_4 = \dfrac{\hat{\gamma}_0 \gamma_1^* T_0 (1 - \beta^* T_0)}{\rho^2 C_E \omega^* c_2^2}$.

We have taken the special case of $\left|(T - T_0)/T_0\right| << 1$i.e., infinitesimal temperature deviation from reference temperature are considered. Therefore $f(T)$can be taken in the form $f(T) = 1 - \beta^* T_0$, where$\beta^*$is an empirical material constant. Also introducing the dimension less scalar potential functions$\phi(x, z, t)$and $\psi(x, z, t)$defined by the relations,

$$u = \frac{\partial R}{\partial x} + \frac{\partial \psi}{\partial z}, \qquad w = \frac{\partial R}{\partial z} - \frac{\partial \psi}{\partial x} \quad (20)$$

Equations (16) in component form will become,

$$(\nabla^2 + a_0 \Omega^2 - a_0 \frac{\partial^2}{\partial t^2}) R + 2\Omega a_0 \frac{\partial \psi}{\partial t} + a_1 \varphi - a_0' T = 0 \quad (21)$$

$$(\nabla^2 + a_2 \Omega^2 - a_2 \frac{\partial^2}{\partial t^2}) \psi - 2\Omega a_2 \frac{\partial R}{\partial t} - a_3 \varphi_2 = 0 \quad (22)$$

$$(\nabla^2 - 2a_4 - a_5 \frac{\partial^2}{\partial t^2}) \phi_2 + a_4 \nabla^2 \psi = 0, \quad (23)$$

$$(a_6 \nabla^2 - a_7 - \frac{\partial^2}{\partial t^2}) \phi^* - a_8 \nabla^2 R + a_9 T = 0, \quad (24)$$

$$[(\varepsilon_2 + \varepsilon_3 \frac{\partial}{\partial t}) \nabla^2 - \frac{\partial^2}{\partial t^2}] T = \varepsilon_1 \ddot{e} + \varepsilon_4 \frac{\partial \phi^*}{\partial t}. \quad (25)$$

Where,

$$c_1^2=\frac{(\lambda_2+2\mu_0+k_0)}{\rho},\, c_3^2=\frac{2\alpha_0^*(1-\beta^*T_0)}{3\rho j},$$
$$c_4^2=\frac{2\lambda_3(1-\beta^*T_0)}{9\rho j},\ ; c_5^2=\frac{2\lambda_0^*(1-\beta^*T_0)}{9\rho j},$$
$$a_0=\frac{c_2^2}{c_1^2(1-\beta^*T_0)},\, a_0'=\frac{c_2^2}{c_1^2},\, a_1=\frac{\lambda_0^*}{\lambda_2+2\mu_0+k_0},$$
$$a_2=\frac{\rho c_2^2}{(\mu_0+k_0)(1-\beta^*T_0)},\ a_3=\frac{k_0}{(\mu_0+k_0)}\ a_4=\frac{k_0 c_2^2}{\gamma_0\,\omega^{*2}},$$
$$a_5=\frac{\rho jc_2^2}{\gamma_0(1-\beta^*T_0)},\quad a_6=\frac{c_3^2}{c_2^2},\quad a_7=\frac{c_4^2}{\omega^{*2}},\quad a_8=\frac{c_5^2}{\omega^{*2}}$$
$$\text{and}\quad a_9=\frac{2\hat{\gamma}_1 c_2^2(1-\beta^*T_0)}{9\hat{\gamma}_0\, j\omega^{*2}}.$$

3 The Solution of the Problem

The solution of the considering physical variables can be decomposed in terms of normal mode as given in the following form:

$$[R,\,\psi,\,\phi^*,\phi_2,\sigma_{il},m_{il},T,\lambda_z]\,(x,z,t) = \left[\bar{R},\,\bar{\psi},\,\bar{\phi}^*,\,\bar{\phi}_2,\bar{\sigma}_{il},\bar{m}_{il},\bar{T},\bar{\lambda}_z\right](z)\,\exp(\omega\, t+i\,bx) \quad (26)$$

Where $[\bar{R},\,\bar{\psi},\,\bar{\phi}^*,\,\bar{\varphi}_2,\bar{\sigma}_{il},\bar{m}_{il},\bar{T},\bar{\lambda}_z](z)$the amplitude of the functions, $\omega$is a complex and b is the wave number in the$x$-direction. Using (26), then (21)-(25) become

$$(D^2-A_1)\bar{R}-a_0'\,\bar{T}+a_1\,\bar{\phi}^*+A_2\bar{\psi}=0, \quad (27)$$

$$(D^2-A_3)\bar{\psi}-a_3\,\bar{\phi}_2-A_4\bar{R}=0, \quad (28)$$

$$(D^2-A_5)\,\bar{\phi}_2+a_4(D^2-b^2)\bar{\psi}=0, \quad (29)$$

$$(a_6D^2-A_6)\bar{\phi}^*-a_8(D^2-b^2)\bar{R}+a_9\bar{T}=0, \quad (30)$$

$$[\varepsilon(D^2-b^2)-\omega^2]\,\bar{T}-\varepsilon_1\omega^2(D^2-b^2)\bar{R}-\varepsilon_4\omega\,\bar{\phi}^*=0. \quad (31)$$

Where

$$D=\frac{d}{dz},\quad A_1=b^2+a_0(\omega^2-\Omega^2),\quad A_2=2\Omega\,a_0\,\omega,$$
$$A_3=b^2+a_2(\omega^2-\Omega^2),\qquad A_4=2\Omega\;a_2\,\omega,$$
$$A_5=b^2+2a_4+a_5\omega^2,\quad A_6=b^2a_6+a_7+\omega^2,\varepsilon=(\varepsilon_2{+}\varepsilon_3\omega).$$

Eliminating,$\bar{\phi}_2,\bar{\psi},\bar{R},\bar{T}and\bar{\phi}^*$in (27)-(31), we get the following tenth order ordinary differential equation

$$[D^{10}-AD^8+BD^6-CD^4+ED^2-F]\{\bar{\phi}_2,\bar{\psi},\bar{R},\bar{T},\bar{\phi}^*\}(z)=0 \quad (32)$$

Equation (32) can be factored as

$$(D^2-k_1^2)\;\;(D^2-k_2^2)(D^2-k_3^2)\,(D^2-k_4^2)\,(D^2-k_5^2)\,\{\bar{\phi}_2,\bar{\psi},\bar{R},\bar{T},\,\bar{\phi}^*\}(z)=0. \quad (33)$$

Where,

$$A=g_{18}/g_{17},\ B=g_{19}/g_{17},C=g_{20}/g_{17},E=g_{21}/g_{17},$$
$$F=g_{22}/g_{17},g_1=\varepsilon_4\omega-a_1\varepsilon_1\omega^2,\,g_2=-\varepsilon_4\omega A_1+a_1\varepsilon_1\omega^2b^2,$$
$$g_3=a_1(\varepsilon b^2+\omega^2)+a_0'\varepsilon_4\omega,$$
$$g_4=A_2\varepsilon_4\omega,\ \ g_5=A_3+A_5-a_3a_4,\ \ g_6=A_3A_5-a_3a_4b^2,$$
$$g_7=g_2-g_1g_5,g_8=-g_2g_5+g_1g_6+g_4A_4,g_9=g_2g_6-g_4A_4A_5,$$
$$g_{10}=-g_3-a_1\varepsilon g_5,\ \ g_{11}=g_3g_5+a_1\varepsilon g_6,g_{12}=g_3g_6,$$
$$g_{13}=a_6(\varepsilon b^2+\omega^2)+A_6\varepsilon,\ \ g_{14}=\varepsilon_4a_9\omega+A_6(\varepsilon b^2+\omega^2),$$
$$g_{15}=\varepsilon_1a_6\omega^2b^2+A_6\varepsilon_1\omega^2-a_8\varepsilon_4\omega,$$
$$g_{16}=\varepsilon_4\,a_8\,\omega\,b^2-\varepsilon_1\,\omega^2A_6b^2,\quad g_{17}=\varepsilon(a_6\,g_1+a_1\,\varepsilon_1\,\omega^2a_6),$$
$$g_{18}=-\,a_6(\varepsilon g_7+\varepsilon_1\,\omega^2g_{10})+g_{13}g_1+a_1\,\varepsilon\,g_{15},$$
$$g_{19}=a_6(\varepsilon g_8+\varepsilon_1\omega^2g_{11})-g_{13}g_7+g_{14}g_1-a_1\varepsilon g_{16}-g_{10}g_{15},$$
$$g_{20}=-a_6\varepsilon g_9+g_{13}g_8-g_{14}g_7+g_{10}g_{16}+g_{11}g_{15}+g_{12}a_6\varepsilon_1\omega^2,$$
$$g_{21}=-\,g_{13}g_9+g_{14}g_8-g_{11}g_{16}+g_{12}g_{15}.$$
$$g_{22}=-\,g_{14}g_9-g_{12}g_{16}.$$

The solution of (33), has the form

$$\bar{R}=\sum_{n=1}^{5}M_ne^{-\,k_nz}, \quad (34)$$

$$\bar{T}=\sum_{n=1}^{5}H_{1n}M_ne^{-\,k_nz}, \quad (35)$$

$$\bar{\psi}=\sum_{n=1}^{5}H_{2n}M_ne^{-\,k_nz}, \quad (36)$$

$$\bar{\phi}_2=\sum_{n=1}^{5}H_{3n}M_ne^{-\,k_nz}, \quad (37)$$

$$\bar{\phi}^*=\sum_{n=1}^{5}H_{4n}M_ne^{-\,k_nz}. \quad (38)$$

WhereM_n are some parameters and$\mathrm{k_n^2}$, $(\mathrm{n}=1,2,3,4,5)$ are the roots of the characteristic equation of (33). Here,

$$H_{1n}=[k_n^4\,a_6\varepsilon_1\omega^2-k_n^2g_{15}-g_{16}]/[k_n^4\,a_6\varepsilon-k_n^2g_{13}+g_{14}],$$

$$H_{2n}=[A_4(k_n^2-A_5)]/[k_n^4-k_n^2g_5+g_6],$$

$$H_{3n}=[-a_4A_4(k_n^2-b^2)]/[k_n^4-k_n^2g_5+g_6],$$

$$H_{4n}=[a_8(k_n^2-b^2)-a_9H_{1n}]/[a_6k_n^2-A_6].$$

4 The Boundary Conditions

The plane boundary subjects to an instantaneous normal point force and the boundary surface is isothermal, the boundary conditions at$z=0$ are

1.The mechanical boundary condition is that the surface of the half-space obeys,

$$\sigma_{xx} = -\,p(x,t)\,, \sigma_{zz} = \sigma_{xz} = \lambda_z = 0. \tag{39}$$

2.The thermal boundary condition is that the surface of the half-space is subjects to a thermal shock,

$$T = f(x,t). \tag{40}$$

We obtain the non-dimensional expressions for the displacement components, force stress, coupled stress and temperature distribution of the microstretch generalized thermoelastic medium as follows,

$$\bar{u} = \sum_{n=1}^{5} (ib - k_n H_{2n}) M_n e^{-\,k_n z} \tag{41}$$

$$\bar{w} = \sum_{n=1}^{5} (-k_n - ibH_{2n}) M_n e^{-\,k_n z} \tag{42}$$

$$\bar{\sigma}_{xx} = \sum_{n=1}^{5} H_{5n} M_n e^{-\,k_n z}, \tag{43}$$

$$\bar{\sigma}_{zz} = \sum_{n=1}^{5} H_{6n} M_n e^{-\,k_n z}, \tag{44}$$

$$\bar{\sigma}_{xz} = \sum_{n=1}^{5} H_{7n} M_n e^{-\,k_n z}, \tag{45}$$

$$\bar{\sigma}_{zx} = \sum_{n=1}^{5} H_{8n} M_n e^{-\,k_n z}, \tag{46}$$

$$\bar{\lambda}_z = -\sum_{n=1}^{5} a_{15} k_n H_{4n} M_n e^{-\,k_n z}. \tag{47}$$

where

$$a_{10} = \frac{\lambda_0^*}{\rho\, c_2^2}, \quad a_{11} = \frac{c_1^2}{c_2^2}, \quad a_{12} = \frac{\lambda_2}{\rho\, c_2^2}, \quad a_{13} = \frac{(\mu_0 + k_0)}{\rho\, c_2^2},$$
$$a_{14} = \frac{k_0}{\rho\, c_2^2}, \quad a_{15} = \frac{\alpha_0^* \omega^{*2}}{\rho\, c_2^4},$$

$$H_{5n} = (1 - \beta^* T_0)[a_{10} H_{4n} + i\,b\,a_{11}(ib - k_n H_{2n}) + k_n a_{12}(k_n + ibH_{2n}) - H_{1n}],$$

$$H_{6n} = (1 - \beta^* T_0)[a_{10} H_{4n} + k_n a_{11}(k_n + ibH_{2n}) + iba_{12}(ib - k_n H_{2n}) - H_{1n}],$$

$$H_{7n} = (1 - \beta^* T_0)[-k_n(ib - k_n H_{2n}) - a_{13} ib(k_n + ibH_{2n}) + a_{14} H_{3n}],$$

$$H_{8n} = (1 - \beta^* T_0)[-ib\,(k_n + ib\,H_{2n}) - k_n a_{13}(ib - k_n H_{2n}) + a_{14} H_{3n}].$$

Applying the boundary conditions (39) and (40) at the surface$z = 0$ of the plate, we obtain a system of five equations. After applying the inverse of matrix method,

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{pmatrix} = \begin{pmatrix} H_{52} & H_{52} & H_{53} & H_{54} & H_{55} \\ H_{61} & H_{62} & H_{63} & H_{64} & H_{65} \\ H_{71} & H_{72} & H_{73} & H_{74} & H_{75} \\ k_1 H_{41} & k_2 H_{42} & k_3 H_{43} & k_4 H_{44} & k_5 H_{45} \\ H_{11} & H_{12} & H_{13} & H_{14} & H_{15} \end{pmatrix}^{-1} \begin{pmatrix} -\bar{p} \\ 0 \\ 0 \\ 0 \\ \bar{f} \end{pmatrix} \tag{48}$$

We obtain the values of the five constants$M_n, n = 1,2,3,4,5$. Hence, we obtain the expressions for the displacements, the force stress, the coupled stress and the temperature distribution of the microstretch generalized thermoelastic medium.

5 Particular Cases

Case 1: The corresponding equations for the generalized micropolar thermoelasticity elastic medium without stretch can be obtained from the above mentioned cases by taking:

$$\alpha_0 = \lambda_0 = \lambda_1 = \varphi^* = 0 \tag{49}$$

After substituting (49) in (1)-(7) and using (22), (28) and (34) we get

$$(D^2 - A_1)\bar{R} + A_2 \bar{\psi} - a_0' \bar{T} = 0, \tag{50}$$

$$(D^2 - A_3)\bar{\psi} - A_4 \bar{R} - a_3 \bar{\phi}_2 = 0, \tag{51}$$

$$[D^2 - A_5]\,\bar{\phi}_2 + a_4\,(D^2 - b^2)\bar{\psi} = 0, \tag{52}$$

$$[\varepsilon(D^2 - b^2) - \omega^2]\,\bar{T} \, = \varepsilon_1\, \omega^2 (D^2 - b^2)\bar{R}. \tag{53}$$

Eliminating$\bar{\phi}_2, \bar{\psi}, \bar{R}and\bar{T}$ in (50)-(53), we get the following eight order ordinary differential equations for$\bar{\phi}_2, \bar{\psi}, \bar{R}$ and $\bar{T}$.

$$[D^8 - AD^6 + BD^4 - CD^2 + E]\,\{\bar{\phi}_2, \bar{\psi}, \bar{R}, \bar{T}\}(z) \; = 0 \tag{54}$$

Equation (54) can be factored as

$$(\,D^2 - k_1^2)\,(D^2 - k_2^2)(\,D^2 - k_3^2)(\,D^2 - k_4^2)\,\{\bar{\phi}_2, \bar{\psi}, \bar{R}, \bar{T}\}(z) = 0. \tag{55}$$

Where

$$A = g_6'/\varepsilon,\;\; B = g_7'/\varepsilon,\;\; C = g_8'/\varepsilon,\;\; E = g_9'/\varepsilon,$$
$$g_1' = a_0' \varepsilon_1 \omega^2 + \varepsilon b^2 + \omega^2 + \varepsilon A_1, g_2' = a_0' \varepsilon_1 \omega^2 b^2 + \varepsilon b^2 A_1 + \omega^2 A_1, g_3' = \varepsilon b^2 + \omega^2, g_4' = A_3 + A_5 - a_3 a_4,$$
$$g_5' = A_3 A_5 - a_3 a_4 b^2,\;\; g_6' = \varepsilon g_4' + g_1',$$
$$g_7' = \varepsilon g_5' + g_1' g_4' + g_2' + A_2 A_4 \varepsilon,$$
$$g_8' = g_1' g_5' + g_2' g_4' + A_2 A_4 (g_3' + \varepsilon A_5),$$
$$g_9' = g_2' g_5' + A_2 A_4 A_5 g_3'$$

The solution of (56) has the form

$$\bar{R}=\sum_{n=1}^{4} Z_n e^{-k_n z}, \tag{56}$$

$$\bar{T}=\sum_{n=1}^{4} H'_{1n} Z_n e^{-k_n z}, \tag{57}$$

$$\bar{\psi}=\sum_{n=1}^{4} H'_{2n} Z_n e^{-k_n z}, \tag{58}$$

$$\bar{\phi}_2=\sum_{n=1}^{4} H'_{3n} Z_n e^{-k_n z}. \tag{59}$$

Wherez_n are some parameters and k_n^2, $(n=1,2,3,4)$ are the roots of the characteristic equation of (55) and

$$H'_{1n}=[\varepsilon_1\omega^2(k_n^2-b^2)]/[\varepsilon(k_n^2-b^2)-\omega^2],$$

$$H'_{2n}=[A_4(k_n^2-A_5)]/[k_n^4-k_n^2 g'_4+g'_5],$$

$$H'_{3n}=[-a_4 A_4(k_n^2-b^2)]/[k_n^4-k_n^2 g'_4+g'_5].$$

This gives the required expressions for the displacement components, force stress, coupled stress and temperature distribution of the medium as follows

$$\bar{u}=\sum_{n=1}^{4}(ib-k_n H'_{2n})Z_n e^{-k_n z}, \tag{60}$$

$$\bar{w}=\sum_{n=1}^{4}(-k_n-ibH'_{2n})Z_n e^{-k_n z}, \tag{61}$$

$$\bar{\sigma}_{xx}=\sum_{n=1}^{4} H'_{4n} Z_n e^{-k_n z}, \tag{62}$$

$$\bar{\sigma}_{zz}=\sum_{n=1}^{4} H'_{5n} Z_n e^{-k_n z}, \tag{63}$$

$$\bar{\sigma}_{xz}=\sum_{n=1}^{4} H'_{6n} Z_n e^{-k_n z}, \tag{64}$$

$$\bar{\sigma}_{zx}=\sum_{n=1}^{4} H'_{7n} Z_n e^{-k_n z}. \tag{65}$$

Where

$$H'_{4n}=iba_{11}\left(ib-k_n H'_{2n}\right)+k_n a_{12}\left(k_n+ibH'_{2n}\right)-H'_{1n},$$

$$H'_{5n}=k_n a_{11}\left(k_n+ibH'_{2n}\right)+iba_{12}\left(ib-k_n H'_{2n}\right)-H'_{1n},$$

$$H'_{6n}=-k_n(ib-k_n H'_{2n})-a_{13}ib\left(k_n+ibH'_{2n}\right)+a_{14}H'_{3n},$$

$$H'_{7n}=-ib\left(k_n+ibH'_{2n}\right)-k_n a_{13}\left(ib-k_n H'_{2n}\right)+a_{14}H'_{3n}.$$

Applying the boundary conditions $\sigma_{xx}=-p(x,t)$, $\sigma_{zz}=\sigma_{xz}=0$, $T=f(x,t)$ at the surface$z=0$ of the plate, we obtain a system of four equations. After applying the inverse of matrix method,

$$\begin{pmatrix} Z_1\\ Z_2\\ Z_3\\ Z_4\end{pmatrix}=\begin{pmatrix} H'_{41} & H'_{42} & H'_{43} & H'_{44}\\ H'_{51} & H'_{52} & H'_{53} & H'_{54}\\ H'_{61} & H'_{62} & H'_{63} & H'_{64}\\ H'_{11} & H'_{12} & H'_{13} & H'_{14}\end{pmatrix}^{-1}\begin{pmatrix}-\bar{p}\\ 0\\ 0\\ \bar{f}\end{pmatrix} \tag{66}$$

We obtain the values of the four constants Z_n, $n=1,2,3,4$.

Case 2: The corresponding equations for the displacement, stresses and temperature distribution functions are obtained for generalized thermoelastic medium with stretch can be derived by using $k=\alpha=\beta=\gamma=0$ in (34)-(38).

Case 3: The equations for displacement, stresses and temperature distribution function for generalized thermo microstretch elastic medium without rotating medium will be obtained by assume$\Omega\rightarrow 0$.

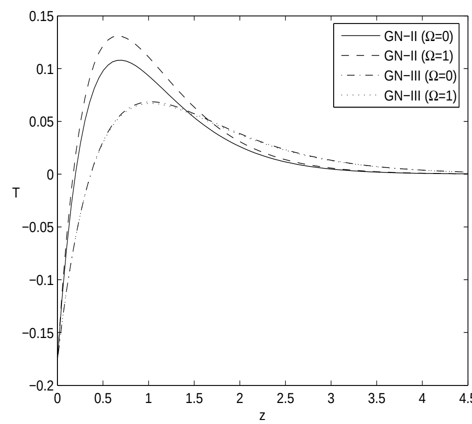

Fig. 1: Temperature distribution against z for rotational frequency

6 Numerical Results and Discussions

In order to illustrate our theoretical results obtained in the preceding section and to compare theories of thermoelasticity, we now present some numerical results. In the calculation, we take a magnesium crystal, the micropolar parameters are taken as Othman and Lotfy[28] , thermal characteristic as Prafitt and Eringen [34]and stretch parameters as Lotfy and Othman . As the material subjected to mechanical and thermal disturbances,

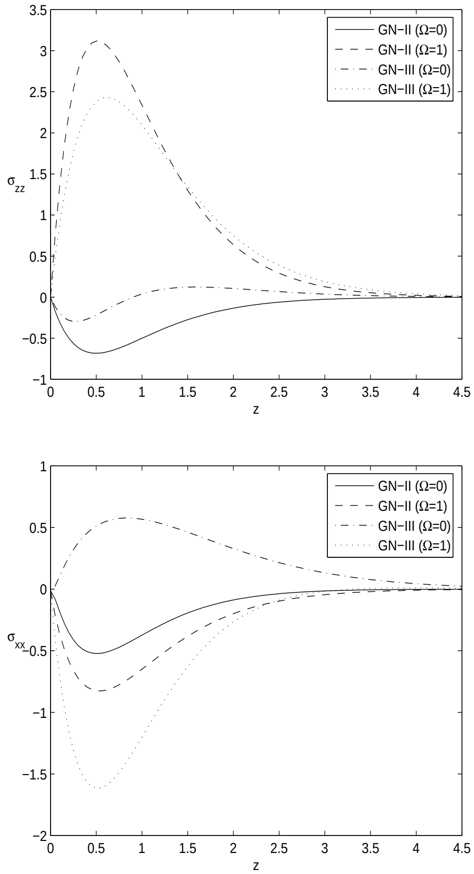

Fig. 2: Normal Stress distribution against z for rotational frequency

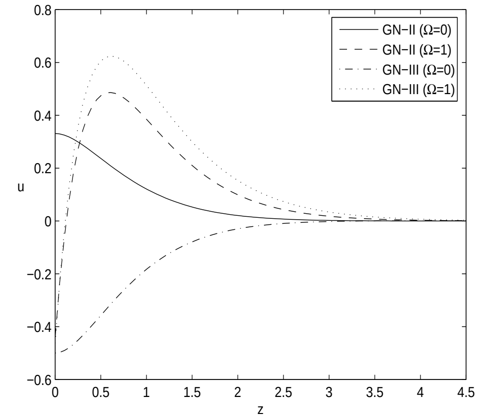

Fig. 3: Displacement distribution against z for rotational frequency

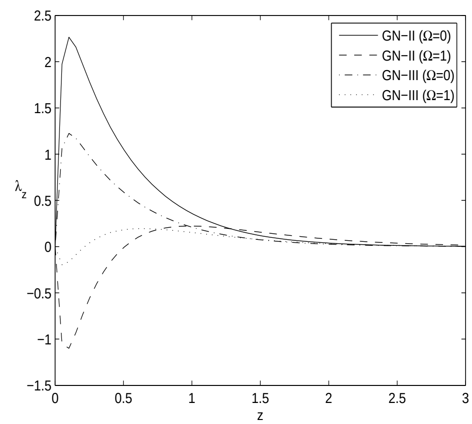

Fig. 4: Microstress distribution against z for rotational frequency

since$\omega$is a complex constant,we take $\omega = \omega_0 + \mathrm{i}\,\zeta$with $\omega_0 = -1$and $\zeta = 1$.The physical constants used are

$$\rho = 1.74 \times 10^3\ kmm^{-3},\ j = 0.2 \times 10^{-19}\ m^2,$$
$$\lambda_2^* = 9.4 \times 10^{10}\ Nm^{-2},\ T_0 = 298K,\ t = 0.1s,$$
$$\mu_0 = 4.0 \times 10^{10}\ Nm^{-2}, k_0 = 1 \times 10^{10}\ Nm^{-2},$$
$$\gamma_0 = 0.779 \times 10^{-9}\ N, \alpha_{t_1} = 0.05 \times 10^{-3} K^{-1}, \overline{f} = 0.5,$$
$$\alpha_{t_2} = 0.04 \times 10^{-3} K^{-1},\ K = K^* = 1.7 \times 10^2\, Jm^{-1}s^{-1}K^{-1},$$
$$\lambda_0 = 2.1 \times 10^{10}\ Nm^{-2}, C_E = 1.04 \times 10^3 J\,kg^{-1}K^{-1}$$
$$\lambda_3^* = 0.7 \times 10^{10}\ Nm^{-2},\ \alpha_0 = 0.779 \times 10^{-9}\ N,$$
$$\varepsilon_1 = 1.8,\quad \varepsilon_2 = 1.7,\ \varepsilon_3 = 1, \overline{p} = 0.2.$$

The change in amplitudes of field variables against a vertical component of distance in the context of Green and Naghdi theory of both types II and III for generalized thermo-micro-stretch medium is represented graphically. Figs. 1-4 show the variation in field variables by the angular frequency in these figures we have adjusted reference temperature by fixingβ^* at 0.0126, while Figs. 5-8 represent the variation in field variables for different value of two temperature parameter. In these figures solid lines and dashed lines for GN-II, dashed with dot and dotted lines are for GN-III at $\Omega = 0$ and $\Omega = 1$respectively, it is more precisely explained in each figure. Fig. 1 explains the amplitude of temperature distribution against a vertical component of distance. In this figure we have considered the effect of rotational frequency on T.It is observed that rotational frequency increases the amplitude of$T$for GN-II, but for GN-III the effect of rotation is negligible in the present selected parameters. Both the curves converge to zero for sufficient large values of vertical distance. Fig. 2 depicts the stress distribution functions against vertical distance components. In both the components of stress $\sigma_{xx}$and σ_{zz} rotation is having an increasing effect on both the theories of GN. Maximum amplitude for$\sigma_{xx}$is obtained for with energy dissipation and in the presence of rotational frequency but for σ_{zz} maximum amplitude of the stress

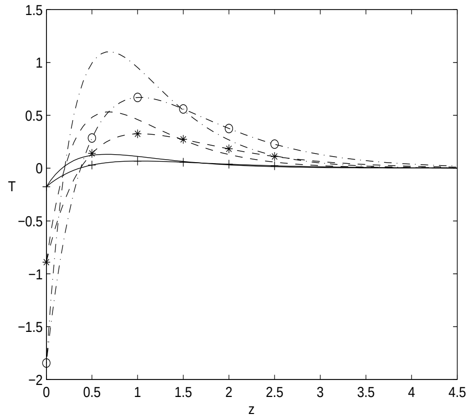

Fig. 5: Temperature distribution against z for different values of $\beta^* = 0.0126, .05, 0.1$

distribution function is obtained from without energy dissipation. Fig. 3 shows the distribution of the horizontal component of displacement becomes oscillating in the presence of rotational frequency. It can also be seen that in the presence of rotational frequency curves of displacement distribution function for GN-III are higher than the curves for GN-II. All curves converge to zero but the curves without rotation converge to zero faster. In the graphs of microstretch distribution function rotational frequency reduces the micro-stress component for GN-II but in the presence of energy dissipation rotation have increasing effect. Fig. 4 depicts the distribution of micro-stress, it can be seen that in the presence of rotational frequency, curves of micro-stress distribution function for GN-III are higher than the curves for the GN-II. All curves converge to zero but the curves without rotation converge to zero faster. Figs. 5-8 are representing the behavior of field variables for different value of empirical constant. In these sets of figures we have fixed the medium on rotation with angular frequency $\Omega = 1$. The figures contain the curves with following presentations

—- is for GN-II$\beta^* = 0.0126$; ....is for GN-II$\beta^* = 0.05$; -.-.-.-. is for GN-II$\beta^* = 0.1$; + + + is for GN-III$\beta^* = 0.0126$; * * *is for GN-III $\beta^* = 0.05$ o o o is for GN-III $\beta^* = 0.1$

Fig. 5 depicts the variations in temperature distribution function for different values of empiric material constant. For GN-II and GN-III amplitude of T is directly proportional to that of β^*i.e., by increasing the intensity of β^* amplitude of the temperature distribution function also increases. It can also be seen that starting point of temperature T for each value of β^* is same for both theories of the GN with curves of GN-II while, having great amplitude as compared to those of GN-III. It indicates that energy dissipation is having decreasing effect on temperature distribution. All curves converge to

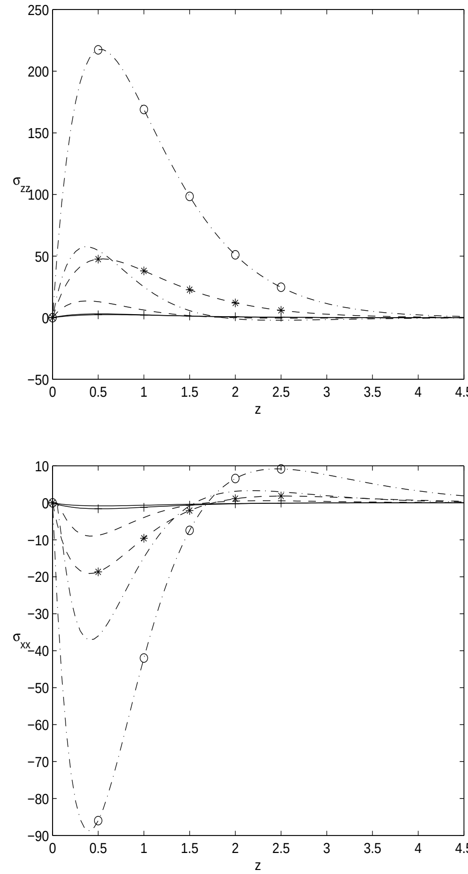

Fig. 6: Stresses distribution functions against z for different values of$\beta^* = 0.0126, .05, 0.1$

zero as the vertical distance from the surface increases satisfying the condition of surface waves. The variation in field variables $\sigma_{xx}, \sigma_{xx}, u, w$ and λ_z is represented in Fig. 6,7 and 8.It can be seen clearly from these figures that curve for each variable increases while increasing the value of intrinsic material constantβ^*For both components of the normal stresses and displacement distribution function maximum amplitude of stress distribution functions for are obtained under GN-III for $\beta^* = 0$.but for micro-stress distribution function maximum amplitude is obtained under GN-II for $\beta^* = 0.1$

7 Conclusion

In this article the effect of rotational frequency on plane waves in a generalized thermo-microstretch elastic media is studied in addition we have also encountered the influence of reference temperature on field variables. By analysing the graphical behaviour we have concluded the following important pionts

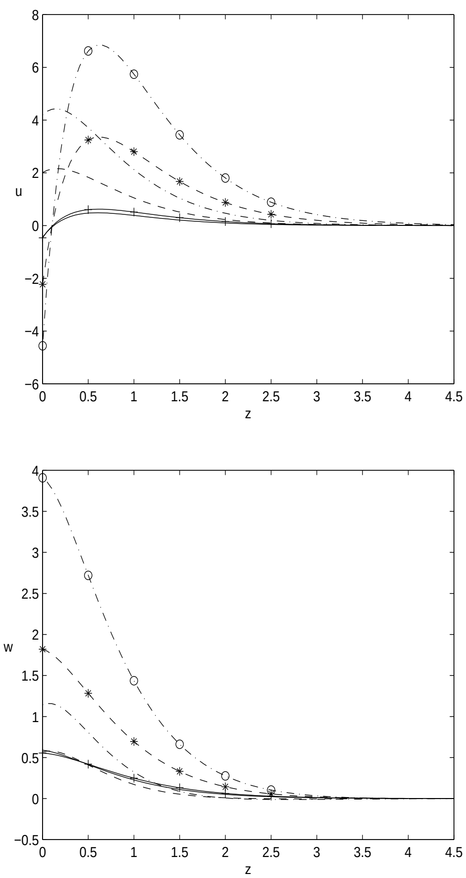

Fig. 7: Displacement distribution function against z for different values of$\beta^* = 0.0126, .05, 0.1$

1- In curves of temperature distribution function it is being observed that the amplitude for obtained by GN-II is less than the amplitude obtained from the GN- III in the absence of rotational frequency, while in the presence of rotational effect behavior of temperature distribution changes. Rotation of medium plays a significant role in the propagation of the wave through the medium.
2- Rotation is having an increasing effect on the distribution function of each field Variable accept micro-stress distribution function in which it has small decreasing effect.
3- By increasing the intensity ofβ^*effect of energy dissipation also increases on field variables. Hence we can say that reference temperature has very significant impact on wave propagation through the medium.
4. Greater the value ofβ^* greater the amplitude of normal stress distribution functions and micro-stress distribution function against z. It also increases the exponential behavior of stress propagating through the medium.
5- All the curves converges to zero as distance from surface of medium increases, this satisfies the condition for surface wave propagation.

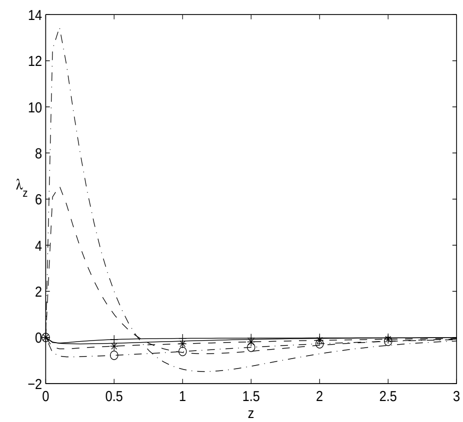

Fig. 8: Microstress distribution against z for different values of$\beta^* = 0.0126, .05, 0.1$

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